STOCHASTIC STABILITY BASED ON DIFFUSSION APROXIMATION

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Let $\xi(t)$ be the homogeneous Markov process in the metric phase space $\mathbb{U}$ with C-infinitesimal operator Q which is given by formula

$$(Qv)(z) = a(z) \int_{\mathbb{U}} [v(u) - v(z)] p(z, du) \tag{1}$$

for a measurable function on $\mathbb{U}$. It is piecewise constant process with intensity of switchings $a(z)$.$(0 < c < a(z) < \infty)$.and embedded Markov chain with transition probability $p(z, du)$ with Feller property. Let us suppose that $\xi(t)$ is stochastically continuous process with transition probability $P(t, z, du)$ and unique invariant measure μ, satisfying the condition of uniform ergodicity in the form

$$\sup_{\substack{A \in \mathfrak{B} \\ z \in \mathbb{U}}} |P(t, z, A) - \mu(A)| \leq e^{(-bt)}, \tag{2}$$

for any $t \geq 0$ with some $b > 0$, where $\mathfrak{B}$ is Borel σ-algebra of subsets of $\mathbb{U}$. Let us suppose that above mentioned Markov process $\xi(t)$ is switching process for dynamical system given by differential equation in $\mathbb{R}^d$:

$$\frac{dy^\varepsilon}{dt} = \frac{1}{\varepsilon} \varphi_1(y^\varepsilon(t), \xi(t/\varepsilon^2)) + \varphi_2(y^\varepsilon(t), \xi(t/\varepsilon^2)). \tag{3}$$

where $\varphi_1(y, z)$ and $\varphi_2(y, z)$ are continuous bounded functions with two continuous bounded y-derivatives. In discrete time moments $t \in \{\tau_{j-1}, j \in \mathbb{N}\}$ of switchings of the Markov process $\xi(t)$ the process $y^\varepsilon(t)$ also has switchings

$$y^\varepsilon(t) = y^\varepsilon(t_-) + \varepsilon g_1(y^\varepsilon(t_-), \xi(t_-/\varepsilon^2)) + \varepsilon^2 g_2(y^\varepsilon(t_-), \xi(t_-/\varepsilon^2)). \tag{4}$$

The process $x^\varepsilon(t)$ we will deal with satisfies the linear differential equation in $\mathbb{R}^n$

$$\frac{dx^\varepsilon}{dt} = B(y^\varepsilon(t), \xi(t/\varepsilon^2))x^\varepsilon, \qquad (5)$$

for $t \notin \{\tau_{j-1}, j \in \mathbb{N}\}$ and has the jumps

$$x^\varepsilon(t) = x^\varepsilon(t_-) + \varepsilon G_1(y^\varepsilon(t_-), \xi(t_-/\varepsilon^2))x^\varepsilon(t_-)+$$
$$+\varepsilon^2 G_2(y^\varepsilon(t_-), \xi(t_-/\varepsilon^2))x^\varepsilon(t_-), \qquad (6)$$

for $t \in \{\tau_{j-1}, j \in \mathbb{N}\}$., where $B(y, \xi)$, $G_1(y, \xi)$, $G_2(y, \xi)$ are cointinuous and bounded together with their two y-derivatives matrices and ε is small positive parameter.

Under the above conditions the triple $\{x^\varepsilon(t), y^\varepsilon(t), \xi(t/\varepsilon^2)\}$ is Markov process on phase space $\mathbb{R}^n \times \mathbb{R}^d \times \mathbb{U}$.

If $\varepsilon \to 0$ the family of processes $y^\varepsilon(\varepsilon t)$ converges to the solution of averaged equation

$$\frac{dy}{dt} = \bar{F}(y) := \int_{\mathbb{U}} (\varphi_1(y, u) + a(z)g_1(y, u))\mu(du). \qquad (8)$$

It is well known [1,2] that under above conditions and the condition $\bar{F}(y) \equiv 0$ the solutions of (3)–(4)–(5)–(6) weakly converge with $\epsilon \to 0$ to the corresponding solutions of equation

$$\frac{d\bar{x}}{dt} = \bar{B}(y(t))\bar{x}, \qquad (9)$$

and equation of diffusion approximation

$$dy = b(y)\,dt + \sigma(y)\,dw(t) \qquad (10)$$

on any finite interval $t \in [0, \frac{T}{\varepsilon}]$, where $w(t)$ is a standard Wiener process in $\mathbb{R}^m$.

Theorem 1. *If the equation (9) with $y(t)$ satisfying (10) is asymptotically stochastically stable, then the equation (5) conserning $y(t)$ from (8) is also exponentially p-stable for any $\varepsilon \in (0, \varepsilon_0)$ and some $\varepsilon_0 > 0$.*

E. Liepa. Uz difūziju aproksimācijas bāzēta stohastiskā stabilitāte. Stohastiskas impulsu diferenciālvienādojumu sistēmas atrisinājuma eksponenciālā p-stabilitāte izriet no sistēmas ar vidējotiem raksturlielumiem asimptotiskās stohastiskās stabilitātes.

References.

1. G. Blankenship and G. C. Papanicolaou, *Stability and control of stochastic systems with wide-band noise disturbances.I*, SIAM J. Appl. Math. **34** (1978), no. 3, 437–476.
2. A. V. Skorokhod, *Asymptotic Methods of Theory of Stochastic Differential Equations*, AMS, Providance, 1994.