

# INDUCTIVE AUTOMATON: OPTIMAL SPIKE FREQUENCY

## INDUKTĪVS AUTOMĀTS: OPTIMĀLA IMPULSU FREKVENCE

A.Pchelkin

*Keywords: neural networks, self-organization, inductive automaton, optimal spike frequency*

### 1. Introduction

In this paper we continue our investigation [1] that relates to a new artificial neural network [2] proposed by Emelyanov-Yaroslavsky in 1990 [3]. We will refer to this neural network as “inductive automaton”. The main idea was not to implement a specific functionality for each class of tasks but to define one basic task – energy consumption minimization and then to obtain all other functionalities as by-products. Our aim is to develop a more complete understanding of complex dynamic biological processes described in the monograph about inductive automaton.

Our opinion is that the explanation of the complex processes, presented by Emelyanov-Yaroslavsky, is not complete. During his investigation the author is trying to understand biological logic of natural neural networks, he investigates different mechanisms observed in natural networks but he is not trying to answer a question: why all these complicated mechanisms are needed? Therefore, there is a need for additional investigation.

Inductive automaton has optimal spike frequency that determines learning rule – associative excitatory links correction rule. According to this learning rule, each neuron in active state is modifying its synapses weights in order to generate spikes at the optimal frequency. Emelyanov-Yaroslavsky introduces this spike frequency as the most profitable for neuron itself in the sense of energy consumption minimization. However, we have found that the optimal spike frequency is strongly connected with neuron assembly functioning consistency. Therefore, we are going to present another explanation of the optimal spike frequency.

In the previous paper [1] we have presented our understanding of emotional center function, whereas the current paper *presents our understanding of optimal spike frequency*. In the next paragraph we will give a short review of biological concept of inductive automaton [3], and then we will describe our understanding, called informational concept of inductive automaton [1].

### 2. Biological concept of inductive automaton

The stability of dead matter is determined by the strength of internal bonds, while the stability of a live matter is provided by repair activities. Any live unit permanently repairs itself, expending matter and energy. Following this ideas the spike in a neuron is understood as an intensive repair state in biological concept of inductive automaton.

The spike frequency of the neuron depends on potential  $Y$  and dynamic threshold  $S(t)$ , where  $t$  – time passed from the last spike instant (see Fig. 1). The spike in the neuron is a probabilistic event that can take place at the fulfillment of condition  $Y > S(t)$ . Dependence  $S(t)$  is well known in neurophysiology. After the spike occurrence the neuron is not able to generate a new spike for a very short period of time – it is the phase of the absolute refractoriness. The next phase is the relative refractoriness – the neuron has low ability to generate a new spike and it can only take place if the neuron potential has a great value. The exaltation phase comes next and the neuron is able to generate a new spike even at negative value of its potential.

The main output parameter of a neuron is spike frequency, therefore the transfer function of the neuron is gradual, but unlike popular sigmoid it has not two, but three stable states. A neuron

can be inactive (no spikes), half active (generating at low frequency – spikes occurs in the exaltation phase) and fully active (generating at high frequency – spikes occurs in the relative refractoriness phase). After full activation of a neuron occurs self-locking and forced deactivation of the neuron because full activation is only a temporal state. However, a neuron can maintain half active state for a longer period of time.

The neuron has two main properties: (1) growing old and (2) an ability to renew itself generating spikes. The neuron can be inactive or active - generating spikes. The neuron is characterized by its conventional age, which grows in the inactive state and decreases in the active state. The age defines energy consumption that is required by the neuron per a time unit. Greater age implies greater energy consumption. External energetic feed for the neuron is limited, and therefore neurons are forced to solve the consumption minimization problem by generating spikes.

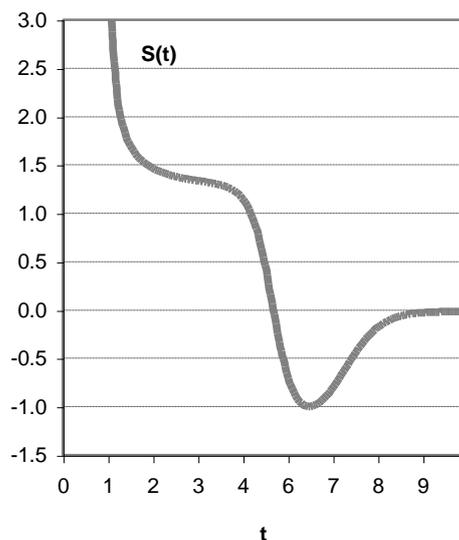


Fig.1. The dynamic threshold  $S(t)$ .

In the active state the energy consumption increases, but due to the decrease of the age, the total energetic effect of spikes could be positive in long-terms: at a prolonged time interval an integral energy consumption could decrease. Since every spike decreases neuron's age, the variation of frequency will vary an energetic cost for decrease of age.

Inductive automaton has optimal spike frequency that is in the interval of high frequencies. This frequency is the most profitable for neuron itself, because it allows energy consumption minimization. This frequency determines a learning rule – associative excitatory links correction rule. According to this learning rule, each neuron in active state is modifying its synapses weights in order to generate spikes at optimal frequency. If the current spike frequency is lower than optimal one, then the strength of excitatory input links must increase, but in the opposite case – decrease. We will refer to this rule as “optimal learning rule”.

### 3. A neuron only as a part of whole unit

Since this paragraph we are going to present informational concept of inductive automaton. It is called so because it could be considered as a complementary to purely biological concept presented by Emelyanov-Yaroslavsky. The central attention will be paid to the role of optimal spike frequency. In biological concept of inductive automaton optimal spike frequency is related

only to the most efficient energy consumption minimization. However, we have found that the optimal spike frequency is strongly connected with neuron assembly functioning consistency.

To provide a needed function, it doesn't matter which neuron in the neural network performs this function, or performs it alone or through collaboration with other neurons, - it only does matter that the needed function is provided. Therefore, a neural assembly, *i.e.* a group of "small" neurons strongly connected only by excitatory links, could be considered as one "big neuron" (see Fig. 2). Usually in artificial neural networks an artificial neuron is considered as such "big neuron" in order to provide a more stable and simple learning procedure, *e.g.* in Kohonen neural network each cluster unit models a neural group, not a single neuron [4]. Emelyanov-Yaroslavsky has also proposed to consider only an assembly as an elementary unit. In contrast, we will consider the neuron only as a part of a computational unit. It means that the properties of a neuron could be studied and understood only through dynamic interactions with another neurons of the same unit. Indeed, there are two possible ways how to think about a neuron: is it a whole consistent independent unit, or a part of it only. If we say that a neuron is only a part of unit, we could only analyze in the context of interaction with another neurons.

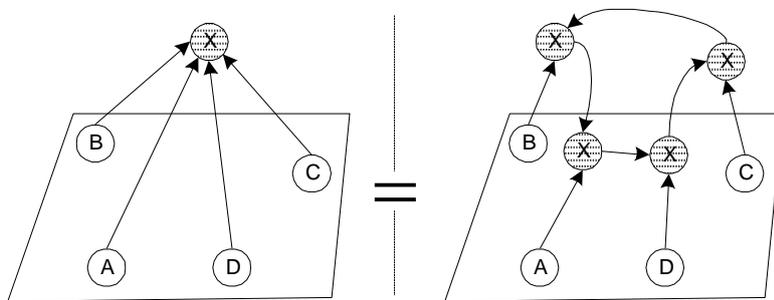


Fig. 2. An ensemble of "small" neurons can do the same as one "big" neuron.

It is generally known that component-based approach, or bottom-up approach, is more efficient in case of building of complex systems. It means that in the beginning the blocks of elementary units are created and only then these blocks are joined together. This method is based on assumption that each complex system has hierarchical structure. In the case of neural network the situation is the same. Representation of "big neuron" as a group of elementary units – "small neurons" could radically improve the speed of self-organizing, because in the beginning the network creates only part, and only then it joins them together. Here, it is possible to use a heuristic that the expedience of a part may approximate the expedience of the whole unit, *e.g.* if a "big neuron" has to recognize the simultaneous activation of neurons  $\{A,B,C,D,E\}$ , then a neuron, that recognizes the simultaneous activation of neurons  $\{A,D,E\}$ , could be considered as an approximation of the "big neuron".

Thus, a "small neuron" could be considered as a detail of the corresponding "big neuron". It allows reusing of the same detail in the future, *e.g.* in case of splitting of a formed neural assembly by inhibitory links, its neurons might be reused to construct new assemblies.

Let us analyze a neuron as a part of whole. What does it mean? It means that the neural assembly must be a group with stable composition: all the neurons in the group are simultaneously fully active or inactive, and their spike frequencies must be synchronized. In the ideal case neurons in an assembly should start spike generating simultaneously, keeping equal frequency for all neurons till self-locking after full activation. Therefore, *full activation must occur only in case of full synchronization of all neurons in a group – even the deactivation of one single neuron in the group must prevent from full activation of the whole group.* Any kind of desynchronization must turn the neurons of an assembly into half active state that could be

understood as desynchronization indicator. We will refer to this property as *consistency* of a neural assembly.

#### 4. A model of neuron activation

We have found that correct optimal frequency setting could provide consistency of a neural assembly. To present the result of our investigation, we are going to analyze the form of dynamic threshold  $S(t)$  dependency on time since the last spike (see Fig. 1). Unfortunately, Emelyanov-Yaroslavsky has presented this dependency only on a picture without defining a precise formula. That's why, we had a need to define this function by ourselves as a linear composition of more simple functions:

$$S(t) = k_{s1} \cdot \frac{1}{t - T_{s1}} + k_{s2} \cdot g\left(\frac{t - T_{s2}}{T_{ds2}}\right) + k_{s3} \cdot g\left(-\frac{t - T_{s3}}{T_{ds3}}\right) + S_c, \quad g(x) = \frac{1}{1 + e^x}.$$

We have selected the coefficient values (see tab. 1;  $S_c = -1.3$ ) of the dependency  $S(t)$  in order to make it similar to the dependency proposed by Emelyanov-Yaroslavsky.

Tab. 1. The selected coefficient values of function  $S(t)$ .

	$k_s$	$T_s$	$T_{ds}$
$S_1$	0.32	1	-
$S_2$	2.5	5.2	0.27
$S_3$	1.25	7	0.45

Let's define a density  $V$  of a neural group as a total strength of all links between all neurons in this group divided by the number of the neurons, and let's analyze the impact of this density on the activation of the corresponding neural group. For this purpose, a simplified learning algorithm of inductive automaton [3] will be used.

The spike in the neuron is a probabilistic event that can take place at the fulfillment of condition  $Y > S(t)$ . Potential  $Y$  can be expressed as  ${}^+Y - P$ , where  ${}^+Y$  is a positive potential, and  $P$  – the sum of all other impacts on a neuron, e.g. a negative potential generated by incoming inhibitory links, different kinds of thresholds that control the neuron. When a spike occurs, a neuron sends impulses to another neurons, and their positive potentials increase: the positive potential of each neuron increases by the strength value of the corresponding incoming link. Simultaneously, the positive potential decreases in geometric progression, converging to zero value.

#### 5. Optimal spike frequency and assembly functioning consistency

To simplify our analysis, it is assumed that  $P$  is a constant, and all the neurons are equally connected – the sum of incoming links' strengths is the same for each neuron. This sum must be equal to the density  $V$ . The dependency  $S(t)$  determines spike frequency  $\gamma$  for each value of potential  $Y$ , let it be denoted by  $f(Y)$ . Function  $f(Y)$  might be interpreted as an activation function – the analysis of it is typical in artificial neural networks. Thus, the dynamics of a neural assembly could be approximately described as follows, where  ${}^+Y$  is moving towards value  $f(Y) \cdot V$ :

$$\frac{d{}^+Y}{dt} = f(Y) \cdot V - {}^+Y,$$

where  $V$  – the density of the assembly.

Let's find a density  $V^*(\gamma)$  that provides the stability condition:

$$\frac{d^+Y}{dt} = f(Y) \cdot V^*(\gamma) - Y = 0 .$$

Here, we can approximately express  $V(\gamma)$  as follows:

$$V^*(\gamma) = \frac{^+Y}{f(Y)} = \frac{Y + P}{f(Y)} = \frac{S(\gamma^{-1}) + P}{f(S(\gamma^{-1}))} = \frac{S(\gamma^{-1}) + P}{\gamma} .$$

This analysis shows that the activation dynamics of our neural assembly could be approximately described as follows: in case  $V > V^*(\gamma)$  spike frequency  $\gamma$  grows, but in the opposite case - decreases. Fig. 3 illustrates this dependency  $V^*(\gamma)$  for the case when  $P=1$ . Let's consider an example of an assembly with density  $V_0$ . This assembly has two stable states: (1) low frequency  $\gamma_1$  and (2) high frequency  $\gamma_2$ . Let  $\varepsilon$  be a small positive value, then: if  $\gamma = \gamma_1 - \varepsilon$ , then  $V^*(\gamma) = V^*(\gamma_1 - \varepsilon) < V^*(\gamma_1) = V_0$  and frequency  $\gamma$  must increase. If  $\gamma = \gamma_1 + \varepsilon$ , then  $V^*(\gamma) = V^*(\gamma_1 + \varepsilon) > V^*(\gamma_1) = V_0$  and frequency  $\gamma$  must decrease. The same is valid for the second stable state – high frequency  $\gamma_2$ .

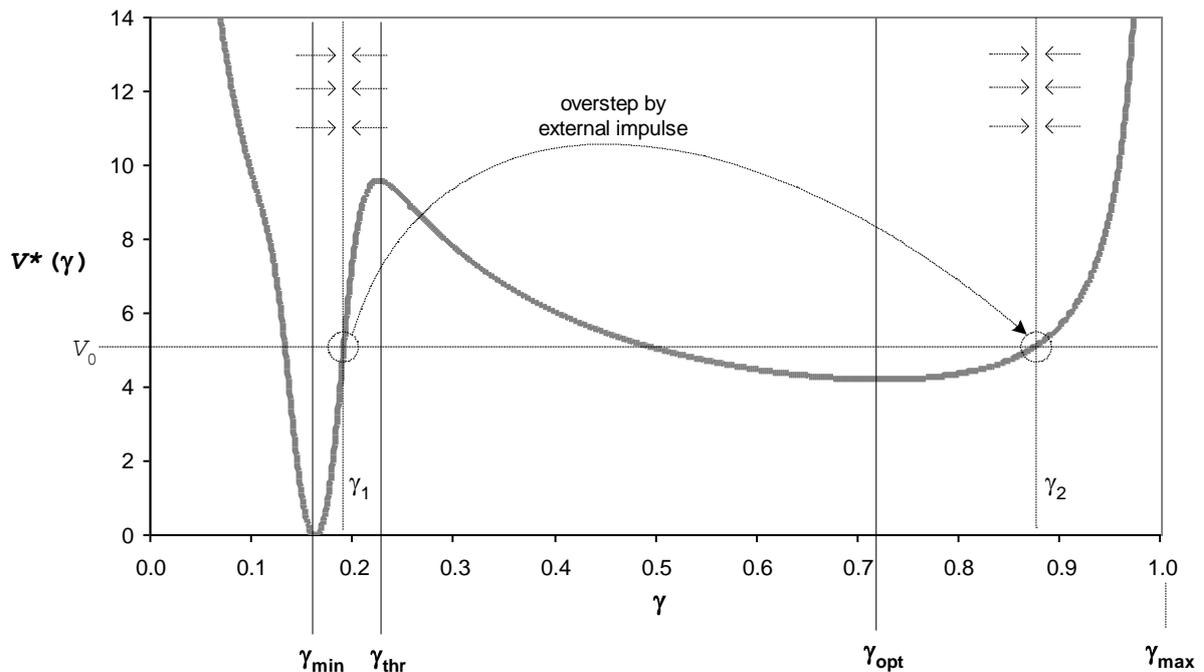


Fig. 3. Dependency of  $V^*$  on spike generating frequency  $\gamma$ .

It should be noted, that the neural assembly needs an additional activity, coming from external sources (perceptions or other assemblies), to overstep from the first stable state – low frequency  $\gamma_1$  into the second stable state – high frequency  $\gamma_2$ . Additionally, we can define threshold frequency  $\gamma_{thr}$  as the local maximum of function  $V^*(\gamma)$  in order to distinguish between high and low frequencies. It means that not a single neuron, but only a group of neurons could be understood as a threshold element. A single neuron is not able to provide the described property. We propose to define optimal frequency  $\gamma_{opt}$  as a minimal stable frequency in the range of high frequencies, and, additionally,  $\gamma_{min}$  - as a minimal stable frequency in the range of low frequencies, but  $\gamma_{max}$  - as a maximal frequency. Defining optimal frequency in this way, we can obtain many interesting and useful properties.

There are only two stable states: (1) half active state – low frequencies in range  $[\gamma_{\min}; \gamma_{\text{thr}})$  and (2) fully active state – high frequencies in range  $[\gamma_{\text{opt}}; \gamma_{\text{max}})$ . According to “optimal learning rule” (discussed above), density  $V$  is changing in this way: if the current spike frequency is lower than optimal one, then density  $V$  must increase, but in the opposite case – decrease. Consequently, this learning rule implies the tendency of density  $V$  to converge to  $V^*(\gamma_{\text{opt}})$  – a minimal density that provides stable fully active state. We will call the density  $V_{\text{opt}}$ , equal to  $V^*(\gamma_{\text{opt}})$ , an optimal density.

We suppose that the tendency of an assembly to have an optimal density must provide the consistency (discussed above) of a neural assembly because a neural assembly with an optimal density can generate spikes at a high frequency only in case of full synchronization between its neurons, e.g. the deactivation of one single neuron in this assembly could prevent from full activation. Each kind of desynchronization must reduce the assembly density below optimal one making itself unable to produce stable spikes generation at high frequencies. It means that the correct setting of optimal frequency allows full activation of an assembly only in case of full synchronization of all neurons in it.

In contrast, the setting to  $\gamma_{\text{opt}}$  a smaller (1) or greater (2) value may produce a negative result: (1) the principal inability of stable generating at high frequencies or (2) very low impressibility to desynchronization and, consequently, the total lost of neural assembly functioning consistency. Thus, we suppose that the correct setting of optimal frequency should provide neural assemblies formation as thresholds units that are able to indicate synchronized impulses from different sources.

## 6. An experiment

In the previous section, a simplified model of neural assembly has been considered. The strong relation between the correct choosing of optimal spike frequency and neural assembly functioning consistency has been found and analyzed. However, there is a need for checking of proposed assumption correctness. For this purpose, we have selected a typical neural assembly with manually controlled impact from external signal sources (perceptions or other neural assemblies). There were 20 neurons inside the assembly, and 20 neurons were used as external sources. Each of neurons in the assembly had incoming excitatory links from each other neuron inside and outside the assembly -  $20 \cdot (19+20)$  excitatory links at all.

We have applied the same simplified model as in the previous sections, except each neuron was modeled distinctly and the stochastic spike generation was applied (a spike occurs at the fulfillment of condition  $^+Y-P > S(t)$ , with probability 1/3). The strength of each link is equal to 0.23, and neuron parameter  $P$ , defined above, was considered as a constant, equal to 1. Fig. 4. shows a dynamics of neurons activation (average spike frequency in the assembly), performed in the experiment.

The assembly has a density equal to  $19 \cdot 0.23 = 4.37$  that implies two stable frequencies: low frequency  $\gamma_1$  and high frequency  $\gamma_2$  (see Fig. 3 and Fig. 4 to compare frequencies  $\gamma_1$  and  $\gamma_2$ ). In this experiment the assembly has been stimulated four times by external sources. The frequency of external neurons was equal to 1/3. In the first two cases (a) and (b) (900...2000; 2900...4000) only 75% of external neurons were turned on, but in the second two cases (c) and (d) (6900...8000; 18900...20000) 100% were turned on.

In the first case (900...2000) the assembly receives only 75% of the external activity needed for full activation. The neurons reach half active state and then continue to generate impulses at low frequency. In the second (2900...4000) attempt we have obtained the same result – after the stimulus the assembly stabilizes at frequency  $\gamma_1$ . In the third case (6900...8000) all 20 external neurons were generating. It allowed overstepping from low frequencies to high frequencies: in

the beginning the assembly reaches a frequency greater than frequency  $\gamma_2$ , but then it stabilizes at this frequency  $\gamma_2$  (actually, under frequency  $\gamma_2$  because of stochastic spike generating).

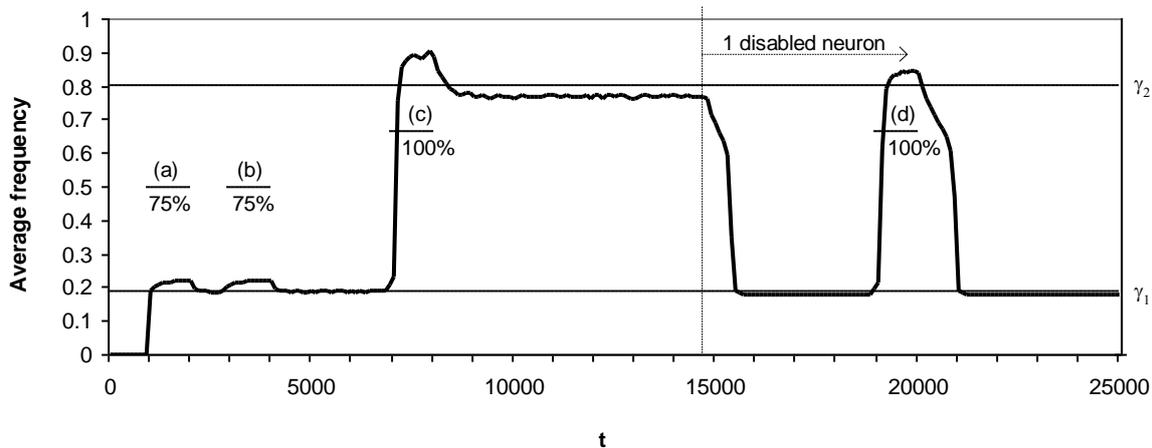


Fig. 4. The dynamics of a neural assembly activity.

However, the stable spike generation at high frequency is only possible in case of full synchronization of all the neurons in the assembly. To check this phenomenon, one of the neurons was disabled (since  $t=14900$ ). The minimal density that can provide spike generating at high frequency is equal to 4.20, but the disabling one neuron reduces the density from 4.37 to 4.14. Therefore, the assembly becomes unable to continue spike generating at high frequencies, and returns to low frequency  $\gamma_1$ . In case (d) an external stimulus returns the assembly to high frequencies, but without external help the assembly is unable to keep high frequency  $\gamma_2$ .

## 7. Conclusions

In this paper we have presented informational concept of inductive automaton focusing on the understanding of optimal spike frequency. In our research, a neuron was considered not as a threshold unit, but as a detail of it. It means that the properties of a neuron could be studied and understood only through dynamic interactions with other neurons of the same assembly.

We have found that the optimal spike frequency is strongly connected with neuron assemblies functioning consistency, and the correct setting of optimal spike frequency can provide consistency of a neural assembly. This consistency means that full activation must occur only in case of full synchronization between neurons in an assembly – even the deactivation of one single neuron must prevent from full activation of the whole assembly.

In original biological concept [3] of inductive automaton optimal spike frequency is considered only in sense of energy consumption minimization but in this paper we have analyzed another important aspect of it – the correct setting of optimal spike frequency has a great impact on consistency of neural assemblies.

**Acknowledgement.** This research was supported in part by the Latvian Science Foundation grant No.02-86d.

## 8. References

1. Pchelkin A. Self-organizing in neural networks based on memorizing // In press: Scientific Proceedings of Riga Technical University: Computer Science. Information Technology and Management Science, RTU, Riga, Volume 5., 2003.

2. Zurada J. M. Introduction to Artificial Neural Systems // In: St. Paul: West Publishing Company, 1992 - 684 p.
3. Emelyanov-Yaroslavsky L.B. Intelligent quasi-biological system (Inductive automaton) // M.: "Nauka", 1990 - 112 p.
4. Allinson, N.M. Self-Organising Neural Maps and their Applications // In: Theory and Applications of Neural Networks, Proceedings of the First British Neural Network Society Meeting, Springer-Verlag, London, 1992 - p. 101-118.

**Artūrs Pčolkins**, Mg. sc. dat.  
Riga Technical University  
Faculty of Computer Science and Information Technology,  
Institute of Information Technology  
Address: Kalku 1, LV-1658, Riga, Latvia  
Phone: +371 7089530  
E-Mail: arturp@balticom.lv

***Pčolkins A. Induktīvs automāts: optimāla impulsu frekvence***

*Šajā rakstā mēs turpinām mūsu pētījumu, saistīto ar induktīvu automātu – jauno mākslīgo neironu tīkls, kuru piedāvāja Emeljanov-Jaroslavski 1990 gadā. Šajā rakstā mēs cenšamies paplašināt piedāvāto tīri bioloģisku koncepciju ar papildus analīzi, fokusējot uzmanību uz optimālas impulsu frekvences sapratnes. Šī optimāla impulsu frekvence nosāka neironu tīkla apmācības principu: katrs neirons aktīva stāvoklī modificē sinoptiskus svarus tā, lai aktīvie neironi ģenerētu impulsus uz frekvences, kura tuvinātos optimālai frekvencei. Emeljanov-Jaroslavski ievēd optimālo impulsu frekvenci kā viss izdevīgāko enerģijas patēriņa minimizēšanai. Tomēr, šajā darbā tika papildus izpētīts, ka optimāla impulsu frekvence ir arī stipri saistīta ar integritāti neironu ansambļu funkcionēšanā. Pareiza optimālas frekvences izvēle var nodrošināt ļoti svarīgu īpašību: pilnīga aktivizācija var notikt tikai visu neironu ansambli saskaņošanās gadījumā, un pat viena neirona nobloķēšana var izjaukt neironu ansambļa pilnīgu aktivizāciju.*

***Pchelkin A. Inductive automaton: optimal spike frequency***

*In this paper we continue our investigation, connected with inductive automaton - a new artificial neural network, proposed by Emelyanov-Yaroslavsky in 1990. We are trying to extend his purely biological concept by an additional analysis, focusing on the understanding of the optimal spike frequency that determines a learning rule: each neuron in active state is modifying its synapses weights in order to generate spikes at the optimal frequency. Emelyanov-Yaroslavsky introduces the optimal spike frequency as the most profitable for neuron itself in the sense of energy consumption minimization. However, in this paper it has been found additionally that, the optimal spike frequency is strongly connected with neuron assemblies functioning consistency. The correct setting of optimal spike frequency can provide consistency of a neural assembly. This consistency means that full activation must occur only in case of full synchronization between neurons in an assembly – even the deactivation of one single neuron must prevent from full activation of the whole assembly.*

***Пчелкин А. Индуктивный автомат: оптимальная частота генерации***

*В этой статье мы продолжаем наше исследование, связанное с индуктивным автоматом – новая искусственная нейронная сеть, предложенная Емельяновым-Ярославским в 1990 году. В данной статье мы пытаемся расширить его чисто биологическую концепцию дополнительным анализом, фокусируя внимание на понимании оптимальной частоты генерации импульсов нейронами. Данная оптимальная частота определяет правило обучения нейронной сети: каждый нейрон в активном состоянии модифицирует синаптические веса так, чтобы его частота генерации импульсов приближалась бы к оптимальной частоте. Емельянов-Ярославский вводит оптимальную частоту генерации как наиболее выгодную с точки зрения минимизации потребления энергии. Однако, в данной работе было обнаружено, что оптимальная частота генерации также связана с целостностью в функционировании нейронных ансамблей. Было выяснено, что правильный выбор величины оптимальной частоты может обеспечить целостность функционирования нейронного ансамбля: полная активация нейронов в ансамбле возможна только в случае полной синхронизации всех нейронов этого ансамбля.*