OPTIMIZATION OF MULTIBODY VIBRATION RESPONSE BY GLOBAL SEARCH PROCEDURE

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Abstract. *This paper presents an efficient approach for vibration response optimization in the case of absence of pronounced optimization criteria. Simultaneously a great number of dynamic indices must be minimized and lot of side constraints must be taken into consideration. Minimized dynamic indices are contradictory and for that reason compromise variant of solution is necessary to search. For this efficient tool is developed by united utilization of the simulation system and the global search code with the dialog procedure for compromise searching. Main ideas of global search algorithm are briefly discussed. Basic relations for obtaining of nonlinear system vibration response in the time domain are given for the case of stationary and stationary bonded random excitations. Developed methodic is demonstrated by formulation of optimization task for the common spatial power unit.*

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1 Introduction

There are many practical tasks for vibration response optimization of multibody systems in the case of absence of pronounced optimization criteria. In such tasks the state variables, used in optimization process, are possible to divide in two groups. The first one has to satisfy side constraints only. Let us call these variables as state variables of pure constraints. The second group usually consists of the dynamic indices that especially characterize dynamic properties of optimized object. These indices must satisfy the side constraints and simultaneously numerical values must be minimal as possible. Let us call these variables as minimized state variables. In the simple cases, when number of minimized state variables and state variables of pure constraints is not so great, for the solution are available well-known multi-objective optimization methods. For example algorithm package that is based on utilization of LP_τ distribution is discussed in [1]. For successful utilization of such type algorithms it is necessary well-defined optimization task, that is very difficult at the initial stage of investigation. For example, search region must be sufficiently localized and number of criteria can not be very large. For case of investigation of non-linear vibrational systems additional problems arise. Due to possible multi-regime motions the objective function has breaks as well as regions where function is not defined at all. There are situations when the region of the design variable variation is so wide, that obtaining the necessary dynamic regime is very difficult by utilization of the specialized methods yet. A vast majority of the tasks are multi-extremal and from the point of view of designer it is especially important to obtain all set of extremes as possible. The last achievements in the field of global optimization are discussed in [2, 3].
The aim of this study is to develop methodic and tools for solution of tasks without pronounced optimization criteria to get possibilities of creation of the alternative variants for the problem optimal solution.

2 Tools of Simulation and Optimization

For solution of the optimization task three ingredients of software are used: 1) Tool for simulation of technical system behavior, 2) Tool for optimization, 3) Code that interlinks the first two ingredients. The last one usually provides development and formation of objective function as well as managing of the search process. These ingredients fully correspond to the requirements of the "Three-Columns-Concept" [4].
Investigations are carried out for the objects, dynamic schemes of which it is possible to construct from rigid bodies that are interlinked by means of non-inertial elastic and damping links. It is considered small spatial oscillations. The standard inertial, elastic, dissipative and excitation elements are used. As the standard elastic and dissipative elements serve element with piecewise characteristic with one switch. The more complex characteristics are obtained by parallel linking of the standard elements with appropriate characteristics. The standard excitation elements are forces and moments, described by polyharmonic series, that can

contain switches controlled by the state variable values and time. As random excitation serves appropriate correlation functions.
For simulation of the described mechanical systems the automated imitation simulation system Imita is used [5]. For dynamics simulation of the considered object class sufficiently effective is so called “stitching method”. In the case of random excitation the time domain methods are used. Utilization of these methods gives real possibility to implement optimization of dynamics of the complex objects without the great expenses of processor time.

2.1 Global Search Algorithm

The global search code Globex [6] is used for solution of the considered type tasks. This algorithm is based on the idea of informative planning and global random search. It solves the tasks successfully, which objective function are multi-extremal, non-analytical and noisy. The global search algorithm is operating in the following way. Based on the results of the initial search, which is done by planning the experiments to cover all the initial region, perspective subregions are singled out in the form of rectangular parallelepipeds. Thus it is the interior of these subregion that is being searched. The probability of an extreme situation is assumed to be higher in those subregions, where the function is of a better value, as well as in those in which there is a larger concentration of better points. The search is conducted in series. During each series the subregions are uniformly filled with an equal number of trial points. From the total number of points the best ones are selected and their number corresponds to the number of the subregions. Let these points be called support points. The support points are located in the centre of the respective subregion. In the course of searching, those subregions, which have not contained support point, are discarded and simultaneously new subregions are formed from several subregions. The sizes of the subregions constantly decrease from series to series.

The lengths of the subregions sides for the next series are obtained by dividing the sides of the corresponding subregion of the current series whose contraction coefficient might for instance, be of the following form: $k_{div} = a^{d-1} m^{b/m}$, where d- is the number of support points in the corresponding subregion of the current series; m - is the number of optimized arguments; a, b - are the numerical coefficients. Parameters d, a, b can be used for the program controlling. Finding of the extremes is accomplished when within a given precision all the subregions have been localized, i.e. when actually the entire summary region is converted to point.

The main stages of the search process for two dimensional case (m = 2) are illustrated in figure 1 a - c. Firstly in the initial stage of search support regions cover each other. Process of extraction of the perspective regions for search is going on. In the second stage the perspective regions are distributed and support regions mainly do not cover each other as well as they compete each with other. In the third stage algorithm already decides for extreme and search is concentrated in the extreme attraction region.

The extremes already found are isolated from further searching by means of penalty parallelepipeds as shown in figure 1 d.

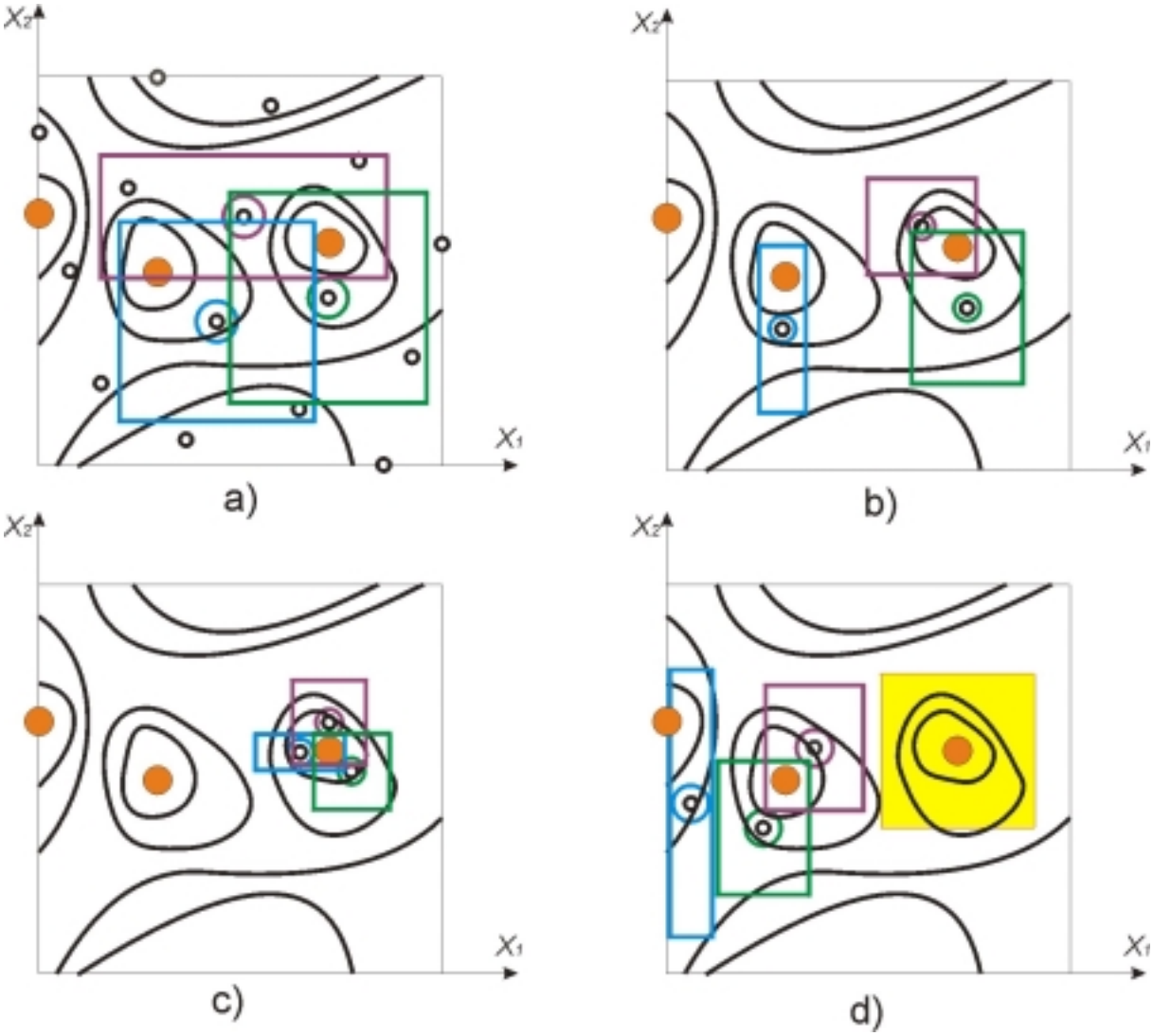

Figure 1: The search stages of optimization algorithm

Subsequent main indices characterize quality of optimization program: 1) In global mean - safety of optimum finding; 2) In local mean - accuracy of optimum finding; 3) Number of the trial points necessary to achieve target, i.e. to find optimum.

In most cases user must solve the global search task. Then he is interested in safe obtaining of the optimum with minimal number of the trial points. Usually in the case when the objective function structure preliminary is unknown these indices are contradictory, that is, greater safety needs more trial points. Such class of the functions

$$f(X)=\sum_{i=1}^{m}(X_i^2+iX_i)+40\prod_{j=1}^{m}\sin(X_j) \qquad (1)$$

extremes were examined to compare safety of obtaining global extreme by Globex and by typical non-linear programming routines [7].

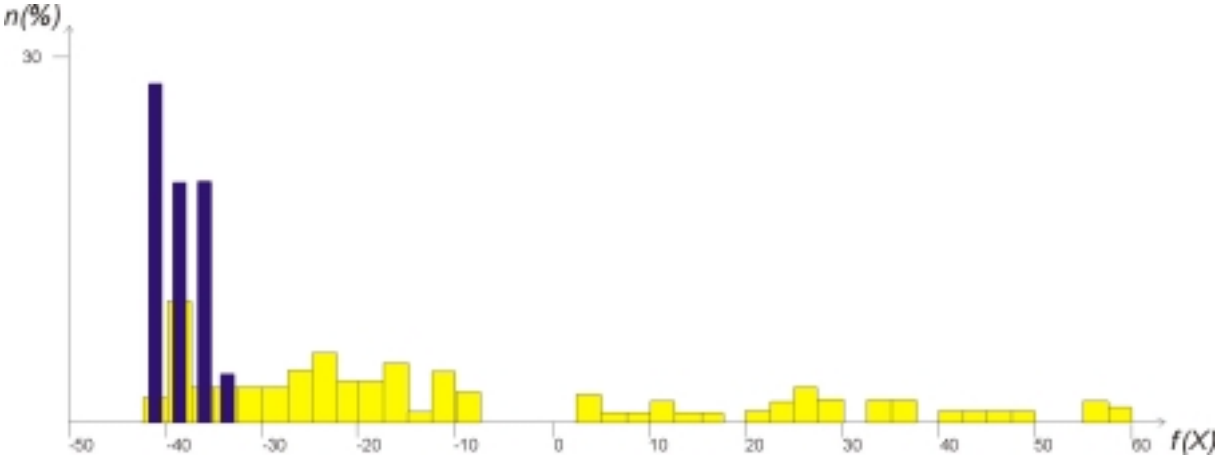

Figure 2: Extremes found by Globex (dark columns) and by Supex (light columns)

For case m = 4 the function (1) has one global extreme (X* = [-1.47; 1.54; 1.57; 1.59] ; f(X) = -42.93) and great number of local extremes. In figure 2 we can see that more than in 30% cases Globex finds the global extreme, but in other cases three nearest local extremes. At the same time non-linear programming routine finds in much cases the local extremes with almost regular random distribution.

2.2 Method for Solution of Equations of Motion

One has solved various types of differential equations to obtain dynamic response of the examined class of discrete mechanical systems. Let us consider one of the methods used in simulation system Imita for investigation of non-linear random vibrations. Suppose that the non-linear M-D-o-F system is described by equation:

$$A\ddot{q} + B\dot{q} + \boldsymbol{\varphi}(q) = f(t), \tag{2}$$

where A, B are inertial and damping matrices of order n,
f, q are column vectors of stationary and stationary bonded excitations and generalized co-ordinates, respectively,
φ (q) is column vector of non-linear restoring forces.
Instead of solution of (2) in the method of statistical linearization it is considered solution of linear equation:

$$A\ddot{q} + B\dot{q} + Cq = f(t), \tag{3}$$

where C is stiffness matrix of order n, whose elements are chosen so that (2) and (3) are statistically equivalent in some sense. Solution of (3) can be obtained as the column vector of the stationary random process q = q(c,t). Let us suppose that we know this solution and substitute it into equation (2) and (3). It has been shown that equations (3) will be identity, but the equations (2) will be identity only for case of introducing of Ψ - error column vector us a supplement item [8]:

$$A\ddot{q} + B\dot{q} + \varphi(q) = f(t) + \Psi . \tag{4}$$

Now by subtraction of (4) from (3) is obtained:

$$\Psi = \boldsymbol{\varphi}(q) - Cq . \tag{5}$$

Thus Ψ depends from matrix C elements values $c_{i,j}$ (i=1,…,n; j=1,…,n) then let us choose they to minimize mean value of the such scalar multiplication:

$$< \Psi^T \Psi >= \min , \tag{6}$$

where "<>" designates operation of averaging. So we obtain equations:

$$\frac{\partial < \Psi^T \Psi >}{\partial c_{i,j}} = 0 . \tag{7}$$

Let us change the sequence of operations and firstly carry out differentiation. Then we obtain:

$$2 < \Psi^T \frac{\partial \Psi}{\partial c_{i,j}} >= 0 \quad \text{or} \quad -2 < \Psi_i q_j >= 0 . \tag{8}$$

By substituting (5) into (8) we obtain:

$$< \varphi(q) q^T > - C < q q^T >= 0 . \tag{9}$$

Let us introduce a force column vector h(z), whose elements are forces and moments, that act on appropriate body at appropriate direction, and **z** is the vector of the subsequent relative co-ordinates. Then it is possible to write:

$$\varphi_i(q) = h_i(z) = \sum_{k(k \neq i)} h_{ik}(z_{ik}) . \tag{10}$$

In this expression summing must be implemented for all the k non-linear elements, that are attached to the subsequent body and give contribution to the force at the direction i. Taking into account expression (10) the equations (9) obtain the form:

$$C < qq^T >=< h(z)q^T > . \tag{11}$$

Averaging by ensemble of vectors multiplication for equal values of argument t, i.e. for τ=0, where τ is difference of time instants for which the correlation bonds are evaluated, correlation matrices diverge to the corresponding dispersion matrices and we obtain:

$$CD_{qq} = D_{hq}, \text{ or } \quad C = D_{hq} D_{qq}^{-1}, \tag{12}$$

where D_{qq} and D_{hq} are matrices of dispersion of generalized co-ordinates and mutual dispersion of the vectors h and q correspondingly. These matrices can be obtained by solving of (3).
Let us illustrate above discussed algorithm for consideration of systems containing such type characteristics of the non-linear springs:

$$r_j(z_j)=c_j z_j+ \varepsilon_j z^3_j , \tag{13}$$

where z_j is corresponding relative co-ordinate and c_j, ε_j are constants.
Equations of motion of such type systems are:

$$A\ddot{q} + B\dot{q} + Cq + Ez^3 = f(t), \tag{14}$$

where C is matrix of linear part of stiffness,
E is matrix of non-linear part of stiffness,
z is the relative co-ordinates vector that can be expressed from generalized co-ordinates by transfer matrix:

$$q=\Pi z. \tag{15}$$

Forces from the elastic elements that act on system bodies at the corresponding directions can be expressed by vector:

$$h(z)=C\Pi z + Ez^3 . \tag{16}$$

After multiplication of (16) by transposed vector of the relative co-ordinates z and averaging by ensemble we obtain:

$$< h(z)z^T > = C\Pi < zz^T > + E < z^3 z^T > . \tag{17}$$

By solving of equivalent equation (3) with the initial values of matrix C elements we obtain first approximation of generalized co-ordinates dispersion matrix, that is:

$$D_{qq}=<qq^T> \tag{18}$$

and similarly

$$D_{zz}=<zz^T>, \tag{19}$$

where appropriate vector values are taken for the equal values of argument, i.e. for $\tau=0$. Taking into account (15) and (18) the expression (19) can be written:

$$D_{zz}=\Pi^{-1}D_{qq}(\Pi^{-1})^T. \tag{20}$$

So matrix D_{zz} can be expressed through previously obtained dispersion matrix D_{qq} of generalized co-ordinates. Now in expression (17) we must obtain only $< z^3 z^T >$. For this we can utilize bond between the characteristic function and the moments of random vector. The central moments of random vector with normal distribution can be expressed through characteristic function that expanded in Maclaurin series, so expression for the even central moment calculation is [9]:

$$\mu_{l_1,\ldots,l_{n_e}} = \frac{l_1!\ldots l_{n_e}!}{2^s s!}\sum k_{p_1q_1}\ldots k_{p_sq_s}\ , \tag{21}$$

where summing is implemented for all possible 2s different rearrangements of indices $p_1,q_1,\ldots, p_s,q_s$ and $2s = l_1+\ldots+l_{n_e}$, n_e – number of vector elements. For example, for $n_e=4$ one of the central moment (element of matrix $< z^3 z^T >$) can be obtained from corresponding correlation functions:

$$\mu_{1,1,1,1} =< z_1^3 z_2 >= 3k_{z_1z_1}k_{z_1z_2}. \tag{22}$$

Similarly all other elements of matrix $< z^3 z^T >$ can be calculated by utilization of the elements of correlation matrix, which are obtained from solution of (3).

Now we can calculate matrix $< h(z)z^T >$ and by using of (13) obtain the first approximation of stiffness matrix for equivalent linear system (3):

$$C = D_{hz}\left(\Pi^{-1}\right)^T D_{qq}^{-1}. \tag{23}$$

By repeating of solution of (3) we can obtain new values for the elements of dispersion matrix $D_{qq}^{'}$. By comparison of values of $D_{qq}^{'}$ with D_{qq} etc., we can manage the convergence of

iterative calculation process of dispersion. If discrepancies of values of the all generalized co-ordinates for current and previous step are less than prescribed value of *e* percents, i.e.:

$$\frac{\left|d_{i,j}^{qq'} - d_{i,j}^{qq}\right|}{d_{i,j}^{qq}} \le \frac{e}{100}; \quad (i=j=1,\ldots,n), \tag{24}$$

where $d_{i,j}^{qq'}$ and $d_{i,j}^{qq}$ are elements of $D_{qq}^{'}$ and D_{qq} correspondingly, then it is supposed that dispersion matrix of issue non-linear system (14) is obtained.

For determining solution of (3) in the time domain one has to solve two differential correlation equations (for example, [10]):

$$\begin{aligned} A\ddot{K}_{qf}(\tau) + B\dot{K}_{qf}(\tau) + CK_{qf}(\tau) &= K_{ff}(\tau) \\ A\ddot{K}_{qq}^{T}(\sigma) + B\dot{K}_{qq}^{T}(\sigma) + CK_{qq}^{T}(\sigma) &= K_{qf}^{T}(\sigma) \end{aligned} \tag{25}$$

where

$K_{ff}(\tau)$ - square matrix of correlation functions of excitations f,

$K_{qq}(\tau)$ - square matrix of correlation functions of generalized co-ordinates q,

$K_{qf}(\tau)$ - square matrix of mutual correlation functions of generalized co-ordinates q and excitations f,

τ- difference of time instants for which the correlation bonds are evaluated ($\sigma = -\tau$).

Suppose that the correlation functions of stationary and stationary bonded random excitations could be approximated by means of expressions of the following type:

$$K_f(\tau) = De^{-\alpha|\tau|}(\cos\beta\tau + \alpha/\beta\sin\beta|\tau|), \tag{26}$$

where parameters D>0, α >0, $\beta \ge 0$. The presence of expressions like (26) in the right - hand part of the first correlation equation (FCE) requires its solution to be obtained in two intervals: I1- for $-\infty \le \tau \le 0$ and I2- for $0 \le \tau \le \infty$ with subsequent stitching for argument value $\tau = 0$.

The following boundary conditions derive from the properties of the correlation functions:

$$K_{qf}(-\infty) = K_{qf}(\infty) = 0. \tag{27}$$

The solution of fce can be obtained in the form:

$$K_{qf}(\tau) = K_{qfc}(\tau) + K_{qfp}(\tau), \tag{28}$$

where $K_{qfc}(\tau)$ and $K_{qfp}(\tau)$ are matrices comprising the appropriate complementary functions and particular solutions. In view of independence of the $K_{qf}(\tau)$ columns, let us consider obtaining only one of them.

The column *l* of $K_{ff}(\tau)$ in I2 has the form of vector column:

$$k_{ff}(\tau)_l = (m_{1l}\cos\beta\tau + m_{2l}\sin\beta\tau)e^{-\alpha\tau}. \tag{29}$$

Then the particular solution is sought for in the form:

$$^{+}k_{qfp}(\tau)_l = (u_{1l}\cos\beta\tau + u_{2l}\sin\beta\tau)e^{-\alpha\tau}. \tag{30}$$

The column vectors u_{1l} and u_{2l} can be found from equation:

$$VP\begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix}_l = \begin{Bmatrix} m_1 \\ m_2 \end{Bmatrix}_l, \tag{31}$$

where VP is a square matrix of order 2n

$$VP = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \tag{32}$$

with square sub-matrices: $V_{11} = (\alpha^2 - \beta^2)A - \alpha B + C$, $V_{12} = \beta B - 2\alpha A$, $V_{21} = -V_{12}$, $V_{22} = V_{11}$.

The column *l* of $K_{ff}(\tau)$ in I1 has the form of vector column:

$$k_{ff}(\tau)_l = (m_{1l}\cos\beta\tau - m_{2l}\sin\beta\tau)e^{\alpha\tau}. \tag{33}$$

Then the particular solution is sought for in the form:

$$^{-}k_{qfp}(\tau)_l = (u'_{1l}\cos\beta\tau + u'_{2l}\sin\beta\tau)e^{\alpha\tau}. \tag{34}$$

The column vectors u'_{1l} and u'_{2l} can be derived from equation:

$$VM\begin{Bmatrix} u'_1 \\ u'_2 \end{Bmatrix}_l = \begin{Bmatrix} m_1 \\ -m_2 \end{Bmatrix}_l, \tag{35}$$

where VM is of a structure similar to VP, but differs in the sub-matrices of order n, namely:

$$V_{11} = (\alpha^2 - \beta^2)A + \alpha B + C, \quad V_{12} = \beta B + 2\alpha A. \tag{36}$$

To obtain the column *l* of $K_{qfc}(\tau)$ appropriate homogeneous equation has to be solved. For case of non- classical distribution of damping $B \neq A\sum a_b A^{-1}C^b$ (b=0,1,...,m), it is convenient to consider equation of the form:

$$\dot{k}_{zf}(\tau)_l - Wk_{zf}(\tau)_l = 0, \tag{37}$$

where $k_{zf}(\tau)_l$ is a vector column of order 2n with structure:

$$k_{zf}(\tau)_l = \begin{Bmatrix} \dot{k}_{qf}(\tau)_l \\ k_{qf}(\tau)_l \end{Bmatrix} \text{ and } W = \begin{bmatrix} -A^{-1}B & -A^{-1}C \\ I & 0 \end{bmatrix}, \tag{38}$$

where I and 0 are unit and zero matrices each of order n.

If W has 2n simple eigenvalues, namely, 2k complex ($\lambda_j = \mu_j \pm i\nu_j$ for j=1,...,k) and k_a real ($\lambda_j = \mu_j'$ for j=1,..,k_a), then, after appropriate normalization, the eigenvectors can be grouped as follows $[K \cdot K \cdot S] + i[L \cdot -L \cdot 0]$,

where K and L are 2n x k matrices whose columns are the eigenvectors corresponding to the real and the imaginary parts of the complex eigenvalues, respectively;

S is the 2n x k_a matrix that contains the eigenvectors corresponding to the real eigenvalues;

0 is the 2n x k_a null matrix.

Thereby the solution of (37) will be:

$$k_{zf}(\tau)_l = \left(K\left|e^{\mu\tau}\right|\left[\cos\nu\tau\right] - L\left|e^{\mu\tau}\right|\left[\sin\nu\tau\right]\right)a_{1l} -$$

$$\left(L\left[e^{\mu\tau}\right]\left[\cos\nu\tau\right] - K\left[e^{\mu\tau}\right]\left[\sin\nu\tau\right]\right)a_{2l} + S\left|e^{\mu'\tau}\right|a_{3l}, \tag{39}$$

where:

$\left|e^{\mu\tau}\right|$- diagonal matrix of order k, whose nonzero elements are $e^{\mu_j\tau}$ for j=1,..,k and μ_j are the real part of the complex eigenvalues;

$\left[\cos\nu\tau\right], \left[\sin\nu\tau\right]$- diagonal matrices of order k, the nonzero elements of which are appropriate trigonometric functions;

$\left|e^{\mu'\tau}\right|$- diagonal matrix of order k_a , whose nonzero elements are $e^{\mu'_j\tau}$ for j=1,.., k_a and μ_j' are real eigenvalues;

a_{1l}, a_{2l}, a_{3l} - vector columns of order k and k_a respectively.

From stitching at $\tau = 0$, the FCE solutions in I1 and I2, n equations can be obtained:

$$\mathrm{Mo}\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}_l = \begin{Bmatrix} {}^{-}\dot{k}_{qfp}(0) \\ {}^{-}k_{qfp}(0) \end{Bmatrix}_l - \begin{Bmatrix} {}^{+}\dot{k}_{qfp}(0) \\ {}^{+}k_{qfp}(0) \end{Bmatrix}_l, \tag{40}$$

where the modal matrix $\mathrm{Mo} = [\mathrm{K} \cdot -\mathrm{L} \cdot \mathrm{S}]$.
By substituting (30), (34) and their derivatives into (40) we obtain:

$$\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}_l = \mathrm{Mo}^{-1} \begin{Bmatrix} r_{\dot{q}l} \\ r_{ql} \end{Bmatrix}, \tag{41}$$

where $r_{\dot{q}l}$ and r_{ql} are vector columns of order n:

$$\begin{aligned} r_{\dot{q}l} &= \alpha\left(u'_{1l} + u_{1l}\right) + \beta\left(u'_{2l} - u_{2l}\right), \\ r_{ql} &= u'_{1l} - u_{1l}. \end{aligned} \tag{42}$$

Now the complementary function of FCE assumes the form:

$$k_{zf}(\tau)_l = \left(\mathrm{Co}\lceil \cos \nu\tau \rceil + \mathrm{Sn}\lceil \sin \nu\tau \rceil\right)e^{\mu\tau} + \mathrm{Ape}^{\mu'\tau}, \tag{43}$$

where $\mathrm{Co} = \mathrm{K}\lceil a_1 \rceil - \mathrm{L}\lceil a_2 \rceil, \mathrm{Sn} = \mathrm{K}\lceil a_2 \rceil, \mathrm{Ap} = \mathrm{S}\lceil a_3 \rceil$,
and $\lceil a_1 \rceil, \lceil a_2 \rceil, \lceil a_3 \rceil$ are diagonal matrices the nonzero elements of which are the appropriate elements of column vectors a_1, a_2, a_3;
$e^{\mu\tau}$ and $e^{\mu'\tau}$ are column vectors whose elements are appropriate nonzero elements of $\left|e^{\mu\tau}\right|$ and $\left|e^{\mu'\tau}\right|$.
After changing the argument the second correlation equation (SCE) assumes the form:

$$\mathrm{A}^{\mathrm{T}}\ddot{\mathrm{K}}^{\mathrm{T}}_{qq}(\tau) - \mathrm{B}^{\mathrm{T}}\dot{\mathrm{K}}^{\mathrm{T}}_{qq}(\tau) + \mathrm{C}^{\mathrm{T}}\mathrm{K}^{\mathrm{T}}_{qq}(\tau) = \mathrm{K}^{\mathrm{T}}_{qf}(\tau). \tag{44}$$

For practical purposes it is sufficient only to obtain the particular solution of (44) for $\tau \geq 0$:

$$\mathrm{K}^{\mathrm{T}}_{qqp}(\tau) = \mathrm{K}^{\mathrm{T}}_{qq1}(\tau) + \mathrm{K}^{\mathrm{T}}_{qq2}(\tau). \tag{45}$$

where $\mathrm{K}^{\mathrm{T}}_{qq1}(\tau)$ and $\mathrm{K}^{\mathrm{T}}_{qq2}(\tau)$ comprise solutions of (44) when in the right - hand of (44) there are: 1) the particular solution of FCE and 2) the complementary function of FCE - both in I2.
In view of independence of the $\mathrm{K}^{\mathrm{T}}_{qq}(\tau)$ columns, let us consider obtaining only one of them.
If the column *l* of $\mathrm{K}'_{qq}(\tau)$ for $\tau \geq 0$ has a shape similar to that of (30) then the particular solution can be found in the form:

$$k_{qq}(\tau)_l = (s_{1l}\cos\beta\tau + s_{2l}\sin\beta\tau)e^{-\alpha\tau}. \quad (46)$$

The column vectors s_{1l} and s_{2l} can be derived from equation:

$$VM\begin{Bmatrix} s_1 \\ -s_2 \end{Bmatrix}_l = \begin{Bmatrix} u_1 \\ -u_2 \end{Bmatrix}_l, \quad (47)$$

where VM is the square matrix of order 2n with above considered structure.
Since the expressions for the FCE complementary function (43) are of a structure similar to that of the elements of $K_{qfp}(\tau)$, likewise $K_{qq2}(\tau)$ can be obtained. Thus, to obtain $K_{qq2}(\tau)$ one has to inverse n matrices of the type VM. Thereupon the necessary dispersion matrices of the output processes can be obtained, for example, of generalized co-ordinates:

$$D_{qq} = K_{qqp}(0). \quad (48)$$

To sum up, the developed method in every iteration requires solving the standard eigenvalues problem for an arbitrary real matrix W of order 2n and inverting (n+3) times of matrices of order 2n, as well as performance of several simple operations.
Despite of the cumbersome algorithm it is very efficient from computational point of view. It allows to implement global optimization procedure for investigation of non-linear random vibrations of complex objects. However further investigations are necessary to broadly utilize this method in the optimization loop, because it is possible to climb over the basic hypotheses of the method during parameters variation.

3 Common Formulation of Optimization Task

Optimization strategy for the considered type tasks we shall explain by using of the simple car power unit dynamic scheme shown in the figure 3. Initially power unit is treated as body with 6 DoF. The body of the power unit is supported on a frame by means of three mountings S.
There are two polyharmonic excitations acting on the power unit: 1) the force P acting vertically, and 2) the moment M_F the vector of which is parallel X axis.
In the common case of optimization the stiffness of the power unit mountings and the attachment coordinates of the mountings serve as the design variables. There are three spatial mountings, therefore the design variables vector is following: [C_{1X}, C_{2X}, C_{3X}, C_{1Y}, C_{2Y}, C_{3Y}, C_{1Z}, C_{2Z}, C_{3Z} ,S_{1X}, S_{2X}, S_{3X}, S_{1Y}, S_{2Y}, S_{3Y}, S_{1Z}, S_{2Z}, S_{3Z}]. So for the given case we have 18 design variables. Let us designate this vector as: $X_i; i = 1,2....,18$. The design variables have subsequent region for the initial search:

$$Xmin_i \le X_i \le Xmax_i. \quad (49)$$

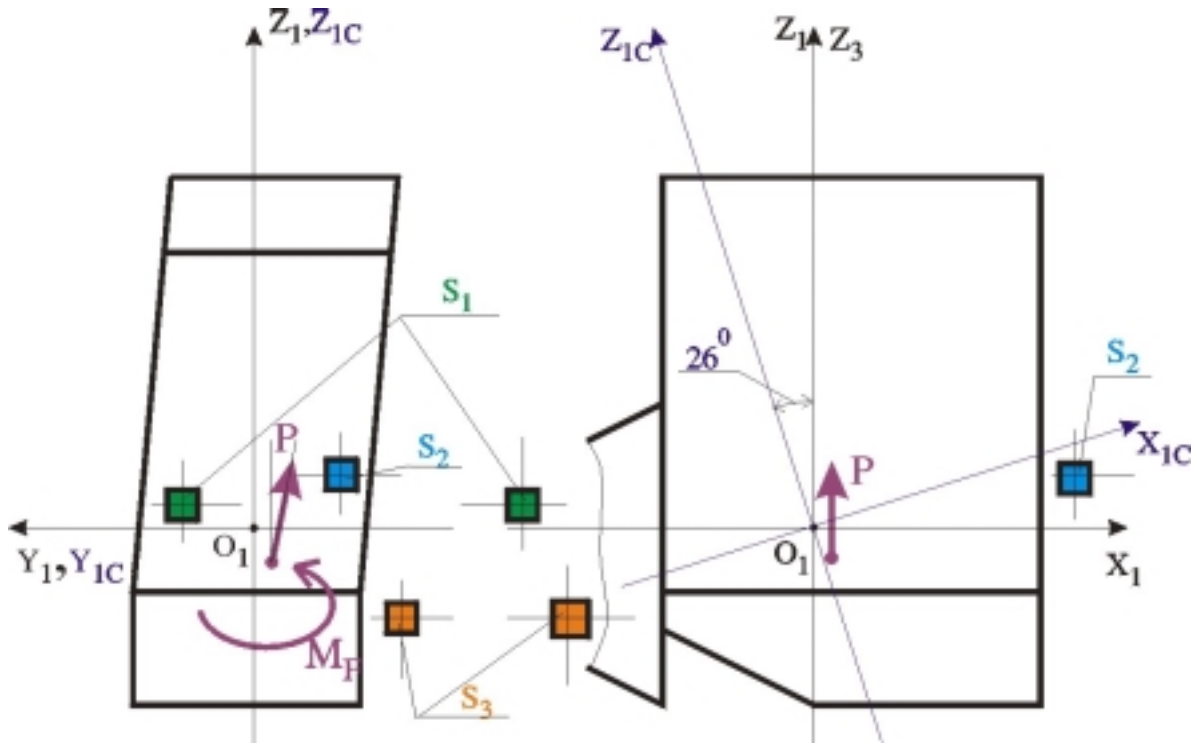

Figure 3: Dynamic Scheme of the Car Power Unit

For the considered task the state variables of pure constraints are eigenfrequencies (f_1, f_2, f_3, f_4, f_5, f_6). They have following side constraints: $10 < f_i < 19$[*Hz*]. Let us call these constraints as pure constraints (PC):

$$PCmin_i \le PC_i \le PCmax_i . \qquad (50)$$

Then minimized state variables follow. They can be divided in the two groups:
a) The angular displacements and the linear displacements of the characteristic points of the power unit SM_i are first one. They have following side constraints: $SMmin_i \le SM_i \le SMmax_i$,
b) The maximal dynamic forces F_{ij} of the suspension mountings have side constraints also: $Fmin_{ij} \le F_{ij} \le Fmax_{ij}$.
These variables we need to minimize and they ideal value will be zero. Usually zero is impossible for dynamic system, therefore we must seek the compromise variant for these variables. SM_i and F_{ij} can oscillate and therefore they have two-sided constraints. In the common case for these parameters let us introduce designation MQ (Minimized Quantities). Then the side constraints are following:

$$MQmin_i \le MQ_i \le MQmax_i . \qquad (51)$$

We can see, that the considered task has many minimized variables and at the same time has not pronounced optimization criteria, therefore this class of the tasks can be named as the tasks without pronounced optimization criteria.

4 Optimization Strategy

Now let us consider the objective function F. According to developed optimization strategy it consists of three parts:

$$F = F_1 + F_2 + F_3 . \tag{52}$$

Item F_1 provides the fast entering of the search process in the regions where pure constraints are satisfied. For the considered in the previous section task it can be expressed in the following way:

$$F_1 = PENALT1_0 + \sum PENALT1_i \tag{53}$$

$$PENALT1_i = \begin{cases} 0; f\min_i \le f_i \le f\max_i \\ (f\min_i - f_i)^2 ; f_i < f\min_i \\ (f_i - f\max_i)^2 \, f_i > f\max_i \end{cases} \tag{54}$$

$$PENALT1_0 = \begin{cases} 0; f\min_i \le f_i \le f\max_i \\ 10^6 \end{cases} \tag{55}$$

When pure constraints are satisfied then the item F_1 practically switches off and in the further search provides filtration of the trial points that do not satisfy these constraints. Numerical value of the constant (in this case 10^6) in (55) has not special meaning. It must provide prevailing of item F_1 under F_2 and F_3 only. So firstly the regions, where the pure constraints are satisfied, are extracted from the search and only after that optimization is continued.
When the algorithm finds the regions, where the pure constraints are satisfied, then the second item F_2 provides the fast entering into the regions, where the minimized state variables satisfy side constraints. So it provides the partial solution of the task and the starting conditions for the last optimization step. This step is mainly interactive and it realizes finding of the set of the compromise solutions. F_2 can be expressed subsequently:

$$F_2 = PENALT2_0 + \sum PENALT2_i \tag{56}$$

$$\mathrm{PENALT2}_i = \begin{cases} 0; \mathrm{MQmin}_i \le \mathrm{MQ}_i \le \mathrm{MQmax}_i \\ (\mathrm{MQmin}_i - \mathrm{MQ}_i)^2; \mathrm{MQ}_i < \mathrm{MQmin}_i \\ (\mathrm{MQ}_i - \mathrm{MQmax}_i)^2 \; \mathrm{MQ}_i > \mathrm{MQmax}_i \end{cases} \tag{57}$$

$$\mathrm{PENALT2}_0 = \begin{cases} 0; \mathrm{MQmin}_i \le \mathrm{MQ}_i \le \mathrm{MQmax}_i \\ 10^4 \end{cases} \tag{58}$$

This item acts like item F_1. When the regions, where minimized state variables satisfy the side constraints, are found, then the item F_2 practically switches off and in the further search provides filtration of the trial points that do not satisfy these constraints. Numerical value of the constant (in this case 10^4) in (58) has not special meaning. It must provide prevailing of item F_2 under F_3 only. Additionally the item F_2 must not prevail under F_1. The permissible borders of constraints (56-58) usually are wider than technically allowable. This provides the easier search of the necessary regions as well as broadens possibilities of the design variable variation in the third optimization step.

In this last optimization step it is necessary to realize purposeful minimization of the minimized state variables as well as to monitor this process and providing correction of the optimization task in the case of necessity. If all minimized state variables are united in one expression as a preference function, for example, by using of appropriate square sums, then there are great difficulties to manage the progress of optimization and the obtained result. The following algorithm is proposed for easier managing and interpretation of the optimization process. The values of lower and upper bounds of the minimized state variables are chosen so that by user opinion they are equally good. Let us call these intervals as binding intervals. In the simplest case they are equal with the technically permissible boundaries. Further in optimization criteria the normalized minimized state variables are used. If values of the minimized state variables are situated in the binding intervals, then normalized values must change within equal borders, for example, from –1 till 1. In common case the relations of linear normalization are following:

$$\mathrm{MQN}_i = \mathrm{A}_i + \mathrm{B}_i \cdot \mathrm{MQ}_i \tag{59}$$

It is possible to use different normalization. For example, normalized and unnormalized quantities can meet requirements to coincide zero values etc. The third item of the objective function is following:

$$F_3 = \max\left(\left|\mathrm{MQN}_i\right|\right) \tag{60}$$

Such arrangement of the objective function item for the condition, when the minimal and maximal border of the binding intervals are normalized to –1 and 1 appropriate, provides

great advantage for case when there is necessity to visualize and to manage optimization process. User gets full notion about optimization process by observing optimization search.

5 Conclusions

The optimization tasks of the dynamical systems without pronounced criteria and with the great number of constraints are possible to solve effectively by using of global optimization system and special three-step strategy for localization of solution. Accordingly to it in the first step the regions are localized where the pure constraints are satisfied. In the second step the further localization of the regions are obtained where the side constraints of the minimized state variables are satisfied. In the last step the minimized state variables are united by means of the binding intervals. Such a binding in dialog mode gives possibility for the users by changing of the appropriate values of the binding interval parameters to easy manage searching of compromise solution in the necessary direction.

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