A Maximum Entropy Analysis of Self-Similar Data Flow

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Abstract - **Important area of investigation of impact of self-similar processes on performance of telecommunication systems is to define which traffic distribution should be utilized when modeling networks with these traffic patterns. For this purpose maximum entropy approach is applied to derive unique solution for arrivals distribution**.

1 Introduction

Statistical analysis of high-resolution traffic measurements from wide range of working packet networks (e.g. Ethernet LANs, WANs, SS7, ISDN and VBR video over ATM) have convincingly shown the presence of self-similar properties in both local and wide area traffic traces. That means that similar statistical patterns may occur over different time scales that can vary by many orders of magnitude (i.e. ranging from milliseconds to minutes and even hours) and have heavy-tailed autocorrelation function (long-range dependence). This discovery calls in question some of the basic assumptions made by earlier research done in control, engineering and operations of modern high-speed data networks. The fact that network traffic is inherently fractal or long-range dependent and many studies assume traffic to be short-range dependent leads to question extent to which the results of these studies are applicable in practice.More information about self-similar traffic can be found in references [1, 2, 3, 4].

The work on self-similarity in network traffic has demonstrated the practical implications for a wide range of traffic management and engineering problems, including network scaling, service levels, overload control etc. The success of modern integrated high-speed technologies and real-time applications may depend on ability to accurately model traffic flows in the networks and ability to make performance forecasting in working environment.

Important area of investigation of impact of self-similar processes on performance of telecommunication systems is to define which traffic distribution should be utilized when modeling networks with these traffic patterns. Some of the criteria, which must be addressed, are:

1. the distribution should generate network traffic which can map to real-world data;
2. the distribution should be applicable to queuing analysis;
3. the distribution should be developed such that it enables researchers to gain insight into dynamics of theses processes.

For purposes to derive distribution that capture desired properties maximum entropy approach is presented. This method is based on concept of entropy functional, introduced in information theory by Shannon [5]. Since maximum entropy corresponds to maximum disorder in the system, these solutions are the least biased of all solutions that satisfy the system constraints. More importantly maximum entropy solutions have been shown to be analytically tractable and can be utilized in queuing models.

In the following sections analysis of self-similar flow of arrivals is presented and generalised maximum entropy approach is directly applied to derive a unique distribution.

2 Self-Similar Stochastic Processes

Consider a discrete time stochastic process $X=(X_1,X_2,\ldots)$ with a constant mean $E\{X_i\}$, variance $E\{(X_i-E\{X_i\})^2\}$ and autocorrelation function

(1) $$r(k)=\frac{E\{(X_i-E\{X_i\})(X_{i+k}-E\{X_i\})\}}{E\{(X_i-E\{X_i\})^2\}}$$

depending on k only. Process X can be interpreted as the traffic volume – measured in packets, bytes or bits at time instance t.

Define another stochastic process $X^{(m)}=(X^{(m)}_1, X^{(m)}_2,\ldots)$ by averaging the original process over non overlapping, adjacent blocks of size m:

(2) $$X_i^{(m)}=\frac{1}{m}\sum_{n=im-(m-1)}^{im}X_n \ .$$

The process X is said to be exact self similar with Hurst parameter $H=1-b/2$ $(1/2<H<1)$ if the following conditions are satisfied:

(3) $$E\{(X^{(m)}{}_i-E\{X^{(m)}{}_i\})^2\}=\frac{E\{(X_i-E\{X_i\})^2\}}{m^{1-H/2}}$$

(4) $$r^{(m)}(k)=r(k)=$$
$$=\frac{1}{2}[(k+1)^{2-b}-2k^{2-b}+(k-1)^{2-b}]$$

where $r^{(m)}(k)$ is the autocorrelation function for process $X^{(m)}$.

The most striking feature of self-similar processes is that their aggregated processes $X^{(m)}$ possess a non-degenerate correlation structure, as $m\to\infty$ .This is in stark contrast to typical packet traffic models in the literature, all of which have the property that their aggregated processes $X^{(m)}$ tend to pure noise, i.e., for all $k\geq 1$, $r^{(m)}(k)\to 0$, *as* $m\to\infty$.

3.Problem Statement

As was shown in our previous work [6]: queue length in single server system with self-similar input traffic depending on utilization increases much faster comparing with classical Poisson models (Fig.1).

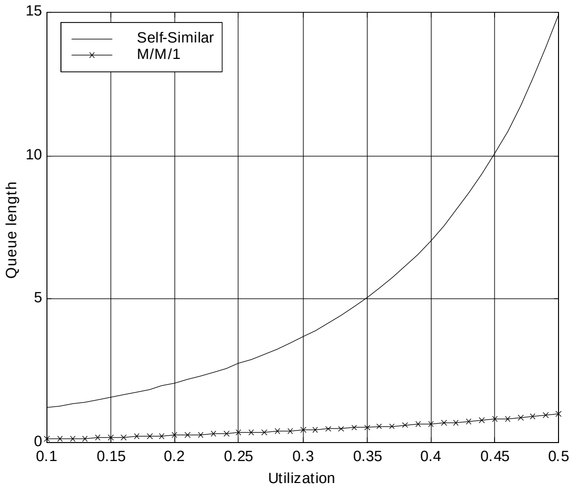

Figure 1. Queue length of M/M/1 system and of single server queuing system with self-similar input traffic

The aim of this work to provide mathematical model of self-similar traffic for purposes of performance analysis of queuing systems. We use maximum entropy technique to describe distribution function of self-similar arrival process. The studying of such non-Markovian queuing system is highly nontrivial and provides fundamental insight into the performance impact question.

A major weakness of existing queuing-based works about self-similar traffic is that they derive asymptotic results for performance measures. Maximum entropy results will be used for performance analysis of single server queuing system with purpose to design traffic control mechanisms. The task is to design control algorithm that allows correlation structure at large time scales to be effectively engaged. Potential problems are:

– arrival process correlation structure exists across multiple time scales;

– information extracted from input traffic is imprecise due to its probabilistic nature.

To resolve theses problems and reach our goal, we need analytically tractable model of self-similar traffic.

4 The Maximum Entropy Formalism

Since the 1970's many attempts have been made to apply the method of maximum entropy in the area of queuing theory and to derive a unique solution for various queuing system [7, 8, 9]. In this section the maximum entropy formalism is introduced.

Consider a system that has a countable set S of possible states with

(5) $p(S_i)>0$

(6) $$\sum_i p(S_i)=1, \qquad i=1,\ 2,...$$

where $p(S_i)$ is the probability of the occurrence of the state S_i. It is desirable to estimate this distribution p based on incomplete information. Assume that information about p is available in the form of expected values of known function f_k: $k=1,2,\ldots,M$ of the states as follows

(7) $$\sum_{S_n\in S} f_k(S_n)p(S_n)=F_k, \qquad 1\le k\le m<\infty$$

Since in general the number of constraints (5) – (7) is less than the number of possible states, its don't precisely identify the distribution p. Actually, exists a number of distribution that satisfy all these constraints. One way of uniquely choose a "right" distribution p is using the principle of maximum entropy. This principle states, that the least biased probability assignment is that which maximizes the system entropy subject to the constraints supplied by the available information. The system entropy is given by

(8) $$H(p,g)=-\sum_{S_n\in S} p(S_i)\ln\left(\frac{p(S_i)}{g(S_i)}\right)$$

where $g(S_i)$ are known a priori information about the distribution. Of course, the values of the prior variables $g(S_i)$ may not all be known apriori. But it may be known that these variables exist. Based on prior knowledge about the states of the system (in addition to constraints) either the value or the type of each $g(S_i)$ may be established This information therefore can be incorporated into the maximum entropy formalism to determine the form of the probability distribution $\{p(S_i)\}$. However this will not determine the numerical values of those still unknown $g(S_i)$ unless they can be computed as aresult of the optimization process.

The maximization of the system entropy subject to constraints can be performed using Lagrange's method of undetermined multipliers leading to the solution:

(9) $$p(S_n)=Z_p^{-1}g(S_n)\exp\{-\sum_k f_k(S_n)b_k\}$$

(10) $$Z_p=\exp\{b_0\}=\sum_{S_n\in S} g(S_n)\exp\{-\sum_k f_k(S_n)b_k\}$$

where $\{b_k\}$ are the Lagrangian multipliers corresponding to the set of constraints (5) – (7).

5 Maximum Entropy Solution for Self-Similar Data Flow

In this section a generalized maximum entropy model is used to derive a unique solution for self-similar data flow. Consider arrival process with *f(n)* as a distribution function of number of arrived packet at time interval *Dt.* This distribution function has following properties:

(11) $$\sum_n f(n) = 1$$

(12) $$\sum_n nf(n) = \bar{n}$$

(13) $$r(k) = \frac{\sum_n (n-\bar{n})(n+k-\bar{n})\, f(n)}{\sum_n (n-\bar{n})^2 f(n)} =$$

$$= \frac{1}{2}[(k+1)^{2-b} - 2k^{2-b} + (k-1)^{2-b}$$

The maximum entropy solution for distribution function *f(n)* is formed by maximizing (8), written as:

(14) $$H[f(n)] = -\sum_n f(n) \ln\{f(n)\}$$

subject to constraints (11)- (13).

For solution estimation we used Langrangian multipliers technique, as also numerical and approximation methods. According to (9) and (10) solution will be

(15) $$f(n) \approx Const \times g(k,b) \sum_{n=0}^{\infty} Const^n \exp\left\{\frac{n \times Const}{Const^n}\right\}$$

Graphically, the result is shown in Fig. 2. For comparison Pareto and Poisson distributions were drawn. The Pareto distribution was chosen, because it is most often used distribution for modeling self-similar traffic.

Comparing with traditional Poisson distribution obtained maximum entropy distribution exhibits long range dependence of corresponding stochastic process.

Since maximum entropy corresponds to maximum disorder in the system, this solution is the least biased of all solutions that satisfy the system constraints.

Difference with Pareto distribution may indicate that some important physical constraints have been overlooked or misunderstanding. To verify this, we will plan to use simulation and measurement. Experimental agreement with maximum entropy distribution will represent evidence the constraints of the system have been properly identified.

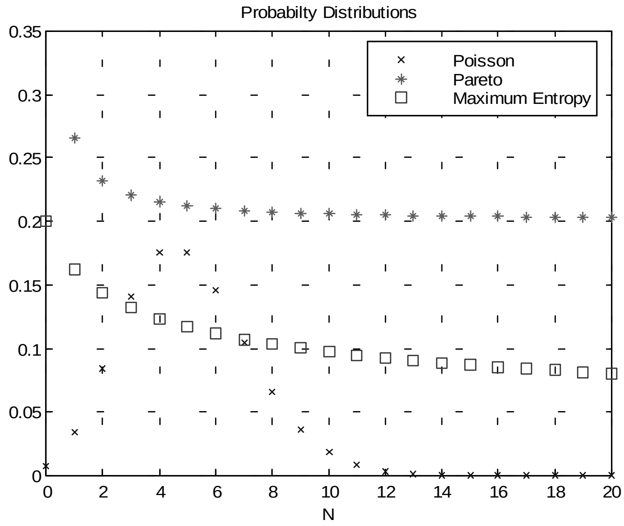

Figure 2. Estimated Maximum entropy distribution for self-similar traffic

6 Conclusions

The purpose of our work to find a means to study impact of self-similar data traffic upon telecommunication system using analytical queuing system. Development of analytical models to study self-similar traffic posses a serious challenge for researchers. The problem lies in the difficulty to obtain exact equation for distribution functions.

In this work, we showed that maximum entropy method can be employed for qualitatively and quantitatively study the performance implications of self-similar data traffic.

The significance of the maximum entropy analysis is that it of allows to build closed form analytical expression for arrival distribution self-similar data flow. In further work is planned to use more constraints, derived from physical backgrounds of self-similar processes.

Obtained results will be employed in queuing system analysis. This approach will help to understand how self-similarity arises in real data communication networks. Also this results provides new insight into the impact of system characteristics such as arrival distribution on performance measures. Moreover, it enables to state the domain of validity of asymptotic results.

The proposed model can be used to simulate self-similar traffic, thus enable network expert to analyze traffic loads, predict and avoid congestion, and test offered quality of service.

Measurements and simulation results still will be used to validate maximum entropy solution. Maximum entropy distribution agreement with these results represents evidence that the constraints of the system have been properly identified.

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