APPLICATIONS OF RESAMPLING APPROACH TO STATISTICAL PROBLEMS OF RELIABILITY SYSTEMS

Alexander Andronov, Maxim Fioshin

Transport and Telecommunication Institute,
Lomonosova 1, Riga, LV-1019, Latvia.
Ph: (+371)-7100650. Fax: (+371)-7100660. E-mail: lora@mailbox.riga.lv, mf@tsi.lv.

Key words: Resampling, Simulation, Nonparametric estimation, Confidence Intervals

Let R is real space, M {1, 2, …, m}- set of integers, I_l and I^r, $l, r \in \{1, 2, \ldots\}$- subsets of M. Also $f_l(x_i: i \in I_l)$, $l = 1, 2, \ldots$ are real-valued functions of real variables $\{x_i: i \in I_l\}$, those define the $s(l)$-th order-statistics ($s(l) = 1, 2, \ldots$) of $\{x_i: i \in I_l\}$:

$$f_l(x_i: i \in I_l) = \min\{x_i \in \{x_i: i \in I_l\}: |\, x_j \in \{x_j: j \in I_l\}: x_i \geq x_j \,| \geq l\}$$

where $|A|$ is cardinal number of A.

In particular if $s(l) = 1$ then $f_l(x_i: i \in I_l) = \min\{x_i: i \in I_l\}$, if $s(l) = x_{k_l}$, where $k_l = |\{ x_i: i \in I_l \}|$, then $f_l(x_i: i \in I_l) = \max\{x_i: i \in I_l\}$.

Let us also define predicates $P_1^{r_1}, P_2^{r_2}, \ldots$,` where upper index r_i means that range of P_i definition is R^{r_i} : $P_j^r(x_i: i \in I_j^r)$. We consider predicates of the following types: 1) "less (great) than": $P^2(x_1, x_2)$ = "$x_1 \leq x_2$"; $P^{r+1}(x, x_1, \ldots, x_r)$ = "$x \leq \min\{x_1, x_2, \ldots, x_r\}$"; 2) "equal": $P^2(x_1, x_2)$ = "$x_1 = x_2$"; $P^r(x_1, \ldots, x_r)$ = "$x_1 = x_2 = \ldots = x_r$". Note that we are able to use constants from R, real variables $x, x_1, \ldots$ and functions $f_1, f_2, \ldots$ as variables of the predicates.

Also, we consider logical function over predicates, for example, disjunction $\vee$, conjunction &, negation $\bar{\ }$, "at least l from k":

$$P^k(y_1, y_2, \ldots, y_k) = \begin{cases} 1 \text{ if } y_1 + y_2 + \ldots + y_k \geq l, \\ 0 \text{ otherwise,} \end{cases}$$

where $\forall\, y_i \in \{0, 1\}$.

Now we construct formula $Q(x_1, x_2, \ldots, x_m)$. It is complex predicate that can contain primary predicates and logical function.

In fact real arguments are independent random variables $X_1, X_2, \ldots, X_m$ with unknown distribution functions $F_1, F_2, \ldots, F_m$. Our aim is to estimate mathematical expectation of formula Q:

$$\theta = E\,Q(X_1, X_2, \ldots, X_m) = \int \ldots \int Q(x_1, x_2, \ldots, x_m)\, dF_1(x_1)\, dF_2(x_2) \ldots dF_m(x_m)$$

on base of sample populations $\{ \mathbf{X}_i = \{X_1^{(i)}, X_2^{(i)}, \ldots, X_{n_i}^{(i)}\}\}$ for all $\{X_i\}$.

Note that various problems of logical control, reliability etc are described by this model. We use resampling approach for our aim. The point estimation of parameters of interest has been considered in the previous papers of authors. Now we consider problem of interval estimation. By this we use approach from paper (Andronov, 2002).

References

1. Andronov A. (2002). On Resampling Approach to a Construction of Approximate Confidence Intervals for System Reliability. In: *Third International Conference on Mathematical Methods in Reliability. Methodology and Practice.* June 17-20, 2002, Trondheim, Norway, Norwegian University of Science and Technology. P. 39-42.