STOHASTISTIC MODELING AND OPTIMIZATION OF INDUSTRIAL STOCK

Vitalijs Jurenoks, Vladimir Janson,
Faculty of Engineering Economics
Riga Technical University
Kalku iela 1, Riga, LV1658, Latvia
vitalijs.jurenoks@rtu.lv
vladjans@latnet.lv

KEYWORDS
Modeling, stock management, optimization, scenario analysis

ABSTRACT

The nonlinear stochastic model of updating of industrial stocks is considered in the paper. The purposes of work are:
1. To define an optimum strategy of updating of industrial stocks and reductions of the expenses connected to their storage;
2. To combine the modeling process with the real information describing the process of industrial stocks management in real time;
3. To take to account the nonlinearity dependences of factors of the model;
4. To construct the management process in online regime.
In the first scenario for incidental values modeling is used traditional methods, in particular, the method of the reverse transformation the Bellman's method (receipt-refusal). In the second scenario of modeling the authors have used modeling methods that allow to consider specific characteristics of changes of value P - order frequency (is measured as units of orders per unit of time), namely, irregularity of consumption intensity, different lengths of intervals between order points, inability to select an appropriate rule of distribution of value P for the whole modeling time interval.
For resolving the problem under given conditions, the method of imitation modeling was applied, which allows to develop (imitate) different options of organization of the process of stock management, taking into account the aforementioned specific characteristics of the particular scenario.

BASIC MODEL OF STOCK MANAGEMENT

In any stock management system the level of stock changes in accordance with the respective cyclical model. The reduction of the level of stock is determined by the demand. At a specific moment, to replenish the stock after a definite period of time, termed delivery lead-time, a new order is made; the order is received and the stock is increased. After that a new stock cycle begins.

To simplify stock management modeling a, number of conditions are established:

1) The demand for products is constant. If the consumption coefficient is constant, then the stock level is also reducing at a constant rate.
2) It is assumed that delivery time is known and is a constant value, which means that the order may be made at the point corresponding to definite time parameters and stock amount (replenishment level) values, which ensure receipt of the required stock at the moment when stock level is 0;
3) Stock-out is not allowed;
4) A definite y amount of raw materials and materials is ordered during the stock cycle time.

The basic model of stock management is presented in fig. 1. All stock cycles are equal. The maximum amount of raw materials and materials available in stock correspond to order amount y.

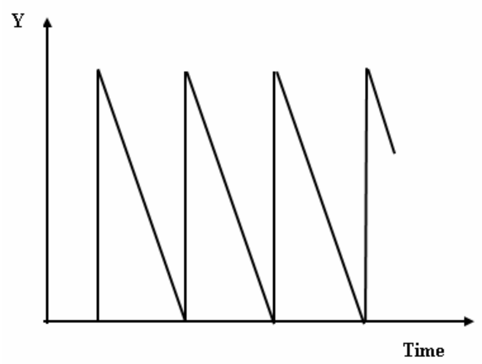

Figure 1. Scheme of the basic stock management model

A model has to be developed to describe costs over the whole stock storage period; the length of the period is not important – it may range from one day up to a year etc. In the particular case a period of one year is chosen and the following system designations are used:

D - annual demand for product;

C_o - variable costs of one order, n units / 1 order;

C_h - variable storage costs per one existing stock unit, n units per product unit per year;

C - acquisition price of one existing stock unit, n units per year;

y - order size in product units.

Proceedings 19th European Conference on Modelling and Simulation
Yuri Merkuryev, Richard Zobel, Eugène Kerckhoffs © ECMS, 2005
ISBN 1-84233-112-4 (Set) / ISBN 1-84233-113-2 (CD)

If the demand for the products is D units per year, but the size of each batch is y number of units, the annual demand is D/y.
The annual cost of an order (C_a) is equal with the costs of one order multiplied with the number of orders made in a year:

$$C_a = C_0 * D/y. \qquad (1)$$

When calculating annual stock storage costs, usually the average amount of products made from the stock during one cycle is taken into account. In the simplest case the level of stock reduces in a straight line from y till 0, thus the average level of stock is y/2.

Different companies use different C_h cost calculation methods, however, on the whole, C_h is expressed as a percentage of the cash loan amount that is frozen in the form of stocks, the costs of ensuring stock security or costs of damages, as well as costs of the stock storage system.

Annual storage costs of stocks C_s are equal with annual variable storage costs of one product unit C, multiplied with the average amount of stock per year:

$$C_s = C_h * y/2. \qquad (2)$$

All the aforementioned allows to state that total annual costs of the stock of one product unit (TC) is established as follows:

$$TC = C_0 * D/y + C_h * y/2. \qquad (3)$$

This equation is called the equation of total costs of the basic stock management model. To establish the optimum order size (y_0), the equation of total costs of the basic stock management model is differentiated. As a result of differentiation and numerous transformations it is possible to obtain the following formula allowing to establish the optimum order size:

$$y_0 = \pm\sqrt{\frac{2C_0D}{C_h}}. \qquad (4)$$

Stock storage costs are proportionate to the size of the order, therefore graphically it is depicted as a straight line going out from the starting point of the coordinate axis. At the same time, order costs are proportionate to the value 1/y.
If the order amount is not very big, then order costs are dominant – in this case orders are placed more frequently, but in small amounts. If the amount of product is comparatively big, the main component of total costs is storage costs – a small number of orders are made for big product amounts.

Annually, after equal time periods, it is necessary to place D/y orders, this means that a new order cycle always starts at point:

$$\frac{1}{D/y} = y/D. \qquad (5)$$

As all order cycles are equal, then interval of repeat orders will also be equal with (y/D) years.
More simple models of stock management are characterized by constant demand, instantaneous replenishment of stock, and non-existence of stock-out.
The designations used are:

P – order frequency (is measured as units of orders per unit of time);

t_o – order cycle time (measured in units of time).

The level of stock changes dependent on the function (fig. 2), which showing also the above-mentioned designations.

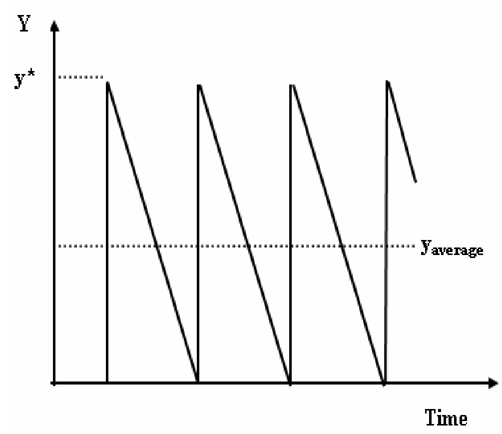

Figure 2. Stock levels

The stock units are replenished instantaneously, when the level of stock reaches 0.
After that stock is used with constant frequency P.
The length of the order cycle in this case may be calculated as follows:

$$t_0 = \frac{y}{P} \text{ units of time.} \qquad (6)$$

Average stock level ($y_{average}$) is established as follows:

$$y_{average} = \frac{y}{2} \text{ units.} \qquad (7)$$

Total cost per unit time (TCU) may be expressed as the following function:

$$\mathbf{TCU} = \frac{C_a + C_h * (\frac{y}{2}) * t_0}{t_0} = \frac{C_a}{\frac{y}{P}} + C_h * \frac{y}{2}. \quad (8)$$

The optimum size of order y is established by minimizing the function TCU by y. Assuming that y is a continuous value, it is possible to obtain the conditional minimum (in the form of an equation), which allows to derive an optimum value y:

$$\frac{dTCU}{dy} = -\frac{C_a P}{y^2} + \frac{C_h}{2} = 0. \quad (9)$$

This condition is sufficient, since the function TCU is a curve. The result of the equation determines the most economic size of the order y*.

$$y^* = \sqrt{\frac{2C_a P}{C_h}}. \quad (10)$$

The optimum stock management strategy for this model is formulated as follows:

It is necessary to order $y^* = \sqrt{\frac{2C_a P}{C_h}}$ product units after every $t_0^* = \frac{y^*}{P}$ units of time.

In real life, replenishment of stock cannot be effected instantaneously at the moment of placing the order, as it was thought earlier.
To make the situation more realistic, there is a condition that the positive time period for fulfilling the order is L (time deflection), which covers the time from placing the order up to physical delivery, as shown in fig.3.
In this case the order point is the instant, when the level of stock has dropped to LP units.

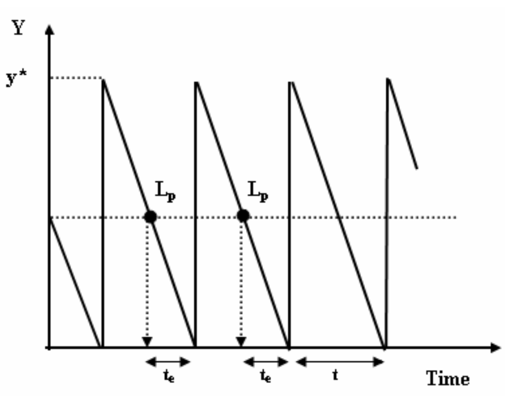

Figure 3. Stock replenishment points

Effective time L_e is established:

$$L_e = L - nt_0^*, \quad (11)$$

where n – the biggest round figure that does not exceed $\frac{L}{t_0^*}$. Such a solution is explained as follows: after n cycles (each of them is $t_0^* = \frac{y^*}{P}$ long) the stock management situation becomes the same as the situation when the interval between placement of one order and receipt of another one were Le.
Consequently, the order point is an instant, when the level of stock reaches LeP product units.
Thus the stock management strategy may be formulated as follows: it is necessary to order $y^* = \sqrt{\frac{2C_a P}{C_h}}$ product units, when the level of stock drops to LeP product units.

The above algorithm of optimization and calculation of order point is traditionally and most frequently used in real-time planning. In reality the function, from the point of view of order TCU, is to a great extent dependent on the value of P – frequency of orders. In order to consider this impact, the authors make an assumption about the incidental character of behavior of value P. Two modeling scenarios are considered:
Given the value P, the rule of distribution is known as F, i.e. P~F (F – distribution function). There is the following interdependence:

$$TCU = \Phi(P, t, t_0, \omega), \quad (12)$$

where w – incidental parameter.

The rule of distribution of value P is not known and it is necessary to model a different character of behavior of value P.
In the first scenario there is a sufficiently well developed mathematical mechanism of modeling and it is necessary only to implement and analyze the data obtained to meet the goals of optimization. When modeling incidental values, traditional methods are applied; in particular, the method of the reverse transformation the Bellman's method (receipt-refusal).
For theoretical modeling the authors have used MS Excel and the program MathCad. The analysis of the modeling results allows to choose the most optimum dynamic mode of replenishing the stock, required raw materials and materials, as well as to minimize total costs TCU, i.e. to choose an optimum strategy for stock management in a stochastic case (the first modeling scenario).
In the second scenario of modeling the authors have used modeling methods that allow to consider

specific characteristics of changes of value P, namely, irregularity of consumption intensity, different lengths of intervals between order points, inability to select an appropriate rule of distribution of value P for the whole modeling time interval.
For resolving the problem under given conditions, the method of imitation modeling was applied, which allows to develop (imitate) different options of organization of the process of stock management, taking into account the aforementioned specific characteristics of the particular scenario.
The process of development and implementation of the imitation model implies the following simplified algorithm (see fig. 4):

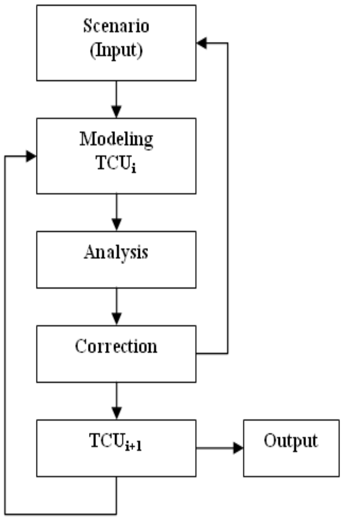

Figure 4. Algorithm of the implementation of the second scenario

Imitation modeling is one of the most widespread methods of research of economic objects and systems.
The selection of methods of modeling of the object under consideration depends on a great number of conditions (modeling components), e.g.
complexity of the object or system being researched:

- the character of behavior of the object or system;
- the character of behavior and the impact of the factors on the changes of the entity or economic system being investigated;
- other similar conditions.

Where the relations between separate components forming the model are comparatively simple and can be accurately described, analytical models can be used for obtaining the required information.
However, most of the economic processes and systems are complex entities, consisting of a great number of interrelated sub-systems (which in their turn also are complex objects and require a detailed study), changing their positions in space and time. For researching such economic systems it is impossible to create an absolutely accurate effective model by applying analytical methods. Fig. 5 presents a process of creation of an imitation model.

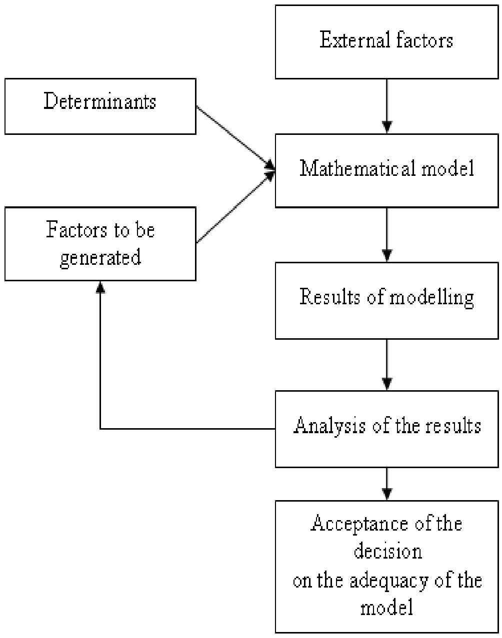

Figure 5. Process of creation of an imitation model

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Imitation modeling is usually applied for researching economic processes and systems. Such factors are called incidental variables or incidental values, and their behavior is described by means of common probability distribution functions.
Imitation modeling may be used for tackling a wide range of economic problems (design and analysis of industrial systems, stock management, balancing of production capacities, allocation of investment funds, optimization of investment funds, optimization of flows of services etc.).
Imitation modeling is frequently associated with the factor of uncertainty, whose description goes outside the confines of the traditional statistical modeling, which, in its turn, complicates the imitation modeling process.
The aim of the research is to find innovative ways of using imitation modeling for investigating economic systems. As a result, it is possible to set the task of creating an efficient procedure for generating incidental parameter values constituting factors of an imitation model, to consider the asymmetric distribution of model factors, to create an adequate

model of non-linear dependence between the factors, to effectively use up-to-date information technologies, to ensure continuous control of the behavior of the specific economic system that is being researched.
The traditional scheme of imitation modeling is the formation (generation) of a mass of incidental parameter values featuring the changes of model factors (see fig. 6).

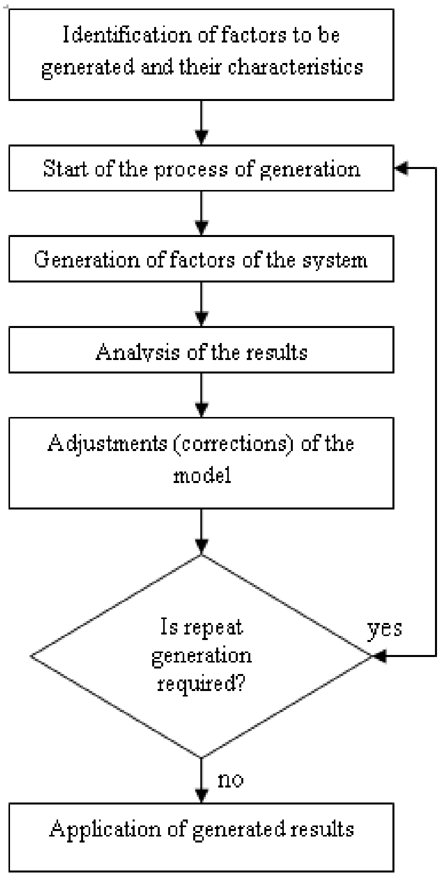

Figure 6. Algorithm of generation of incidental parameters

The algorithm of generation of incidental continuous value X, having continuous distribution function F, can be described in the following steps:

1. Let us generate, within an interval (0.1), an evenly distributed incidental parameter u ~U(0.1).
2. Let us calculate $X = F^{-1}(u)$.

The value of $F^{-1}(u)$ will always be definite, since $0 < u < 1$, but the area of defining the function F is the interval [0,1]. The figure below presents the essence of the algorithm graphically; here incidental value may be assumed to be either positive or negative. This depends on the specific value of parameter u. In the figure, the value of parameter u_1 produces a negative incidental value X_1, but parameter u_2 yields a positive incidental value X_2 (see fig. 7).

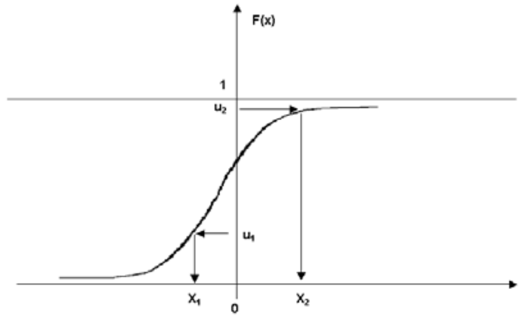

Figure 7. Scheme of reverse transformation

The method of reverse transformation may be also used if value X is discrete. In this case the distribution is as follows:

$$F(x) = P\{X \leq x\} = \sum_{x_i \leq x} p(x_i), \quad (13)$$

where p(xi) is probability $p(x_i) = P\{X = x_i\}$.
It is admitted that incidental parameter X may have only such values as $x_1, x_2, ...$, for which $x_1 < x_2 < ....$
Thus the algorithm of developing the values of incidental parameter X will have the following consequences:

- Let us generate, within the interval (0.1), uniform distributed incidental parameter u ~ U (0. 1);
- Let us establish the least positive round value I, for which $u < F(x_i)$, and assume that $X=x_i$.

Both options of the method of the reverse transformation for continuous and discrete values (at least formally) can be combined in one formula:

$$X = \min P\{x : F(x) \geq U\}, \quad (14)$$

which is true also for mixed distributions (i.e., containing both continuous and discrete components).
In contrast to commonly used direct methods of generating incidental values (the method of the reverse transformation composition and implosion), for imitating the factors of the imitation model it is recommended to use the so-called indirect methods, namely, the acceptance-refusal method. This method may turn out to be suitable if due to certain reasons it is impossible to apply direct methods or if these methods are inefficient.
The “acceptance-refusal” principle is rather common. If the aforementioned algorithm is looked upon from a slightly different perspective, it is clear that it may be extended for generating incidental points in areas having higher dimensions – i.e. in multi-dimensional areas.
This is relevant in modeling real economic systems by applying the Monte Carlo method.
When using the MS Excel program, users usually apply standard functions for modeling incidental parameter values in a dynamic regime.

The variances of the random variables must be finite for the correlation to exist, and for fat-tailed distributions this cannot be the case (a bivariate t-distribution with 2 degrees of freedom, for example). Independence between two random variables implies that linear correlation is zero, but the converse is true only for a multivariate normal distribution. This does not hold when only the marginals are Gaussian while the joint distribution is not normal, because correlation reflects linear association and not non-linear dependency. Correlation is not invariant to strictly monotone transformations. This is because it depends not only on the joint distribution but also on the marginal distributions of the considered variables, so that changes of scales or other transformations in the marginals have an effect on correlation.

In order to overcome these problems we can resort to copula theory, since copulas capture those properties of the joint distribution which are invariant under strictly increasing transformation. A common dependence measure that can be expressed as a function of copula parameters and is scale invariant is Kendall's tau. It satisfies most of the desired properties that a dependence measure must have and it measures concordance between two random variables: concordance arises if large values of one variable are associated with large values of the other, and small ones occur with small values of the other; if this is not true the two variables are said to be discordant. It is for this reason that concordance can detect nonlinear association that correlation cannot see.

The main problems of the research are:

1. To combine the modeling process with the real information describing the process of industrial stocks management in real time.
2. To take to account the nonlinearity dependences of factors of the model
3. To construct the management process in online regime.

CONCLUSION

Application of the imitation modeling allows:

1. To implement a continuous dynamic control of parameter TCU and, consequently, allows to optimize the process of industrial stock management;
2. To consider the inconsistent and frequently unpredictable behavior of value P, which is especially relevant for small and medium-size enterprises, which do not have guaranteed orders and are heavily dependent on the market conjuncture fluctuations.

The received theoretical results of work can be used in practical activities of the industrial enterprises, using standard programs of modeling.

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AUTHORS BIOGRAPHIES

VITALIJS JURENOKS was born in Riga, Latvia and went to Riga Technical University to the Faculty of Engineering Economics when he obtained his degree in 1976. He worked ten years for industrial company in Riga. From 1986 he is working in Riga Technical University where he obtained doctor degree in economical science in 1987. The main field of research is planning and optimization of economical processes and systems. His e-mail address is: vitalijs.jurenoks@rtu.lv.

VLADIMIR JANSON was born in Daugavpils, Latvia and went to Latvian University of Latvia, where he studied mathematical science and obtained his degree in 1970. He worked eight years for Latvian University Computational Centre. From 1978 he worked in Latvian Technical University where he obtained doctor degree in mathematical science in 1983. The main field of research is simulation and optimization of economical systems. His e-mail address is: vladjans@latnet.lv.