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**INVESTIGATION OF THE ELECTRICAL MACHINES  
SYSTEM'S DYNAMICS WITH A METHOD OF  
MATHEMATICAL SIMULATION**

**Extended Abstract of Doctoral Thesis**

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## **TOPICALITY OF THE WORK**

Creation and study of mathematical models of electric machines open new perspectives in the electric machines study. Possibility to substitute a real object by its mathematical model offers great advantages for the electric machines study. Computational experiment is meant for the complicated multiparameter nonlinear processes study and optimization, the study of which by traditional methods is difficult or impossible. Field observations of the electric machines demand creation of pilot samples, considerable financial expenditures, and they do not secure those potentialities, which a computational experiment offers.

Mathematical modeling assisted by modern computer technologies allows studying, with adequate accuracy, steady-state and transient processes in the separate elements as well as in the system as a whole.

It is known that the transient processes study even in the separate element of the electric power system (EPS) - generator, motor etc. - is connected with considerable difficulties since the behavior of each element of the system in dynamics is described by the system of the nonlinear differential equations of high order. These difficulties increase when the elements are united in the power system.

Today, improving of the project design quality can be reached only on the basis of wide application of computer engineering. The mathematical model of the electric power system allows, on the basis of the use of modeling aids and computer engineering, to assess the dynamic properties of the electric power system in the normal mode of operation as well as malfunction, which often cannot be reproduced experimentally in real plants because of economic considerations or exceptional working conditions.

In view of the tendency to increase the operating speed of various kind of switching and protective equipment there increases the necessity for a more precise definition of the electric machines parameters in order to choose protective, control and commutation apparatuses.

The analysis conducted has shown that the well-known software complexes, which are oriented towards modeling of the transient processes of the electrical machines systems, do not satisfy the goals of the refined definition of the electric machine parameters, which are included into the system. Majority of programs is oriented towards the analysis of a long-term mode (from some seconds) but they do not reflect sufficiently passing of the electromagnetic transient processes in the stator circuit at the initial stage of the transient process.

## **OBJECTIVE OF THE WORK**

The objective of this work is:

- > creation of the functional models of the alternating current electric machines, which structurally consist of generators and consumers united in a uniform system;
- > development of the research techniques of the electrical machines system dynamics, which is based on the application of the method of structured modeling, using complete Park-Gorev's equations, that offers an opportunity to study the electromagnetic transient processes in the electric machine stator circuits, taking into consideration their mutual influence;
- > introducing of the elements models into the functional model of the power system in order to study dynamics of the electrical machines power system by means of mathematical modeling.

It has been necessary to solve the following tasks in order to achieve the objective:

- selection of the way how to unite the elements in system;
- re-arrangement of the system of the differential equations of the electric machines in concordance with the form that is convenient for the elements inclusion into the system;
- development of the functional models of the alternating current electric machines;
- selection of the method of numerical integration of the system of nonlinear differential equations.

## **SCIENTIFIC NOVELTY**

- logical design of the algorithm how to set up the multimachine system equation on the basis of the elements complete equations, using the structural principle of the model construction of the studied multimachine system;
- development of the method for the generation of the equations of the element connection in the system;
- the applied principle of algorithmization allows composing and study the multimachine power systems of any configuration.

## **RESEARCH METHODS**

The following means and methods have been applied in the research:

- > Structural modelling;
- > Mathematical models of electrical machines have been worked out on the basis of similarity theory methods, electric machine theory;
- > Park-Gorev's differential equations;
- > Universal methods of numerical integration of non-linear differential equation systems;
- > Programming language FORTRAN and Microsoft Exel as well.

## **CONTENTS OF THE WORK**

### **1. MODELING OF THE ELECTROMECHANICAL SYSTEMS**

Two present methods of modeling as well as the integrated mathematical systems for solution of a quantity of tasks in the EPS processes modeling have been compared in the first part of the work. There are general and special principles of the algorithmization processes in the alternating current electric power system, given in the work and the well-known methods of the electric power system equation generation as well as difficulties that emerge during this process, like selection of the coordinate axes system, realization of the coordinate's transformation.

#### ***1.1. Review of the Present Methods of Modeling***

A series of software complexes, for has been developed modeling of various operating modes of the electric power systems, including autonomous electric power systems. Among them one can mention: EUROSTAG, MATLAB SIMULINK, MUSTANG etc.

One should note that during designing of the programs algorithms it is necessary to meet the requirements which are contradictory in a great degree. They include:

- sufficient completeness of the model of the mathematical modeling of the objects;

- considerable size of the network, including some hundreds of generating stations and thousands of junctions and branches;
- maximal operating speed of the programs;
- satisfactory accuracy of numerical methods.

Since it is impossible to meet all these requirements, program developers have to resort to various kinds of limiting and compromises. In this connection, there have come forth two directions in the development of transient processes. To the first one, we can attribute universal programs design meant for the analysis of stability of the power systems electrical machines models. They are meant mainly for design and working organizations. When they are created, the principal factor is a possibility to model bulky structures and the programs operation speed. These qualities can be achieved owing to the use of the simplified generators models, exciting regulators, load of electric system as well as application of the comparatively crude methods of numerical integration.

The second direction is connected with the analysis of the simple structural models of the power systems, when there is a possibility to give a more precise definition to some part of the mathematical models and get a clearer idea about the separate details of the transient process behavior. Such solutions are used for comparative assessments as well as for workout of algorithmical solutions, which are used later in complicated programs.

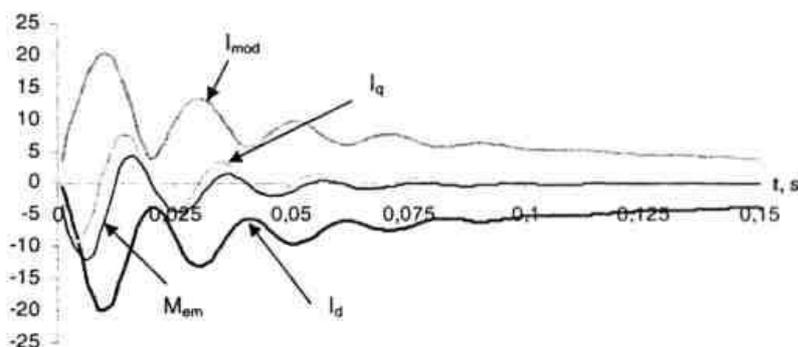
The transient processes study in the big electric power systems, when there are jumping disturbances in the system, generally, are connected with the solution of an ample quantity of nonlinear differential equations of high order. The equations become complicated in a great degree when there are taken into consideration the magnetic circuit saturation. For this reason, in the majority of programs, one can try to introduce the simplifications, which lower the order of the differential equations. As a result of this:

- > the equations of the synchronous generator are given without taking the aperiodic component of the stator current into consideration, it is admissible to have equality of subtransient resistances along the longitudinal and transverse axes;
- > the equations of the asynchronous motor are given without taking the active resistance of the stator circuit into consideration, or ignoring the aperiodic component of the stator current, or disregarding the electromagnetic transient processes.

In this way, the change of the currents at the initial stage of the transient process is not taken into consideration.

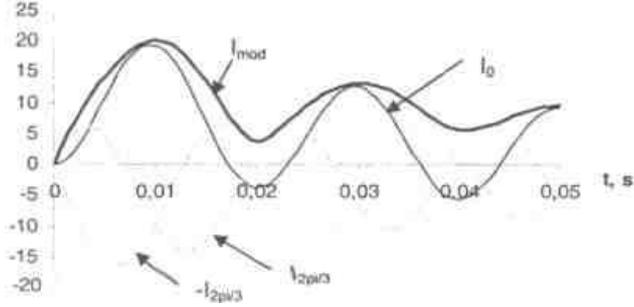
Fig. 1.1. demonstrates  $I_d$ ,  $I_q$  — the curves of current components of stator of a synchronous generator,  $I_{mod}$  is the module of stator current and electromagnetic torque, obtained from the short-circuit transients calculations under the complete Park-Gorev's equations by numerical integration according to Runge-Kutt.

Hereafter the values of all the diagrams are given in relative units (r.u.).



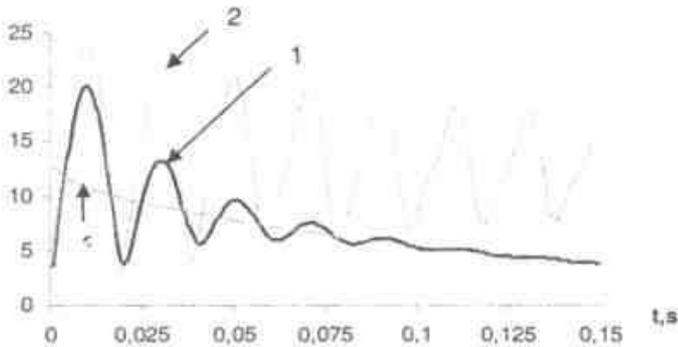
**Fig. 1.1. Components of stator current, current module and electromagnetic torque from the short-circuit transients.**

Figure 1.2 presents the current comparison, which was calculated in the model according to the complete Park-Gorev's equations with the current change in the phases, calculated analytically. From the comparison one can draw a conclusion that the current change character, which was acquired in the model, practically there is an enveloping curve of the phase currents, and that proves the calculations accuracy in the model. Divergence of the results can be explained by computational miscalculation, and it lies in the allowed limits.

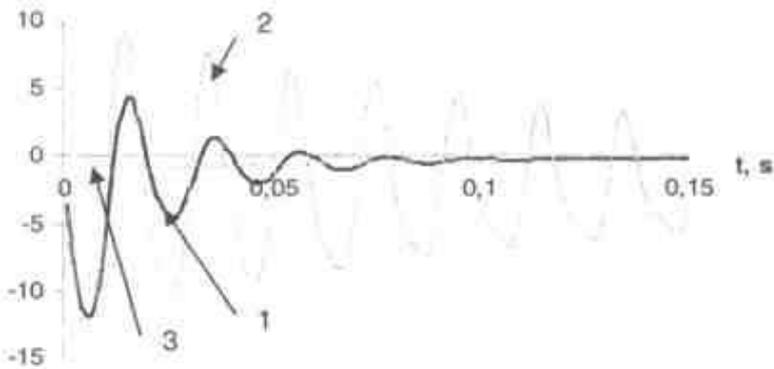


**Fig. 1.2. The current comparison, which was calculated according to the complete equations with the current change in the phases, calculated analytically.**

Possibility of the simplified models application comes from the analysis of the diagrams of currents in fig. 1.3 demonstrating for comparison the curves of currents changes by according to full equations, simplified equations and with disregarding of a stator windings active resistance. Similar characteristics of electro-magnetic torque are given in fig.1.4.



**Fig. 1.3. The curves of variation of stator current module by the full algorithm (1), ignoring the active resistance (2), provided a periodical component of stator's current (3).**



**Fig. 1.4. The curves of variation of electromagnetic moment by the full algorithm (1), ignoring the active resistance (2), provided a periodical component of stator's current (3).**

Disregarding of the stator active resistance completely distorts a quantitative and qualitative picture of the process. Disregarding of the aperiodic component offers an opportunity to receive the qualitative process assessment as well as the steady-state value of the short-circuit current, if the fault was not cleared in time.

This way, in the work, on the basis of the conducted comparison it is proposed to make up the elements models according to the complete Park-Gorev's equations and do not use ready-made, standard programs sets. All the above mentioned software packages solve their own problems of modeling of the electric power systems but they do not offer opportunity to track the electromagnetic subtransient processes at their initial stage (processes with duration starting from some fractions of a second), and these processes are very important because they influence the choice of the high-performance protection equipment. Furthermore, it is necessary to take into consideration the direct influence of one machine on the other one, which are connected with the stator circuits, when various transient processes are modeled, i.e. to study *the dynamics of the multimachine system*.

## ***1.2. Selection of the Coordinate Axis System***

If we know the differential equations of the separate elements of the electric power plant we can set up a general system of equations, which describes behavior of the transient and steady-state processes. Before we directly set up a general system of equations of the multimachine system, it is necessary to determine in which axes it is expedient to write the equations of the separate elements.

When we choose the system of rotating coordinate axes the general principle is maximal simplification of the whole system of the transient processes equations in the electric power system. The totality of the synchronous machines equations, asynchronous motors, static load and constraint equations should be as simple as possible (minimal number of periodic coefficients as well as terms that characterize rotation electromotive force).

As rule, the synchronous machines are those elements, which determine an efficient choice of coordinate system. The equations of the synchronous machines should be written in axes  $d, q, 0$ , which are rigidly bound with the appropriate rotors of the machines. That circumstance allows unambiguously to decide a question about inexpediency of introduction of the EPS synchronous machines equations in general coordinate axes  $d, q, 0$ , which rotate at synchronous speed, as well as in coordinate axes  $d, q, 0$ , which are rigidly bound with one of synchronous machines.

## ***1.3. Align of the variables given in different co-ordinate systems***

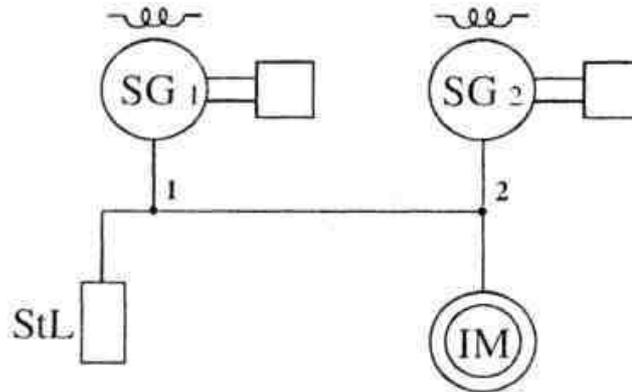
The EPS structured modeling is realized by means of connection of the mathematical models of the system elements into the united whole. With all this going on, there should be secured boundary conditions: the appropriate transformation of the coordinates, it is necessary because of application of the structured modeling method. As is well known, the coordinates matching, when we use the modified equations, is secured in the constraint equations.

Generally, considering electromechanical transient processes of a system, represented with an arbitrary number of synchronous machines there is a possibility of appearance of an error angle of the rotors of the synchronous machines. At the same time their angular rotational frequencies will also differ. Modeling an arbitrary multimachine system the number of coordinate systems is defined with the number of synchronous machines independently on that whether they operate in generator or motor regimes.

This fact excludes the realization of Kirchoff's law in a mode point in the following ready:

$$\sum |I|_i = 0.$$

The direct sum of the currents could be possible if the currents of all elements were represented in one and the same system of co-ordinate axes.



**Fig.1.6. Design of an investigated system (SG-synchronous generator, ID-induction motor, StL-sattic load).**

The research suggests to unite the elements on the basis of equation system of the stator circuits elements. The main principle of the method development is given in Chapter 3. with this aim the method of detachment of static load mode on the output of synchronous generator is suggested. This method allows to define the voltage components on the input of the following next element of the multimachine system.

If to suppose that the current of the static load is attached to the first co-ordinate system ( $SG_1$ ), then there is a necessity of reducing of the second generator's current to the first co-ordinate system (Fig.1.6).

In such a way, the most difficult part of the algorithm composition is the coordinates aligning, build-up of the constraint equations and their solution with regard to the voltages in nodes.

## **2. GENERAL QUESTIONS OF ALTERNATING CURRENT ELECTRICAL MACHINES SIMULATION**

In the second part of the work general issues concerning simulation, the structure of simulation process, the history of the development of the electric machines mathematical models as well as the grounds of the choice for the numerical integration method of the systems of the nonlinear differential equations are considered.

The tasks of the mathematical formulation of the processes going in the alternating current machines can be solved only with some approximation. With all this going on, the study in real complicated conditions is substituted by one with simpler ideal conditions. If we do not make any assumptions, the analytical treatment becomes much more complicated, and sometimes it becomes too bulky, even if we use modern computer technologies. And in the contrary, when we make crude assumptions, it is possible to distort the principle phenomena in the process under study. It is not always possible to conduct the analytical treatment in a real electric machine.

Therefore, for mathematical modeling it is necessary to take into consideration only principal factors, ignoring minor ones. Many years' experience of the study of mathematical models of the electric machines offers to use so-called idealized EM. However, certain assumptions, which idealize the machine, within the limits of acceptable deviations allow retaining, a real picture of processes, which go by in a real machine. At that, one can manage to receive sufficiently precise equations for the electrical machine, which are quite acceptable in a sense of adequacy of solution for engineering calculations.

When we carry out the research, calculate or model the transient processes in the EM, or when we solve the systems of nonlinear differential equation, problem definition determines the choice of the rational method according to its properties. The basic properties of the numerical methods, which determine expediency of some algorithm usage, solving some analysis or synthesis problem, are accuracy, numerical stability, machine time consumption on calculation, the algorithm structure, which allows to use this algorithm to solve a widespread tasks class that are different in a sense of their problem definition. In mathematical support, there are a lot of numerical methods, each of them has its own computational algorithm.

For the cases, when we have to carry out a detailed analysis of the transient processes in the generators, taking into consideration the influence of damper windings, the regulation systems of the turbine capacity and the generator excitation, the calculations are made by means of high order integration methods. They are more stable, and they do not allow considerable error accumulation in the process of calculation. When we describe the processes in the generators by Park-Gorev's equations, application of Runge-Kutt's method of IV order as well as the forecast method and correction of II order at a pace equal to 0,001 s. is recommended.

### **3. THE METHOD OF THE MULTIMACHINE SYSTEM DYNAMICS STUDY BY MEANS OF MATHEMATICAL SIMULATION**

The third part of the work demonstrates the principle of development of method and equations of the elements relations on the scheme example (Fig. 3.1.). There are the models of synchronous and asynchronous machines and static load presented in a way that is convenient for modeling.

#### ***3.1. The Method of the Equations Generation of the Multimachine System on the Basis of the Complete Sentences of the Elements***

The analysis of any system displays, that it can be divided into components: synchronous generator (SG), induction motor (IM), static load (StL) and if to have a possibility of the definition of voltage  $U$  or its components  $U_d$ ,  $U_q$  in a point of connection, it is possible to utilize the offered mathematical models SG, IM and SL, resolved concerning currents. The following algorithm can be in this case offered:

- In central node the voltage is evaluated;
- According to the laws of an electrical engineering this voltage is similar for all elements;
- The calculated voltage will be used in the models of all elements, connected to this node, for an evaluation of parameters of a regime including components of the currents;

- According to the Kirchoff's law (the sum of currents in a node is equal 0) there is the current of an assigning element. Such element can be an active - inductive load, which always can be selected in any system.

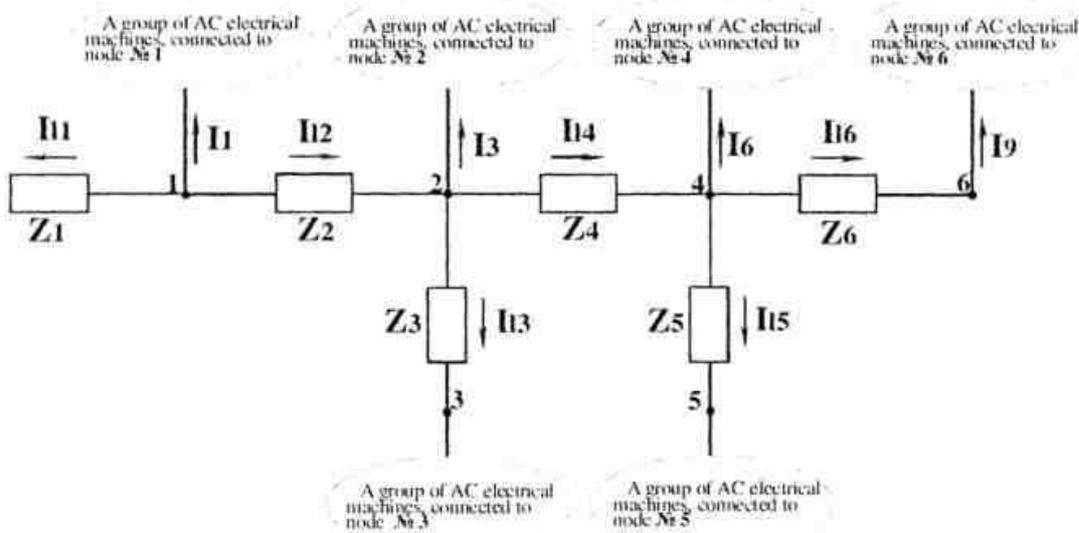


Fig.3.1. Design of an investigated system.

Differential equations are written for all the synchronous generators, taking into consideration the system parameters up to the point of the equivalent load connection.

The differential equations of asynchronous motors are also introduced, taking into consideration the network parameters up to the point of the feeder connection to the bus-bar. The torque of resistance on the drive axes are taken according to the drives nature.

When we use the complete equations of the elements, they are joined into the system on the basis of the equations of the stator circuits.

A voltage of a node point where all the elements of the system are connected, can be defined at active-inductive load:

$$\begin{cases} U_{d1} = R_1 I_{d1} + L_1 \frac{dI_{d1}}{dt} - L_1 \omega I_{q1} \\ U_{q1} = R_1 I_{q1} + L_1 \frac{dI_{q1}}{dt} + L_1 \omega I_{d1} \end{cases}, \quad (3.1)$$

or in matrix form:

$$\begin{pmatrix} U_{d1} \\ U_{q1} \end{pmatrix} = \begin{pmatrix} R_1 & 0 \\ 0 & R_1 \end{pmatrix} \begin{pmatrix} I_{d1} \\ I_{q1} \end{pmatrix} + \begin{pmatrix} L_1 & 0 \\ 0 & L_1 \end{pmatrix} \frac{d}{dt} \begin{pmatrix} I_{d1} \\ I_{q1} \end{pmatrix} + \begin{pmatrix} 0 & -L_1 \omega \\ L_1 \omega & 0 \end{pmatrix} \begin{pmatrix} I_{d1} \\ I_{q1} \end{pmatrix},$$

The same quality in reduced form:

$$\begin{pmatrix} U_{d1} \\ U_{q1} \end{pmatrix} = \begin{pmatrix} L_1 & 0 \\ 0 & L_1 \end{pmatrix} \frac{d}{dt} \begin{pmatrix} I_{d1} \\ I_{q1} \end{pmatrix} + \begin{pmatrix} R_1 & -L_1 \omega \\ L_1 \omega & R_1 \end{pmatrix} \begin{pmatrix} I_{d1} \\ I_{q1} \end{pmatrix}.$$

In the reduced designation in letters the same equality will have the following form:

$$U_1 = L_1 \frac{d}{dt} I_{11} + Z_1 I_{11} \quad (3.2)$$

$$\text{where } z_1 = \begin{vmatrix} R_1 & -L\omega \\ L\omega & R_1 \end{vmatrix}.$$

Let's switch on a master resistance defining the voltage in  $Z_1$  the node point. Resistances  $Z_2 - Z_6$  correspond to the resistances of jumpers between the node points. We suppose that the coordinate system in every node point is the only one. Moving from the last point to the first one, we will formulate the lines currents values through the currents of the nodes elements:

$$- I_{16} = I_{\Sigma 6}, \quad (3.3)$$

$$- I_{15} = I_{\Sigma 5}, \quad (3.4)$$

$$I_{14} = I_{\Sigma 4} + I_{15} + I_{16} = I_{\Sigma 4} + C_{45} I_{\Sigma 5} + C_{46} I_{\Sigma 6}, \quad (3.5)$$

$$I_{13} = I_{\Sigma 3}, \quad (3.6)$$

$$I_{12} = I_{\Sigma 2} + I_{13} + I_{14} = I_{\Sigma 2} + C_{23} I_{\Sigma 3} + C_{24} I_{\Sigma 4} + C_{25} I_{\Sigma 5} + C_{26} I_{\Sigma 6}, \quad (3.7)$$

$$I_{11} = -I_{\Sigma 1} - I_{12} = -(I_{\Sigma 1} + C_{12} I_{\Sigma 2} + C_{13} I_{\Sigma 3} + C_{14} I_{\Sigma 4} + C_{15} I_{\Sigma 5} + C_{16} I_{\Sigma 6}), \quad (3.8)$$

where  $C_{ij}$  - the matrixes of the values transformation, which are obtained in the coordinate axes of point 1, reduced to the coordinate axes of point 2, from point 2 to point 3 and so on:

$$[C_{ij}] = \begin{bmatrix} \cos \delta_{ij} & -\sin \delta_{ij} \\ \sin \delta_{ij} & \cos \delta_{ij} \end{bmatrix}.$$

Using equation (3.8), we will take the time derivative of the current:

$$\begin{aligned} \frac{d}{dt} I_{11} = & -\left( \frac{d}{dt} I_{\Sigma 1} + C_{12} \frac{d}{dt} I_{\Sigma 2} + C_{13} \frac{d}{dt} I_{\Sigma 3} + C_{14} \frac{d}{dt} I_{\Sigma 4} + C_{15} \frac{d}{dt} I_{\Sigma 5} + C_{16} \frac{d}{dt} I_{\Sigma 6} \right) - \\ & - (C_{12} I_{\Sigma 2} + C_{13} I_{\Sigma 3} + C_{14} I_{\Sigma 4} + C_{15} I_{\Sigma 5} + C_{16} I_{\Sigma 6}), \end{aligned} \quad (3.9)$$

$$\text{where } C_{ij} = \frac{d}{dt} C_{ij}.$$

The expression (3.9) differentiation operation is quite admissible since each summand is a continuous differential function under the condition of complete differential equations using.

Let's find time derivatives from other currents, using expressions (3.3)-(3.7):

$$\begin{aligned} \frac{d}{dt} I_{12} = & \frac{d}{dt} I_{\Sigma 2} + C_{23} \frac{d}{dt} I_{\Sigma 3} + C_{24} \frac{d}{dt} I_{\Sigma 4} + C_{25} \frac{d}{dt} I_{\Sigma 5} + C_{26} \frac{d}{dt} I_{\Sigma 6} + \\ & C_{23} I_{\Sigma 3} + C_{24} I_{\Sigma 4} + C_{25} I_{\Sigma 5} + C_{26} I_{\Sigma 6}, \end{aligned} \quad (3.10)$$

$$\frac{d}{dt} I_{13} = \frac{d}{dt} I_{\Sigma 3}, \quad (3.11)$$

$$\frac{d}{dt} I_{14} = \frac{d}{dt} I_{\Sigma 4} + C_{45} \frac{d}{dt} I_{\Sigma 5} + C_{46} \frac{d}{dt} I_{\Sigma 6} + C_{45} I_{\Sigma 5} + C_{46} I_{\Sigma 6}, \quad (3.12)$$

$$\frac{d}{dt} I_{15} = \frac{d}{dt} I_{\Sigma 5}, \quad (3.13)$$

$$\frac{d}{dt} I_{16} = \frac{d}{dt} I_{\Sigma 6} \quad (3.14)$$

The circuit node voltages we will express through the currents in lines:

$$U_2 = C_{21} U_1 - z_2 I_{12} - L_2 \frac{d}{dt} I_{12}, \quad (3.15)$$

$$U_3 = C_{32} U_2 - z_3 I_{13} - L_3 \frac{d}{dt} I_{13}, \quad (3.16)$$

$$U_4 = C_{42} U_2 - z_4 I_{14} - L_4 \frac{d}{dt} I_{14}, \quad (3.17)$$

$$U_5 = C_{54} U_4 - z_5 I_{15} - L_5 \frac{d}{dt} I_{15}, \quad (3.18)$$

$$U_6 = C_{64} U_4 - z_6 I_{16} - L_6 \frac{d}{dt} I_{16}, \quad (3.19)$$

One should pay attention that in the long run the expressions for the node voltages do not depend on the accepted direction of the lines currents. Furthermore, from the received relations one can see the following regularity:

- reference nodal voltage (in the place of engaging of the master element) in any case will be expressed like a product of the master resistance and the stator circuits currents of all the elements of the allocation scheme;
- voltage drop on the connecting line  $z_{n+1}$  between  $n$  and  $n+1$  node points is in proportion to the currents of all the elements, that are found after  $n+1$  node point.

This rule is true for an arbitrarily complicated allocation scheme of radial type, since any following voltage will be expressed through the previous node point voltage, less voltage drop in the connecting line.

If we insert the value of the current in line  $I_{li}$ ; from (3.8) in equation (3.2) and its derivative from (3.9), and in the equations of node voltages (3.15)-(3.19) the values of the currents (3.3)-(3.7) and their derivatives (3.10)-(3.14), we receive the following equations system, which is presented as cellular matrixes:

$$\begin{pmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \\ U_6 \end{pmatrix} = \begin{pmatrix} 0 \\ C_{21} U_1 \\ C_{32} U_2 \\ C_{42} U_3 \\ C_{54} U_4 \\ C_{64} U_4 \end{pmatrix} \times \begin{pmatrix} z_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & z_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & z_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & z_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & z_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & z_6 \end{pmatrix} \times \begin{pmatrix} 1 & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ 0 & 1 & C_{23} & C_{24} & C_{25} & C_{26} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & C_{45} & C_{46} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} L_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & L_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & L_6 \end{pmatrix} \times \begin{pmatrix} 1 & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ 0 & 1 & C_{23} & C_{24} & C_{25} & C_{26} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & C_{45} & C_{46} \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \times$$

$$\times \frac{d}{dt} \begin{pmatrix} I_{\Sigma 1} \\ I_{\Sigma 2} \\ I_{\Sigma 3} \\ I_{\Sigma 4} \\ I_{\Sigma 5} \\ I_{\Sigma 6} \end{pmatrix} \times \begin{pmatrix} 0 & C_{12}' & C_{13}' & C_{14}' & C_{15}' & C_{16}' \\ 0 & 0 & C_{23}' & C_{24}' & C_{25}' & C_{26}' \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{45}' & C_{46}' \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \times \begin{pmatrix} I_{\Sigma 1} \\ I_{\Sigma 2} \\ I_{\Sigma 3} \\ I_{\Sigma 4} \\ I_{\Sigma 5} \\ I_{\Sigma 6} \end{pmatrix}. \quad (3.20)$$

On the basis of the obtained equation there cannot be calculated the node voltages of the circuit in the function of the node elements summed current, the initial values of which are established at every integration step. This happens because the right part of the equation contains the terms that are in proportion to the time derivatives of the currents of the stator elements.

One should note that we have established the systems of differential equations for each element of the system, solved for the currents time derivatives.

If we accept a certain form how to solve the equations of the system elements, relatively to the currents time derivatives, the dependence that expresses the time derivatives from the vectors projections of the stator currents looks like this:

$$\frac{d}{dt} I = QU + H, \quad (3.21)$$

where

$[H] = [A][I] + [B]$ - for synchronous mashine;

$[H] = [A][I]$ - for induction mashine and static load.

Matrix  $Q$  depends only on the machine parameters, but matrix  $H$  is calculated on the basis of indicated parameters and initial values of the variables. In such a way, the expression (3.21) is a general formula of the differential equations of the elements of the stator circuits, for this reason the derivative of the sum vector of the node point  $n$  with  $k$  elements can be expressed by dependence:

$$\frac{d}{dt} I_{\Sigma n} = \left( \sum_{i=1}^k Q_{in} \right) U_n + \sum_{i=1}^k H_{in} = Q_{\Sigma n} U_n + H_{\Sigma n}. \quad (3.22)$$

If we put the dependence found (3.22) in the expression (3.20), we obtain the algebraic matrix expression of the stator circuits of the allocation scheme, from which we can find the nodal voltages. Applying the reduced record of matrixes, which are included into the expression of voltage, we obtain the expression for the vector of the nodal voltages in the following way:

$$[U] = -([K_1] + [L] [K_2] [Q_{\Sigma}])^{-1} \{ [L] [K_2] [H_{\Sigma}] + ([L] [K_2'] + [Z] [K_2]) [I_{\Sigma}] \}, \quad (3.23)$$

where  $[K_2'] = \frac{d}{dt} [K_2]$

$[K_1]$ ,  $[K_2]$  - the matrixes of coordinates, which reflect the structure of the multimachine system;

$[L]$  - the diagonal matrix of the inductive jumpers;

$[Z]$  - the matrix of the jumpers resistances;

$[Q]$  - the matrix of the parameters of the nodes  $Q_{\Sigma n}$  elements;

$[H_{\Sigma}]$  - the column vector of the elements  $H_n$  which are calculated on the basis of initial values of the currents;

$[I_{\Sigma}]$  - the column vector of the currents of the stator circuits of the machines and loads, which adjoin the nodal points.

With the help of the proposed algorithm, one can model and analyse the normal operation conditions as well as an emergency state, up to the mode close to solid fault ( $z_I=0$ ). When  $z_I=0$  we have solid fault in point 7. Imitation of the analogous failure in other node points, without changing of the point of connection of master resistance, is impossible. However, the modes, which are close enough to solid faults, we can obtain, if we foresee connection in the necessary node points of the passive circuits  $r-L$ .

### ***Special features of basvalues choise modelling of a multimachine system***

The research presents the equations of rotating elements in the system of reference units  $X_{ad}$  or in the system of reference units with the equal resistances of mutual inductivities and magnetizing forces. At a presence of few generators in the system, in most of the cases, their full power is accepted as a base value. This approach results in the necessity to recalculation of the parameters of the EES elements with the changing of the number of generators.

A method of base values choice when such when such necessity is excluded is suggested in the work. For each of the elements, independently on their total quantity, its full power is accepted as a base value. For example, when two generators supply three a motors at a full rated load of all elements, the double current of electric power sources will be equal to the triple current of the load. Obviously, for excluding of this contradiction each of the currents should be reduced multiplying them by some rating factors, the values of which should be defined as a ratio of the full power of the element to the full power of power sources:

$$m_i = \frac{S_i}{\sum S_{SG}}$$

If there is one power source in the simulated system, then a scale factor for the sychror system with more then one generator, a total generated power is to be determined and a scal fractions of this power:

$$m_{SG1} = \frac{S_{SG1}}{\sum S_{SG}}; m_{SG2} = \frac{S_{SG2}}{\sum S_{SG}}; etc.$$

The use of scale factors enables simulating a system, where the commutation of elements takes place. Disconnection of any element of the scheme in the algorithm corresponds to the assignment to scale factor of value equal to zero. To restore the operation mode of these elements or to switch them it is again necessary to set a scale factor value in conformity with an element power fraction from generated power of a system.

Scale factors of load elements:

$$m_{AD} = \frac{S_{AD}}{\sum S_{SG}}; m_{SD} = \frac{S_{SD}}{\sum S_{SG}}; m_{SIL} = \frac{S_{SIL}}{\sum S_{SG}}$$

In order to use the parameters of the electric machines during simulation, calculated in their own basic units, in the expressions for the vector of the node voltages, the matrixes terms are multiplied  $[Q]$ ,  $[H]$  and  $[I]$  by the scale (weighting) parameters. In the reduced designation the equality (3.27) will have the following form:

$$[U] = -([K_1] + [L] [K_2] [mQ_{\Sigma}])^{-1} \{ [L] [K_2] [mH_{\Sigma}] + ([L] [K_2] + [Z] [K_2]) [mI_{\Sigma}] \}. \quad (3.24)$$

The considered method has in addition the advantage of possibility to change the load of the station, varying the values of coefficients (factors).  $m_i = 0$  the influence of a correspondent element on the system is excluded, it is disconnected from the system.

### 3.2. Mathematical model of synchronous machine and calculation of operation modes

As it was noted before, it makes sense to construct the transient processes algorithms on the basis of modified equations, when in the system there are jumping disturbances. The algorithm, which is developed on the basis of modified equations, is simpler because after transformation the number of equations diminishes, and periodic coefficients are substituted by constant ones.

In order to study the transient processes of the rotating electrical machines, including the synchronous, it is necessary to obtain the equations of stator and rotor electric circuits and the equation of rotor movements in the differential form. The form of these equations depends on the choice of coordinates axes as well as the positive direction of current. One can take a generator or motor mode for a principal operating mode. As the usual use of synchronous machines is connected with power generation, let's take the generator mode for a principal mode.

Model of alternating current electric machine in a phase coordinate system:

$$\left. \begin{aligned} u_a &= R i_a + \frac{d\Psi_a}{dt} \\ u_b &= R i_b + \frac{d\Psi_b}{dt} \\ u_c &= R i_c + \frac{d\Psi_c}{dt} \\ u_f &= R_f i_f + \frac{d\Psi_f}{dt} \\ 0 &= R_D i_D + \frac{d\Psi_D}{dt} \\ 0 &= R_Q i_Q + \frac{d\Psi_Q}{dt} \end{aligned} \right\} \quad (3.25)$$

where  $\Psi_a, \Psi_b, \Psi_c$  - are the components of full flux linkages with phase windings  $a, b, c$ ;

$u_a, U_b, u_c$  - are the components of instantaneous values of phase voltages;

$i_a, i_b, i_c$  are the components of instantaneous values of phase currents

$R_a, R_b, R_c$  - are the active resistances of phase windings (in case of stator symmetry

$R_a = R_b = R_c = R$ );

$u_f, i_f, \Psi_f, R_f$  - are the components of voltage at excitation winding, current in it, flux linkage with it and active resistance of this winding.

The equation of machine rotor movement is:

$$M_M = M_{em} + J \frac{d\omega}{dt} \quad (3.26)$$

where

$M_M$  - torque of mechanical force on the shaft;

$M_{em}$  - electromagnetic torque;

$J \frac{d\omega}{dt}$  - dynamic torque;

$\omega$  - angular rotation frequency;

$J$  - torque of inertia of rotating masses.

At transition from equations comprising periodic factors, to equations with constant factors, the phase coordinate system is substituted units  $d, q, 0$  coordinate system; thus, phase variables (for example, voltages  $u_a, u_b, u_0$ ) are transformed into new variables connected to  $d, q, 0$  coordinate system ( $u_d, u_q, u_0$  voltages). To carry out such transformation, the method of space vectors is used. All equations are written in relative units (r.u.) systems ( $X_{ad}$  system) with equal resistances of mutual induction and magnetizing forces.

Mathematical model of synchronous machine in  $d, q, 0$  axes:

$$\left. \begin{aligned} u_0 &= \frac{d\Psi_0}{d\tau} + i_0 R; \\ u_d &= \frac{d\Psi_d}{d\tau} - \Psi_q \omega + i_d R; \\ u_q &= \frac{d\Psi_q}{d\tau} + \Psi_d \omega + i_q R; \\ \dot{u}_f &= \frac{d\Psi_f}{d\tau} + i_f R_f; \\ 0 &= \frac{d\Psi_D}{d\tau} + i_D R_D; \\ 0 &= \frac{d\Psi_Q}{d\tau} + i_Q R_Q. \end{aligned} \right\} \quad (3.27)$$

where  $\tau = \omega t$  - time in electrical radians or synchronous seconds.

Equation of rotor movement:

$$T_M \frac{d\omega}{d\tau} = M_M - (\Psi_q \cdot i_d - \Psi_d \cdot i_q), \quad (3.28)$$

where  $T_M$  - inertial constant of the machine in electrical radians.

Flux linkage of all machine circuits, written in  $d, q, 0$  axes, contain only constants independent on an induction time:

$$\left. \begin{aligned} \Psi_0 &= X_0 i_0; \\ \Psi_d &= X_d i_0 + X_{ad} i_f + X_{ad} i_D; \\ \Psi_q &= X_q i_q + X_{aq} i_Q; \\ \Psi_f &= X_{ad} i_d + X_f i_f + X_{ad} i_D; \\ \Psi_D &= X_{ad} i_d + X_{ad} i_f + X_D i_D; \\ \Psi_Q &= X_{aq} i_q + X_Q i_Q. \end{aligned} \right\} \quad (3.29)$$

To obtain the final algorithm of SM simulation in alternating currents, we shall substitute (3.29) for (3.27) in the set of equations and solve it relatively to the derivatives of currents. Then we receive in matrix form:

$$\frac{d}{d\tau} \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_D \\ i_Q \end{bmatrix} = \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 \\ 0 & 0 & 0 & Q_4 & 0 \\ 0 & 0 & 0 & 0 & Q_5 \end{bmatrix} \times \begin{bmatrix} u_d \\ u_q \\ u_d \\ u_d \\ u_q \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \times \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_D \\ i_Q \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} =$$

$$= \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 \\ 0 & 0 & 0 & Q_4 & 0 \\ 0 & 0 & 0 & 0 & Q_5 \end{bmatrix} \times \begin{bmatrix} u_d \\ u_q \\ u_d \\ u_d \\ u_q \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix},$$

$$\text{where } Q_1 = \frac{X_D X_f - X_{ad}^2}{\Delta d}; \quad Q_2 = \frac{X_Q}{\Delta q}; \quad Q_3 = -\frac{X_{sD} X_{ad}}{\Delta d}; \quad Q_4 = -\frac{X_f X_{ad}}{\Delta d}; \quad Q_5 = -\frac{X_{aq}}{\Delta q};$$

$$\Delta d = X_d X_f X_D - X_{ad}^2 (X_d + X_D + X_f - 2X_{ad}); \quad \Delta q = X_q X_Q - X_{aq}^2;$$

$$a_{11} = -R_a Q_1; \quad a_{12} = X_q \omega Q_1; \quad a_{13} = -R_f Q_3; \quad a_{14} = -R_D Q_4; \quad a_{15} = X_{aq} \omega Q_1;$$

$$a_{21} = -\omega X_d Q_2; \quad a_{22} = -R_a Q_2; \quad a_{23} = -X_{ad} \omega Q_2; \quad a_{24} = -X_{ad} \omega Q_2; \quad a_{25} = -R_Q Q_5;$$

$$a_{31} = -R_a Q_3; \quad a_{32} = X_q \omega Q_3; \quad a_{33} = -\frac{X_d X_D - X_{ad}^2}{\Delta d} R_f; \quad a_{34} = \frac{X_s X_{ad}}{\Delta d} X_D; \quad a_{35} = X_{aq} \omega Q_3;$$

$$a_{41} = -R_a Q_4; \quad a_{42} = X_q \omega Q_4; \quad a_{43} = \frac{X_s X_{ad}}{\Delta d} R_f; \quad a_{44} = -\frac{X_d X_f - X_{ad}^2}{\Delta d} R_D; \quad a_{45} = X_{aq} \omega Q_4;$$

$$a_{51} = -X_d \omega Q_5; \quad a_{52} = -R_a Q_5; \quad a_{53} = -X_{ad} \omega Q_5; \quad a_{54} = -X_{ad} \omega Q_5; \quad a_{55} = -\frac{X_q}{\Delta q} R_Q.$$

$$B_1 = Q_3 u_f; \quad B_2 = 0; \quad B_3 = \frac{X_d X_D - X_{ad}^2}{\Delta d} u_f; \quad B_4 = -\frac{X_s X_{ad}}{\Delta d} u_f; \quad B_5 = 0.$$

Or

$$\frac{d}{dt} [I_{SG}] = [Q_{SG}] [U_{SG}] + [H_{SG}], \quad (3.30)$$

where

$$[H_{SG}] = [A_{SG}] [I_{SG}].$$

In order to take into account the saturation *on the way of the magnetic flux* use of the value of mutual induction resistance  $x_m$  that depends on saturation is necessary in the expressions for currents  $i_a$  and  $i_\beta$  and this value can be found from the machine magnetization curve depending on the resultant flux linkage in the machine air-gap:

$$X_{ad} = f(\Psi_\delta).$$

For this purpose one can use the characteristic of the concrete machine idling, if such a characteristic is not available, one can use the normal characteristic of idling.

The differential equations of synchronous and induction machines, which describe the dynamic processes, are written in the system of relative units (system  $X_{ad}$ ). And that

allows to re-count easily the normal characteristic of idling for the concrete machine, taking into consideration the fact that in "the system  $X_{ad}$ " when  $i_f = 1,0$ , EDS  $E_f = i_f * X_{ad}$  is equal to the resistance  $X_{ad}$  unsaturated value in the relative units.

The acquired list-form characteristic of idling is approximated. The obtained analytic expression allows to receive the linear saturated characteristic for each magnetic motive force of the air-gap  $F_\delta$  and calculate the resistance  $X_{ad}$  saturated value:

When we enter the program, the program module allows to determine the resistance  $X_{ad}$  saturated value at each integration step, depending on the machine current values. Using the mathematical model of synchronous machine. Fig. 3.2 presents the results of calculation of the current change and rotating moment in the mode of the synchronous generator static load-on, when  $\cos\phi = 0,8$  taking into account saturation, or without taking it. Comparing these curves, one can conclude that if we take into account saturation we obtain smaller values of current and torque, that responds to physical representations of the machine operation.

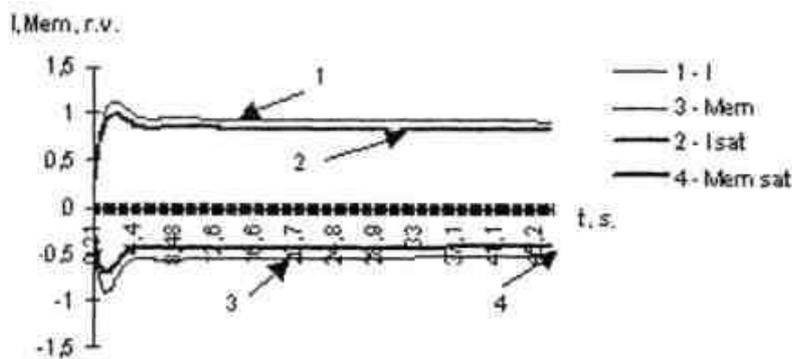


Fig.3.2. The curves of variation of current and electromagnetic moment of synchronous generator in the mode of load.

In such a way, the module of calculation of the mutual induction  $X_{ad}$  resistance saturated value, which is included into the general program, allows to take into account the saturation of magnetic circuit on the way of the main magnetic flux. The obtained analytic dependence  $E_\delta = f(F_\delta)$  allows calculating the mutual induction resistance saturated value at each step of the differential equations integration.

### 3.3. Mathematical model of induction motors and calculation of operation modes

AM equations can be obtained as a particular case of equations for synchronous singlepole machine. A motor operation mode is considered as the main operation mode. An induction motor model in  $d, q, 0$  axes is obtained analogously to a synchronous one and having been transformed has the following form:

$$\left. \begin{aligned} u_{sd} &= R_s i_{sd} - \omega_k \Psi_{sq} + \frac{d\Psi_{sd}}{d\tau} \\ u_{sq} &= R_s i_{sq} + \omega_k \Psi_{sd} + \frac{d\Psi_{sq}}{d\tau} \\ 0 &= R_r i_{rd} - (\omega_k - \omega) \Psi_{rq} + \frac{d\Psi_{rd}}{d\tau} \\ 0 &= R_r i_{rq} + (\omega_k - \omega) \Psi_{rd} + \frac{d\Psi_{rq}}{d\tau} \end{aligned} \right\} \quad (3.31)$$

$$T_M \frac{d\omega}{d\tau} = [X_{ad}(i_{rd}i_{sq} - i_{rq}i_{sd}) - M_{St.}]; \quad (3.32)$$

$$\left. \begin{aligned} \Psi_{sd} &= X_s i_{sd} + X_{ad} i_{rd}; \\ \Psi_{sq} &= X_s i_{sq} + X_{ad} i_{rq}; \\ \Psi_{rd} &= X_r i_{rd} + X_{ad} i_{sd}; \\ \Psi_{rq} &= X_r i_{rq} + X_{ad} i_{sq}; \end{aligned} \right\} \quad (3.33)$$

The reduced equations (3.31) - (3.33) give an opportunity to calculate any electromechanical processes in AM, at that it is necessary to establish the mode parameters  $u_{sd}$ ,  $u_{sq}$  and choose  $\omega_k$  - angular frequency of the coordinate system  $d, q, 0$ .

A set of equations can be resolved in appliance to current derivatives being reduced to:

$$\frac{d}{dt}[I_{IM}] = [Q_{IM}][U_{IM}] + [H_{IM}], \quad (3.34)$$

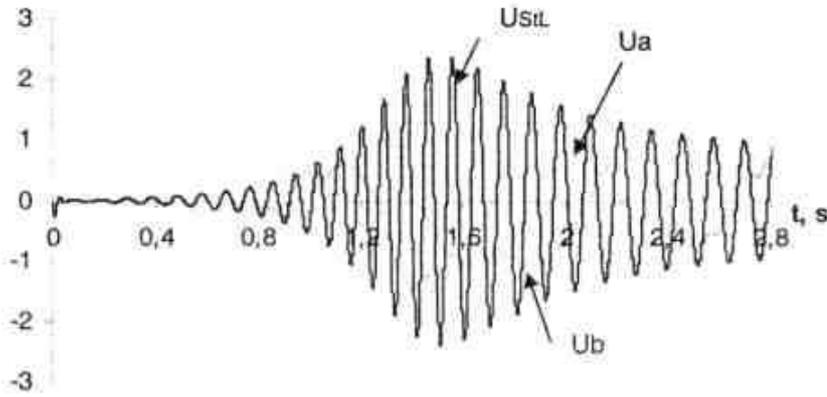
where

$$\begin{aligned} [H_{IM}] &= [A_{IM}][I_{IM}]; \\ [I_{IM}] &= \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix}; \quad [Q_{IM}] = \begin{bmatrix} Q_1 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 \\ 0 & 0 & Q_3 & 0 \\ 0 & 0 & 0 & Q_4 \end{bmatrix}; \quad [U_{IM}] = \begin{bmatrix} u_{sd} \\ u_{sq} \\ u_{rd} \\ u_{rq} \end{bmatrix}; \quad [H_{IM}] = \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix}; \\ [A_{IM}] &= \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}; \\ a_{11} &= -\frac{R_s}{X_d'}; \quad a_{12} = \left( \omega_k - \frac{X_{ad}^2 \omega}{X_r X_d'} \right); \quad a_{13} = \frac{X_{ad} R_r}{X_r X_d'}; \quad a_{14} = \frac{X_{ad} \omega}{X_d'}; \\ a_{21} &= -\left( \omega_k + \frac{X_{ad}^2 \omega}{X_r X_d'} \right); \quad a_{22} = -\frac{R_s}{X_d'}; \quad a_{23} = -\frac{X_{ad} \omega}{X_d'}; \quad a_{24} = \frac{R_r X_{ad}}{X_d' X_r}; \\ a_{31} &= \frac{R_r X_{ad}}{X_r X_d'}; \quad a_{41} = \frac{X_{ad} X_s}{X_r X_d'} \omega; \\ X_d' &= X_s - \frac{X_{ad}^2}{X_r}; \quad Q_1 = Q_2 = \frac{1}{X_d'}; \quad Q_3 = Q_4 = -\frac{X_{ad}}{X_r X_d'}. \end{aligned}$$

Saturation on the way of the main magnetic flux is taken into account in the same way like for the synchronous machine. The program module, which determines the saturated value  $X_{ad}$ , allows to calculate resistance  $X_{ad}$  at each integration step, depending on the machine current value.

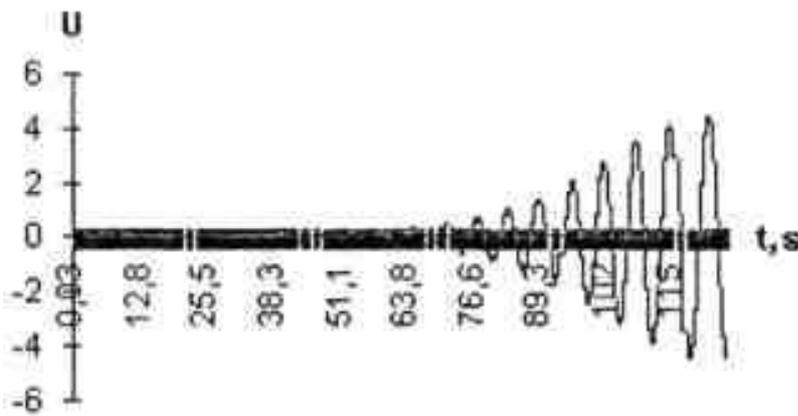
The magnetic circuit saturation account on the way of the main magnetic flux is shown, using the example of the asynchronous generator self-excitation with the capacitor banks.

The process has been considered of the asynchronous generator self-excitation with the capacitor banks in load conditions there. Fig. 3.3 shows the changes of voltage in time during the self-excitation process.



**Fig.3.3. Characteristics of the loaded asynchronous generator in the process of self-excitation during saturation.**

Particularly, one should note that the study of the capacitor self-excitation dynamic modes of the asynchronous generator, without taking saturation into account, is practically impossible. Considering this process without saturation, the values of voltages do not reach the stabilizing values (Fig.3.4), and that does not correspond to the physical representations of the processes in the machine.



**Fig.3.4. Characteristics of the loaded asynchronous generator in the process of self-excitation without saturation.**

### 3.4. Mathematical model of static load

The differential equations of the combined active and inductive load in the coordinates system  $d, q, 0$  can be represented analogously to the equations of the SG and IM stator currents. After transformation of the static load equation in the form of (3.34), we obtain the equations of static load in the form that is convenient for modeling:

$$\frac{d}{dt} [I_{StL}] = [Q_{StL}] [U_{StL}] + [H_{StL}], \quad (3.35)$$

where

$$[H_{StL}] = [A_{StL}] [I_{StL}],$$

#### 4. REALIZATION OF THE METHOD FOR INVESTIGATION OF THE ELECTRICAL MACHINES SYSTEM WITH A METHOD OF MATHEMATICAL SIMULATION

In the fourth part of the work are considered the examples of the use of the algorithm for generation of the multimachine system equations on the basis of the complete equations of the elements.

We considered the modes of run-out of important mechanisms of IM (single and group) and self-starting as well as the mode of motors (induction and synchronous) working from source.

##### 4.1. Single and Group run-out of inductions motors of important mechanisms and repeated switching

All process can be represented as composed of two stages:

- The first stage - run-out of plant items (single or group).
- The second stage - self-starting and operation mode restoring.

##### *Run-out of single induction motors*

Investigating a number of specific operation modes one should know a character of speed change at power supply switching-off. First of all it could be explained by the fact that in IM practical operation often a condition, when repeated connection of drive to a network takes place in a very short interval after being switched-off arises. An example could be the load transfer to a reserve network at voltage drop of main network. In this case the time of power supply interruption is defined by automatic feed switch (AFS) action.

Investigation of IM operation dynamic process at power supply interruption (open feeding circuit) allows to define correct operation mode of IM itself, and at subsequent connection to select corresponding devices of relay protection, which on the whole increases reliability of load power supply.

There is an example of the asynchronous machine switching mode simulation: start in order to obtain the parameters of the steady-state mode of operation on the tailored mechanical load: after that breaking conditions or severe voltage reduction follow, it is the asynchronous machine stop way; then the mode of repeated switching on the principal or spare power source follows.

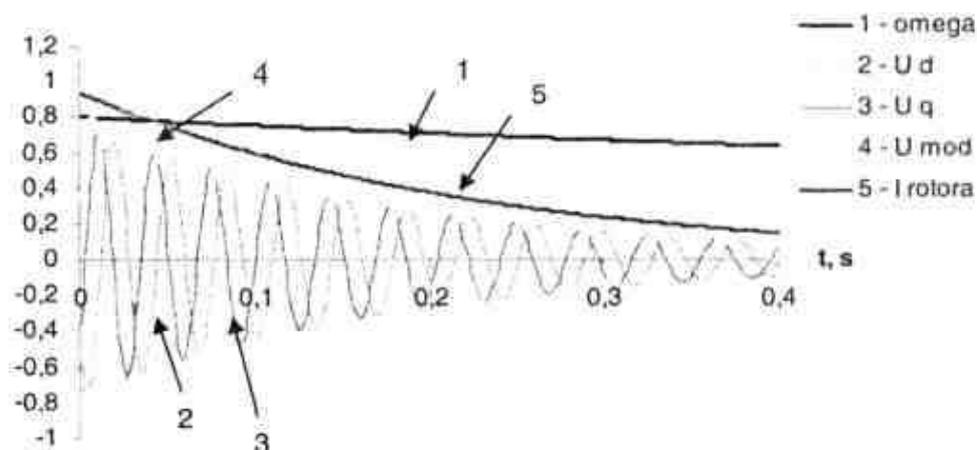
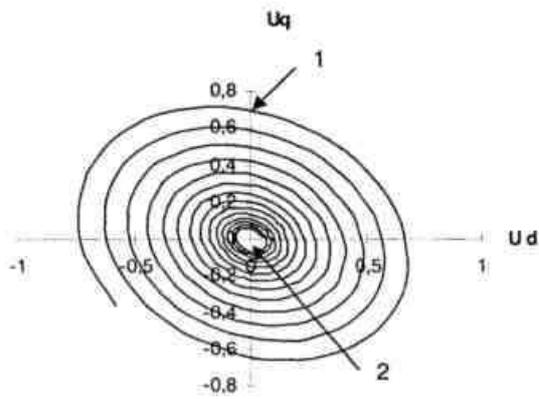
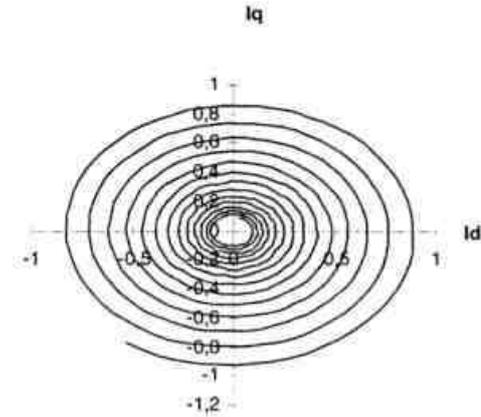


Fig.4.1. Character of rotors  $\omega$ , U, I, at switching-off regime.

Hodographs of current and voltage at run-out of a single motor can be seen in Fig.4.2 and 4.3.

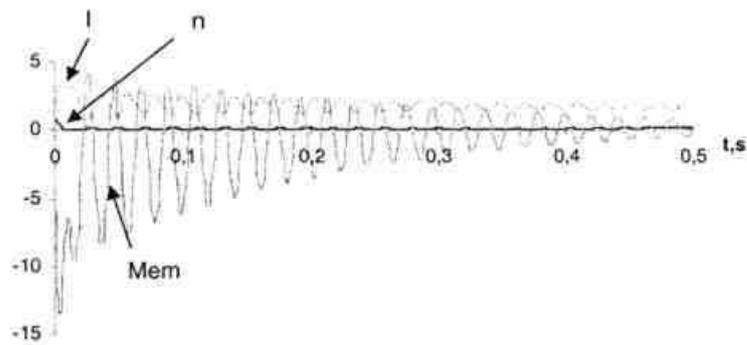


**Fig.4.2. Hodograf of residual voltage at IM switching-off.**

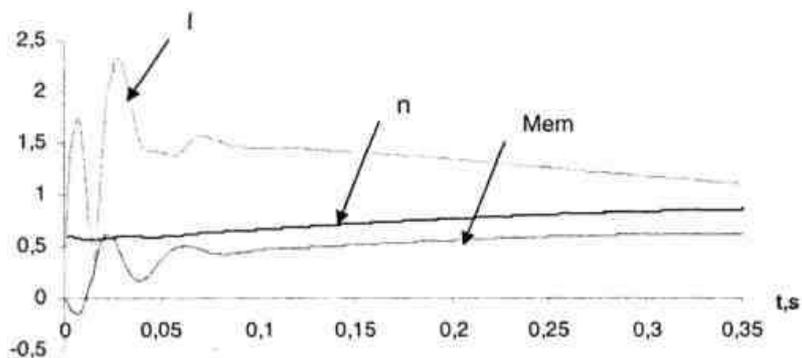


**Fig.4.3. Hodograf of rotor current at IM switching-off.**

It is possible using these calculations to determine permissiveness and good progress of the subsequent motor self-starting regime. Curves of current, rotation speed and moment changes at repeated connection could be seen at point 1 in fig.4.4. and at point 2 in fig.4.5.



**Fig. 4.4. Character of rotation speed, current and moment at repeated connection at p.1.**



**Fig.4.5. Character of rotation speed, current and moment at repeated connection at p.2.**

Carrying out a series of calculations of induction motor run-out, with varying of the motors power and mechanical time constant, has allowed making the following conclusions:

- > with power increasing, a residual voltage amplitude increases;

- > increase of mechanical constant of motor unit results in increase of voltage curve damping factor and delays process;
- > to define the most advantageous time of transfer, when a total EMF at repeated switching should not exceed nominal EMF, a corresponding individual calculation is required.

*IM repeated switching* occurs in regime of transfers such regime is required when there are the main loads that are not allowed prolonged interruptions of power supply or feed voltage drop due to specific character of their operation. When network voltage drops, these IM are transferred to a spare network (when the voltage is restored, a back transfer takes place). Due to the vast use of the microprocessor systems to create new devices of automatic control and protection, the trend to increase their action speed is observed, and transfer occurs so fast that electromagnetic processes originated in IM at lost feeding, have no time to be damped a repeated switching does not occur at initial zero conditions, but in the presence of rotor current and not-damped EMF at stator terminals. At fast transfer with interruption of powersupply up to hundredth fractions of second, it is impossible to ensure a good progress of the process of self-starting, especially it is a characteristic feature of low-powered induction motors with the fast time mechanical constant. Calculations of repeated switching and self-starting regime have been carried out for various IM with different mechanical constants.

The results of simulation of are repeated switching regime after feed interruptions of different duration are reduced to the table 4.1.

**Tab. 4.1.**

Feed interruptions time, sec.	0.001	0.0412	0.06	0.08	0.1
Rotation speed at switching, r.u.	0.991	0.918	0.887	0.857	0.830
Residual voltage, module, r.u.	0.807	0.430	0.325	0.248	0.188
Component $U_d$ , r.u.	0.191	-0.068	-0.046	0.191	0.013
Component $U_q$ , r.u.	-0.784	-0.424	-0.322	-0.158	-0.187
Stator current $I_{max}$ , r.u.	84.392	21.519	19.042	13.546	14.531
Electromagnetic torque $M_{max}$ , r.u.	-77.675	-29.153	-19.696	-16.149	-10.641
Outcome of repeated switching	Unsuccessful	Unsuccessful	Unsuccessful	Successful	Successful

As the calculations and experience of operation show, a repeated switching represents a dangerous regime of IM operation, especially at plugging (connection in opposition) with residual voltage. Negative moment surges resulted from it are indicative of the increased influence of stator electromagnetic transients on the whole transient process causing the decrease of speed (Fig.4.4.-4.5.).

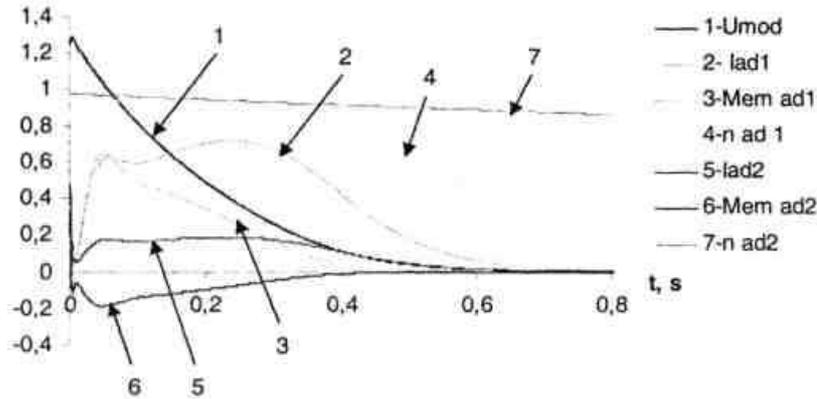
### **Group run-out of inductions motors of important mechanisms**

Group run-out takes place at simultaneous disconnection of various motors, supplied from the same substation or another power source. At group run-out during power supply interruption, the electric motors remain electrically connected one to another via the common buses, even when they are disconnected from the network. Thus, the drives having the greater reserve of a kinetic energy, are transferred to generator operation mode and have additional braking torque in comparison with free run-out. The motors having a smaller reserve of a kinetic energy, receive an additional movement moment (at the expense of replenishment from the first group of machines).

For simulation of the system operation modes the following sequence of design has been accepted:

- induction motors starting from a synchronous generator ( $m_{SG} = 1$ );
- simulation of power source de-energization regime ( $m_{SG} = 0$ ).

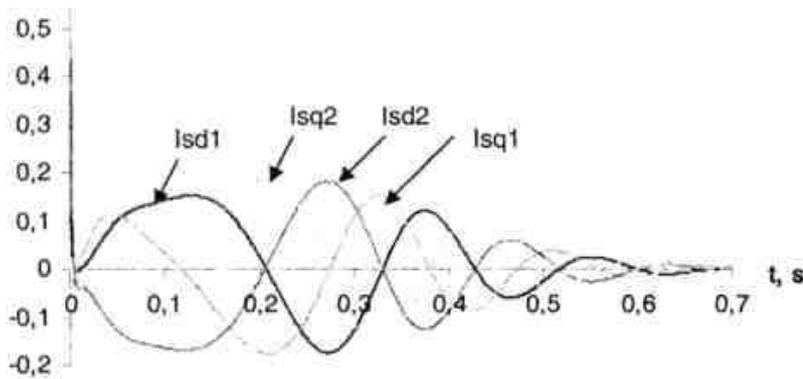
$U, I_{AD1}, M_{emAD1}, n_{AD1}, I_{AD2}, M_{emAD2}, N_{AD2}$  dependence on time for the joint operation of inductions motors after disconnection with sharply differing parameters can be seen from Fig.4.6 ( $P_{emIM2} = 4P_{emIM1}$  and  $T_{mIM2} = 10T_{mIM1}$ ).



**Fig.4.6. Dependence of residual voltage, current, moment and rotation frequency of two induction motors.**

The character of voltage change at system buses can be seen from the results of the simulation. Electromagnetic torque, developed by the second motor, is generated, and the first IM continues working in the motor regime.

The curves of changing of IM 1 and IM2 stator currents components at group runout regime can be seen from fig.4.7.

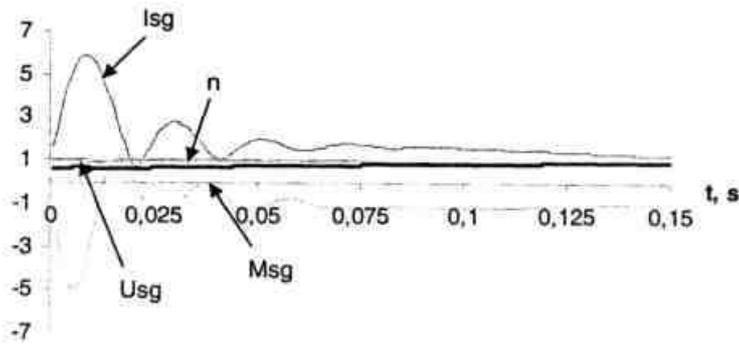


**Fig.4.7. Curves of changing of IM1 and IM2 stator currents components at run-out regime.**

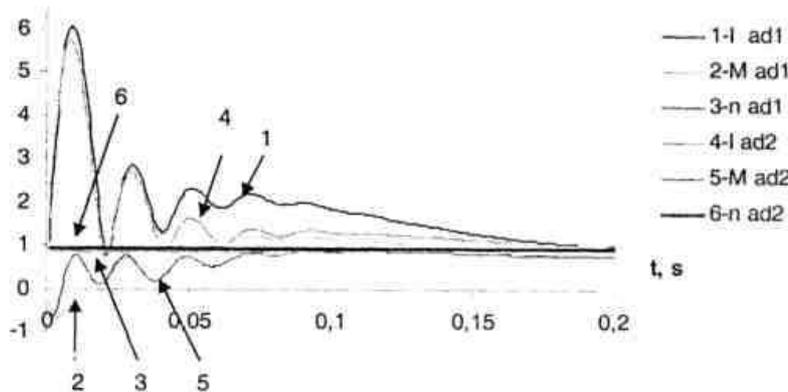
Reconstructing the value ( $m_{SG}=0$  substitute by  $m_{SG}=1$ ), we have an opportunity to realize the asynchronous motors repeated switching on the power source in the proposed model.

The character of voltage, current, moment and frequency change in SG after repeated switching of IM can be seen from fig.4.8.

The character of voltage, current, moment and frequency change in IM1 and IM2 after repeated switching can be seen from fig.4.9.



**Fig.4.8. The character of voltage, current, moment and frequency change in SG after repeated switching of IM.**



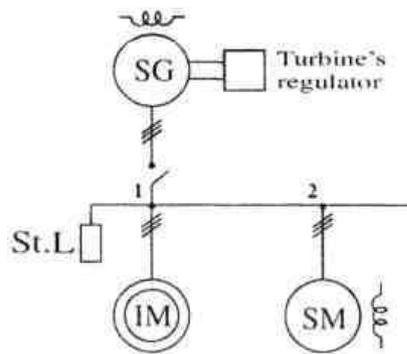
**Fig.4.9. The character of voltage, current, moment and frequency change in 1M1 and IM2 after repeated switching.**

As we can see from the results of the simulation, inductions motors repeated switching at incomplete group run-out is not a dangerous regime, in contrast to the repeated switching regime of a single drive after run-out.

#### ***4.2. Investigations of the Change-Over Modes of the Induction and Synchronous Motors working from one Source***

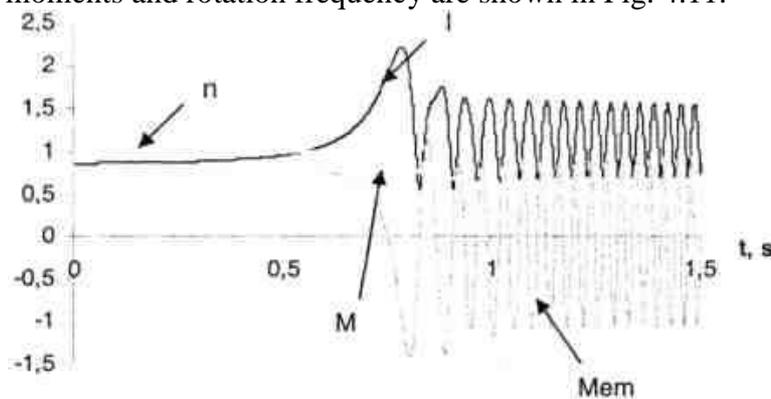
Opportunities how to use the proposed approach for solving of the practical tasks of the electrical machine systems simulation which includes synchronous generator (SG), static load (StL) and dynamic loads: synchronous (SM) and induction motor (IM) are presented, using an example of the study of the motor switching modes, which work from one power source (Fig.4.10), by means of mathematical modeling. With the help of the developed model, it is planned to investigate interference of the asynchronous and synchronous motors that are cut off the power sources. In order to simulate this mode there was a system chosen with one power source, one asynchronous and one synchronous motor and static active and inductive load.

One should note, when voltage disappears (supply is cut off), the motors remain to be connected with the common bus-bars. The motor magnetic flux during a short time period is supported owing to the currents, which are induced in the rotor windings at the moment of disconnection. The motors continue to rotate owing to stored vise viva. In the motor stator windings, there is generated voltage, damping because of diminishing of the currents of rotation angular frequency. This voltage determines interdependence of all the motors, which are connected to the common bus-bars.



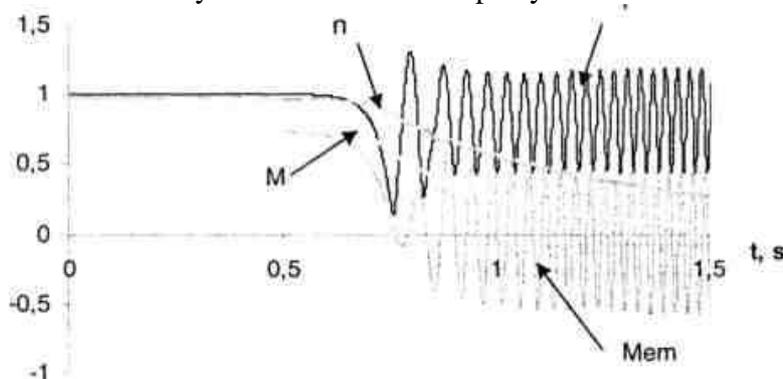
**Fig. 4.10. Design of the investigated system.**

In order to model the mode of the electrical customers change-over and disconnection (asynchronous and synchronous motors) it is necessary to model the initial steady-state mode. In order to facilitate this process it is proposed to use as an initial conditions the "acceleration" of the system starting from the value of the machine rotors rotation frequency equal to one. The synchronous generator has been modeled by the exciting regulator. The synchronous motor has a constant excitation voltage. The diagrams of the changing of currents, moments and rotation frequency are shown in Fig. 4.11.



**Fig.4.11. Diagrams of changing of currents, rotation frequency, electromagnetic torque and torque of resistance of SM at a disconnection moment.**

In Fig. 4.12 are shown analogous dependences for the asynchronous motor in the same mode. Availability of the common circuit for the stator windings results in deceleration of the process of the asynchronous motor stopway.



**Fig.4.12. Diagrams of changing of currents, rotation frequency, electromagnetic torque and torque of resistance of IM at a disconnection moment.**

In Fig. 4.13-4.15 is shown the system elements (the curves of changing of currents, rotation frequency and electromagnetic torque of SG, SM and IM) behavior when voltage is restored after 0,5 seconds. From the given figures one can see that owing to the availability of the SM excitation the process of the motor stoppage is delayed and there is an opportunity to restore the operational mode.

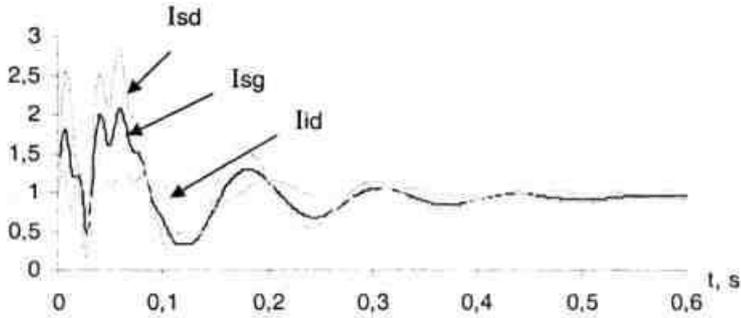


Fig.4.13. The curves of changing of currents of SG, SM and IM after restoring of the voltage.

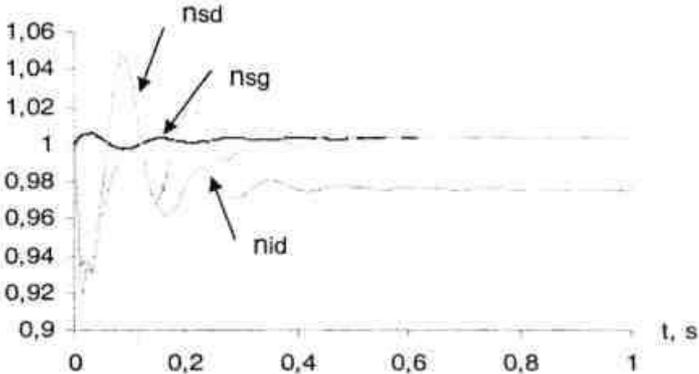


Fig.4.14. The curves of changing of rotation frequency of SG, SM and IM after restoring of the voltage.

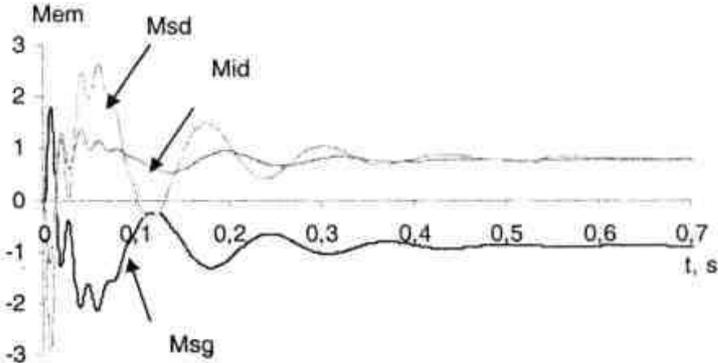


Fig.4.15. The curves of changing of electromagnetic torque of SG, SM and IM after restoring of the voltage.

In such a way, the modeling results show that the repeated voltage restoration after a short break in power supply to the asynchronous and synchronous motors, chiefly, is not a dangerous mode, owing to the SM residual excitation, which is connected to the common bus-bars.

## **PRINCIPAL WORK RESULTS AND CONCLUSIONS**

- Development of the research method of the electrical machines power system dynamics on the basis of complete equations of the elements, using structural principle of the investigated electrical machines system model construction. There are created the functional mathematical models developed for the alternating current electric machines, united in the uniform system.
- In the work, proposes the methods and algorithm of generation of the element constraint equations in the system of the alternating current electric machines, by means of introduction of the static load node. An opportunity for investigation of various operating modes with different commutations in the circuit is presented. The used principle of algorithmization allows composing and studying of the electrical machines power systems of any configuration.
- It is proposed to realize modeling in the system of relative units, where the elements parameters are used in their own basic units.
- The proposed model allows studying of various operating modes of motor load, taking into considerations their interference that is important for the choice of apparatuses of commutation, protection and automation. The model reflects the transient electromagnetic processes behavior in the stator circuits at the initial stage of the transient process.
- The proposed mathematical models of the SM, AM and static load can be used as separate program modules for the operation modes analysis as well as for determination of the machine optimal parameters, which ensure their stable operation at any fluctuation in voltage and frequency.
- The AM model can be used as a constituent part of any system of the automatized drive, substituting for the control object, or power system, presenting generalized asynchronous load.
- Studying the transient modes according to the developed methods, one can define the values, which are necessary for the electric machines designing and exploitation with the purpose to choose the appropriate protection apparatuses and automatic machinery, and their correct offset.

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<sup>1</sup>The last name Zimina was changed for Klujevskā in 2004.