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MODELLING OF DURABILITY OF COMPOSITE MATERIAL

Author's abstract of doctoral thesis

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Topicality of research. Composite materials are widely used in the structure of airframe of sport airplanes, special use light airplanes, as well as heavy passenger airliners: Boeing-777, A380-800. To ensure reliability and durability of these aircrafts it is necessary to know the connection of static strength and fatigue life characteristics of corresponding composite materials and to be able to predict how they change due to the changes of characteristics of their components. This is the main focus area of the research undertaken and reflected in this doctoral thesis.

Goal and objectives of research: The development of a mathematical phenomenological model of fatigue life of a unidirectional composite, the parameters of which are in some connection with the distribution parameters of static strength of composite components. This model may be used to describe the fatigue curve and fatigue damage accumulation in program loading. To achieve these objectives it is necessary to find a solution to the following problems:

1. To study distribution functions of static strength of composite components (fibers, strands etc.); to investigate existing models of static strength;
2. To develop new mathematical models for addressing the problem under investigation;
3. To develop a procedure for estimation of the parameters of these models.
4. On the basis of these new models to develop the fatigue curve description and a prediction of fatigue life in program loading;
5. To verify the developed models using experimental data.

Method of research

Theoretical methods:

- Theory of probabilities and theory of Markov chains.
- Mathematical statistics.
- Mechanics of materials and mechanics of structures.

Experimental methods:

- Tests of static strength and fatigue tests.

Scientific novelty

1 The author has developed, based on Markov chains theory, new mathematical models of fatigue life of an unidirectional composite. These models allow to form a description of the fatigue curve and formulate a rule of accumulation of fatigue damage in program loading.
2. The models allow to develop formulae both for mean fatigue life and for its distribution function.
3. At a fixed number of cycles the conditional distribution function of limited fatigue limit can be obtained by using this function.
4. The parameters of the models are connected with the parameters of static strength distribution.
5. The author has elaborated a procedure of estimation of model parameters on the basis of test dataset.
6. The author has developed a package of program for modelling static strength and fatigue life.

7, The author has also developed a package of program for estimating model parameters.
Innovations with regard to methodology are:

1. The author offers a new kind of probability paper, where inverse function values are substituted by expectation values of corresponding order statistics;
2. The author suggests a new kind of fatigue curve by comparing experimental order statistics and their calculated theoretical expected values;
3. The author recommends a new test for goodness of fit for a family of distributions with location and scale parameters, as well as a program package for calculation of the power of test allowing to estimate the required number of observations.

Practical relevance

The developed models can be used:

- for non-linear regression analysis of fatigue curve;
- for prediction of fatigue curve changes due to some changes of static strength distribution parameter of the composite component.
- for prediction of fatigue life in program loading.

The program package can be used both for scholarly research and educational purposes.

Most relevant results are published in 17 papers and were presented as reports in 13 international conferences.

Chapter 1 presents an overview of the most important works devoted to the problem of investigation of strength and fatigue life of mechanical structures and, in particular, composite materials. The first investigation of the theory of brittle destruction of material was made by Weibull (1939), Frenkel and Kontorova (1941). Further investigations of related mathematical problems were made by Gnedenko (1941). Weibull distribution is one of the versions of stable distribution of smallest extreme value: the strength of a chain of identical items has the same type of distribution as the strength of one item. Only parameters change. This phenomenon is used for the explanation of the so-called "scale factor": subject to brittle destruction ultimate tension stress decreases if the size of specimens increases.

Freudenthal, Gumbel (1956) have applied Weibull's approach to the fatigue life. A similar approach to the fatigue problem was used also by Bolotin (1971, 1977, 1981, 1990). On the basis of these ideas the statistical theory of fatigue curve was developed. A similar model was studied in detail by Pascual and Meeker (1999).

The model of the weakest link is based on the assumption that in a material there are a lot of week items with independent and similar processes of accumulation of fatigue damages. The fatigue life of a specimen is equal to the fatigue life of the weakest item, and therefore it is applied Weibull's distribution approach. An alternative approach is based on the assumption that there is reallocation of stresses and damage accumulation between these week items. Destruction takes place when accumulated damage exceeds a certain limit. The simplest model of this type is offered by Druzhinin (1967), who assumed normal distribution of the rate of fatigue damage accumulation. Bastenair (1972) offered a similar model. Some more sophisticated model suggests a distribution function offered by Bernstein (Беррштейн) (1927), and then by Birnbaum and Saunders (1969). Until now, these hypotheses have not been thoroughly tested but the lognormal distribution is widely used. Kordonskiy (1964) offered a mathematical model of lognormal distribution. According to the Pirson's test, this distribution (with two parameters) has much better fits fatigue dataset than Weibull distribution. Later on Kordonskiy and Fridman (1976) also used the so-called diffusion distribution, earlier studied, for example, by Bartlett (1958).

There are a lot of papers devoted to processing of data about the strength and fatigue life of composites and their components. A review of these papers given in the book published by Nemets, Serensen, Strelyaev (1970) as well as in the book by Tamuzh, Protasov (1986). Statistical description of strength of fibers is developed by Kontorova (1945), Bartenev (1960,1964,1966), Sedrakyan (1958) and other authors. Usually Weibull and lognormal distributions are used for experimental data processing.Tamuzh, Gutans (1984), Padget et al.(1995), Watson and Smith (1985), Curtin (2000) offered some modification of Weibull's distribution to change dependence of its parameters on the length of specimen. Andersons, Joffe, Hojo, Ochiai (2002) used Weibull distribution and its modification investigating glass fiber strength distribution determined by common experimental methods. There is also an offer to use an expansion in the Gram-Sharle series. Sedrakyan offers to use a four-parameter generalization of Weibull distribution (lower and upper limits are introduced).

The development of distribution function of composite characteristics using distribution function of composite component characteristics was initially made by Peirce (1926) and Daniels (1945,1986). It was Daniels who obtained a fundamental result. It was proved that the distribution function of strength of N bundle of fibers converges to a normal distribution function, if N is large enough. And it does not depend on the distribution function of composite components.

Gucer D., Gurland J. (1962) offered to consider a composite as chains of bundles of fibers of some critical length. After failure fibers can be subdivided into fragments of critical length. This idea was developed by Rosen (1964). He studied the destruction of composite as destruction of a chain of bundles of fibers of limited (critical) length. Zweben (1968) takes into account the stress concentration and sequence of destruction of fiber near the group of destructed fibers. Zweben shows that destruction of 2-3 fibers sometimes is enough for destruction of the whole composite.

Tamuzh, Kuksenko (1978) in their book deal with the theory of dispersion destruction at complex loading. This theory is to some extent a generalization of Kachanov's (1958) ideas about the gradual growth of damage parameter and Afanasyev's (1953) statistical theory of fatigue. This theory includes in itself also the statistical theory of strength offered by Volkov (1960).

A general approach of composite destruction based on the use of kinetic models is given also in the work of Bolotin (1990). But a lot of initial information is needed to calculate fatigue life by the use of this theory. It is rather difficult to apply this theory with regard to some specific problem.

Bogdanoff and Kozin (1989) considered application of Markov chains theory to the statistical theory of fatigue damages. They have developed formulae for the distribution function of time to absorption and estimates for the so-called *one jump model.* But the parameters of the model considered do not have connections with the parameters of static strength distribution and the parameters of cycling loading. So the model cannot be used neither for the construction of the fatigue curve nor for the calculation of the fatigue life distribution for the program loading.

Chapter 2 presents a critical analysis of modern models of static strength of composite. Fig. 1 demonstrates the process of destruction of a composite with three parallel components having three different strengths: 10 N, 15 N, 25 N. It is shown that as result of the scatter of strength they can carry only 30 N as if all of them have strength only 10N. Real scatter of static strength of components and strength dependence on the structure

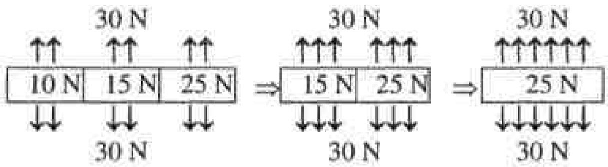


Fig.1.

of specimens is shown in the lognormal probability paper where dataset of strength of fibers, strands, 10-strand specimens and real laminated composite specimens are shown. Fiber has maximum of strength. Mean strength and its variance for a more complex structure decreases. Daniels proved the normal distribution of strength of "bundle of fibers" with standard distribution inverse to $\sqrt{n}$. But for dataset processing usually lognormal or Weibull distributions are used. Decreasing of standard deviation inverse to did not take place either. In the considered work it is shown that Daniels' assumption about uniform distribution of loads between still alive items is probably not true. It can be true for a bundle of disconnected threads but in a composite a new damage appears near the previous damage. This process begins, as a rule, on the surface, then grows along cross-section of specimens and finishes by a catastrophic failure of specimens. This means that even an approximate uniform distribution of stress can be expected only in a very limited volume with drifting location. For example, the calculation of standard deviation of strength of 10-strand specimens (by the Monte Carlo method) is very near to the experimental data, if we assume that in these specimens there are only 5 strands. A relevant illustration is given in Fig. 2

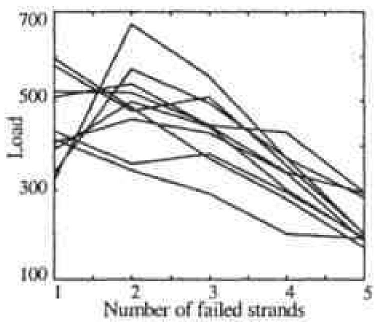


Fig.2.

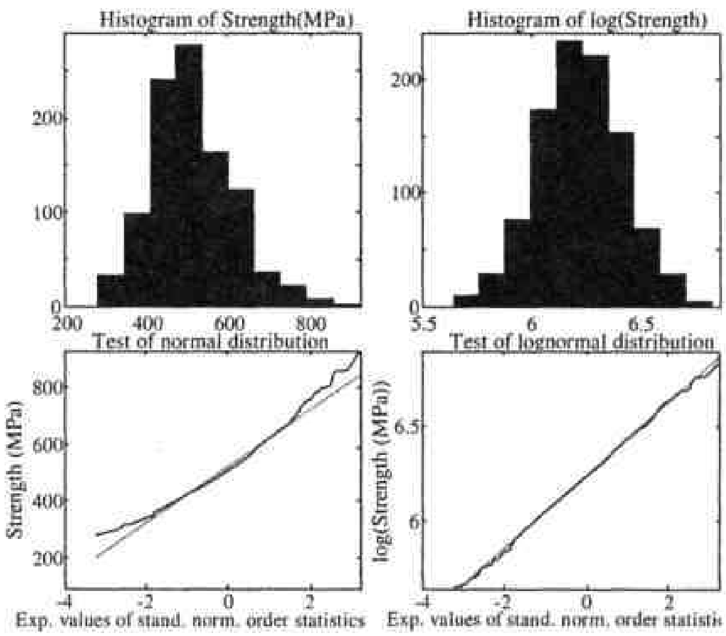


Fig.3.

(for 10 Monte Carlo trials) and in Fig. 3 (for 1000 Monte Carlo trials): the histograms, results of processing using modified normal and lognormal papers. It can be seen that the mean value and standard deviation of calculated strength is very near to the experimental data. It is worth to mention that the lognormal distribution significantly better fit the data of strength of 5-strand specimens.

Chapter 3 is devoted to the 'brittle' model of fatigue and the attempt to develop a fatigue curve equation taking it as the basis. The parameters of this curve should be connected with static strength parameters. This problem is of current importance. In the paper of Pascual and Meeker (1999) there are seven fatigue curve equations, which are offered instead of the widely known Wholer's equation. The authors of this paper have chosen the best equation, but its parameters have no connection with static strength parameters. In this chapter an attempt was made to disclose this connection by "unwrapping" the Daniels' model in time.

The sequence of local stress $\{s_0, s_1, s_2, ...\}$ corresponding to the destruction of part, *F(s)*, of fibers (for which the strength is lower than local stress) is described by the formula

$$s_{(i+1)} = \frac{S_0}{1+F(s_i)} \quad i = 0, 1, 2, \ldots \quad .$$

where S_0 is initial (maximum stress in the cycle) stress in loading with a constant external load. The examples of the corresponding sequences $\{s_0, s_1, s_2, ...\}$ for different S_0 are shown in Fig.4 . It is very important that the curves in this figure are very similar to the curves of changes of some composite characteristic in cycling loading.

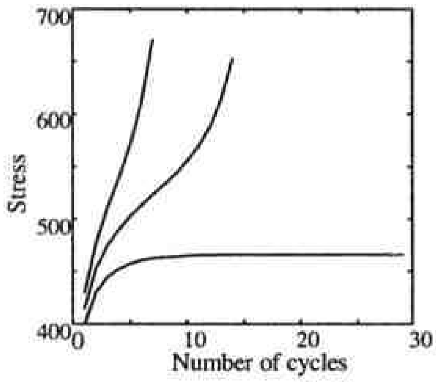


Fig.4

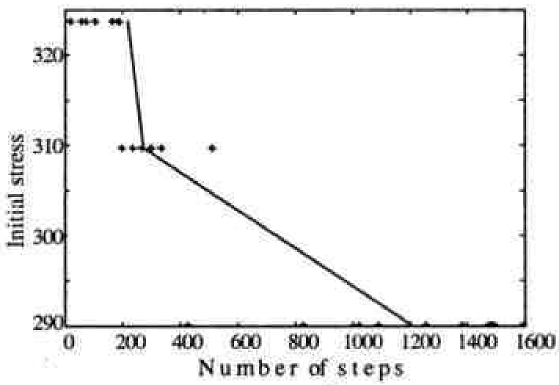


Fig.5.

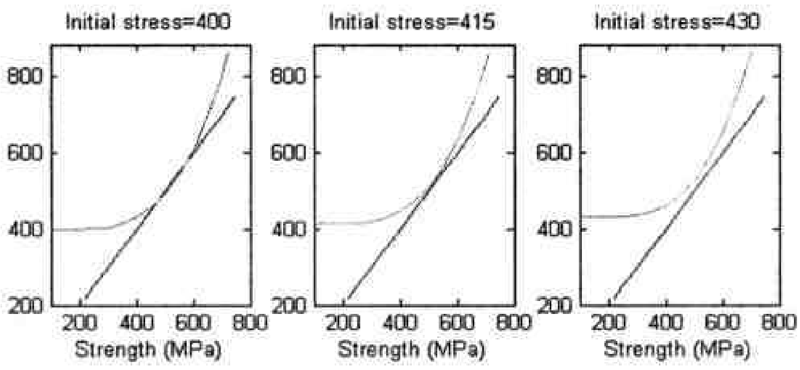


Fig. 6

Fig.6 presents the solution of the equation S_0 /(1 - $F(s)$). If there is a solution of this equation, then the sequence considered converges with this solution and local stress ceases to grow. This solution depends on S_0 . The maximum value of S_0 for which such a solution was obtained, can be considered as the fatigue limit (see the left part of Fig. 6). But if S_0 is more than the fatigue limit then the curves corresponding to the functions $S' = s$ and

$S'' = \frac{S_0}{1-F(s)}$ have no intersection and local stress growth up to destruction of specimens. So we can calculate the number of cycles to a failure and can get the fatigue curve. It is shown that the calculated fatigue curve can be approximated by the equation

$$S = C_1 - C_2 \tan(N(S) / C_3 - \pi / 2) .$$

The use of this equation for non-linear regression analyses is much more simple than the initial model (sequences $\{s_0, s_1, s_2, ...\}$ for different S).

In Fig. 4 we see very drastic changes of the number to a failure from several units (the upper two curves) and up to the infinity (for the lower curve) in the same figure. So this model can be used only as a model of the non-linear regression analyses of the fatigue curve but in a very narrow interval of stress. The example of the corresponding fatigue curve is shown in Fig. 5. Here Kleinhof s (1983) dataset (the results of the fatigue test of carbon-fiber reinforced plastic specimens, pulsating cycle) are shown by (+) but the calculation result is shown by the segments of straight lines.

An attempt was made to study the model, which takes into account the residual deformation of matrix. But calculation of fatigue life, by using this model requires too much time. So a much more productive approach to the problem under consideration is studied in following chapters.

Chapter 4 considers the model of fatigue damage accumulation and the model of fatigue curve based on Markov's chains theory. In accordance with this model, specimens are a set (bundle) of parallel items (fiber or strands). A change in the state of Markov chain (MC) means destruction of one or several of these items. There are *r* non-recurrent states and one absorbing state corresponding to failure of *r* items. If the initial state is the first state, then cdf (cumulative distribution function) and pmf (probability mass function) of a (random) number of steps to absorption *T* are defined by the formulae

$F_T(t) = aP'b$, $p_T(t) = F_T(t) - F_T(t-1)$, where the matrix of transition probabilities is as follows:

$$P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{23} & \cdots & p_{1r} & p_{1(r+1)} \\ 0 & p_{22} & p_{23} & p_{24} & \cdots & p_{2r} & p_{2(r+1)} \\ 0 & 0 & p_{33} & p_{34} & \cdots & p_{3r} & p_{3(r+1)} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & p_{rr} & p_{r(r+1)} \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix},$$

a - (100...0) is the row vector, *b* - (00...01)' is the column vector.

The main problem is to define the connection of transition probabilities with both static strength of composite component distribution parameters and the value of applied stress.

Two versions of this connection are considered. In the first version it is assumed that a failure of only one item can be in one step of MC. We call this version the One-step-Markov-model or Bernoulli-Markov-model (OSMM or BRMM) (Note: the random variable - the number of items failed in one step of MC has Bernoulli distribution). If there are (R - i) items still alive with the same distribution function of static strength, F(s), then there is probability of at least one additional failure (when MC is in (i+l)th state)

$$p_{i+1} = 1 - (1 - F_{i+1}(s_{i+1}))^{R-i},$$

where R is the initial number of items in some 'critical volume' (initially, on the surface of specimens).

F_{i+1} (•) -is the conditional distribution function of the ultimate strength of yet alive rigid items after i items have failed (we make an assumption that $F_{i+1}(s) = F(s)$); s_{i+1} is the stress (corresponding to (i +1)th state of MC) applied uniformly to all $(R-i)$ items:

$$s_{i+1} = \frac{SR - S_f i}{R - i} = \frac{S(1 - S_f i / SR))}{1 - i / R},$$

S_f is the stress (force), which an already failed item can still carry (because at least at the beginning of damage accumulation the failure of (strands) fibers can take place in different cross sections).

In a general case it is assumed that the distribution function of static strength has location and scale parameters. In some papers it is shown that lognormal distribution can be used.

Then $F(s)=\Phi((g(s)-\theta_0)/\theta_l)$, where $\Phi(.)$ is cdf of standard normal distribution; $g(s)=log(s))$; θ_0 , θ_1 are expectation value and standard deviation of logarithm of strength.

For the second version of the model, which we call the Binomial-Markov-model (BMM) we consider a possibility of MC "jumps" from state i-th to any other j- th state, $j = i, i + l, i + 2,...,r +1$ Thus it is natural to use here the binomial distribution

$$p\ (i,j) = \binom{r+1-i}{k} p_{i,1}^{k} (1-p_{i,1})^{r+1-k}, \qquad k = j - i$$

where $p_{i,1} = F(s_i)$,

In a general case also one step of MC can be considered as equal to k_M cycles. Now the considered model is defined by the parameter $\eta = (\theta_o, \theta_1, r, R, k, S_f)$ with 6 components, which have already described "physical" interpretation. The formulae for cdf of fatigue life and its moments can be used in both directions: for calculating mean and p-quantile fatigue curves or the cdf of limited fatigue limits at a fixed cycle number, if the parameters are known (this is a problem of the probability theory or a probability problem), or for the non-linear regression analysis for estimating model parameters, if the fatigue life dataset is known (the problem of mathematical statistics or statistical problem). For example, if the BRM version is used, then the mean and p-quantile fatigue curves are defined by the formulae

$$E(T(S_j)) = \sum_{i=1}^{r} 1/p_i(S_j,\eta), \qquad t_p(S_j) = F_T^{-1}(p;S_j,\eta).$$

where $E(T(S_j))$, $t_p(S_j)$ are mean values and *P*- quantile of fatigue life for stress S_j. Chapter 4 considers the example of an approximate solution of the probability problem. Some of the model parameters were chosen to be approximately equal to the distribution parameters of static strength. Other parameters were chosen to get the minimum difference between the calculated and experimental fatigue curves when this difference is estimated visually.

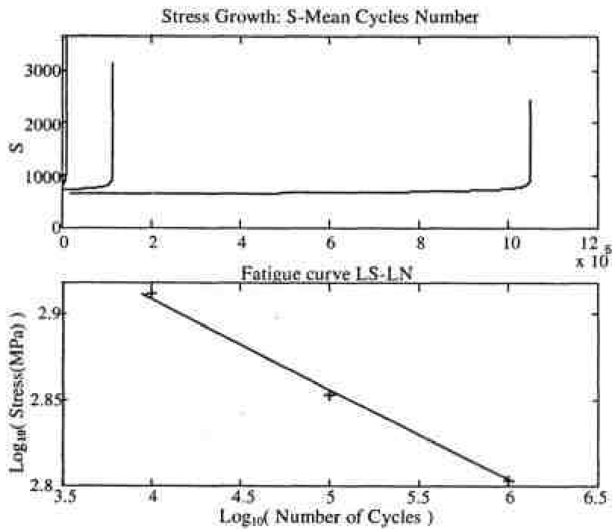


Fig.7

In the upper part of Fig. 7 t the local stress growth in the fatigue test by a pulsating cycle of a composite with organic fibers is shown for three different initial stresses. The time of transition in MC is a random variable, so the mean time corresponding to some stress level is shown the x-axis. It is worth to pay attention to the similarity of these curves with the curves of some physical parameter changes during the fatigue test of a composite. In the lower part of the figure the experimental (+) and calculated (continuous line) fatigue curves are shown. They are very near to one another.

Chapter 5 focuses on the statistical problem: estimation of model parameters processing dataset of Kleinhof (1983, the fatigue tensile test of carbon-fiber reinforced specimens) and the dataset taken from Pascual and Meeker (1999; the fatigue bending test of carbon-fiber reinforced specimens) by using the Maximum likelihood (ML) method. The formulae obtained in the previous chapter defined a non-linear regression model. But the estimation of the 6-dimension parameter η is too complicated. So we confined with approximate estimation of parameters θ_0, θ_1, r, R and (in one example) k_M also provided that $S_f = 0$. In the regression analysis the decreasing parameter number decreases the variance of the estimates of others parameters. So parameters S_f and k_M should be considered as some reserve in too difficult problems of experimental dataset processing (without using these parameters).

It is offered to find MP-estimates using the method of consecutive approximation. Static strength parameters or the parameters of the non-linear regression model can be used as preliminary estimates of parameters θ_0 , θ_1:

$$y_i = F_0^{-1}(D_f / E(T(S_i))) = -\theta_0/\theta_1 + (1/\theta_1)g(S_i) = \beta_0 + \beta_1 x_i \text{ , } x_i = g(S_i) \text{ , } i = 1,2,...,n \text{ ,}$$

of "gamma-approximation (of cdf of fatigue life and)" of the fatigue curve

$$E(T(S)) = \frac{D_f}{F_0((g(S)-\theta_0)/\theta_1)} \text{ .}$$

Parameter r can be estimated using an estimate of the coefficient of variation, parameter R - using an estimate of the corresponding ratio r/R (based on the Daniels model). To avoid tiresome calculations (we should calculate P^t for very large t) the cdf F_T ($t; s, \eta$) is approximated by the lognormal distribution. The time of the fatigue test is limited and usually we have censored data, so the maximum likelihood function on the logarithm scale is defined as follows:

1(TJ) = ln(L(/7)),

where $L(\eta) = \prod_{i=1}^{n} f_i^{A_i}(1-F_i)^{1-A_i}$ A_i is equal to 1, if the fatigue test is finished by the failure of specimens, and , A_i is equal to 0, if the time of the test is limited (right censored observation); f_i F_i are the probability density function and the cumulative distribution function of random variable T (for fixed η and S). If we use the lognormal approximation of cdf, then

$$F_i = \Phi(\frac{\log(t_i) - \theta_{0LT}}{\theta_{1LT}}) \text{ , } f_i = \frac{1}{t_i\theta_{1LT}}\varphi(\frac{\log(t_i) - \theta_{0LT}}{\theta_{1LT}}) \text{ .}$$

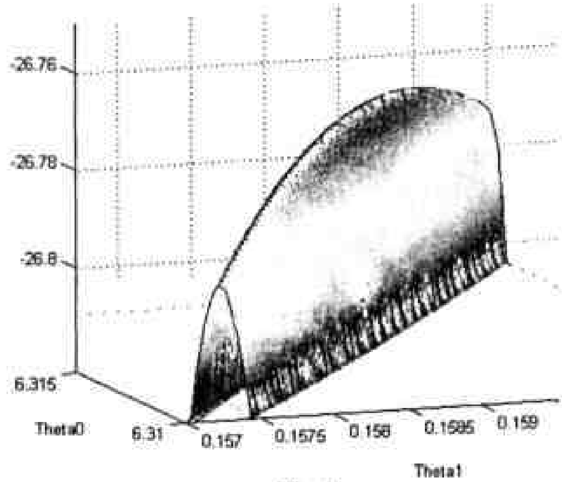


Fig.8

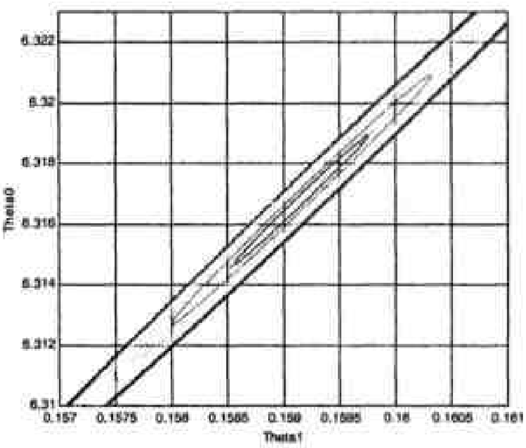


Fig.9

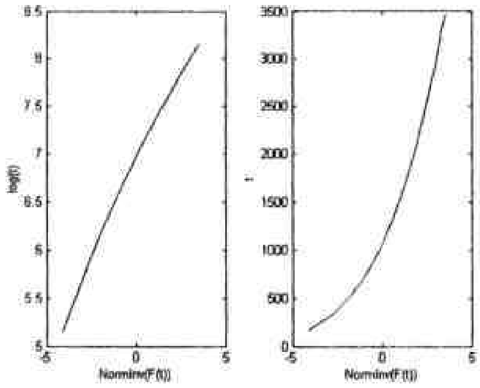


Fig. 10

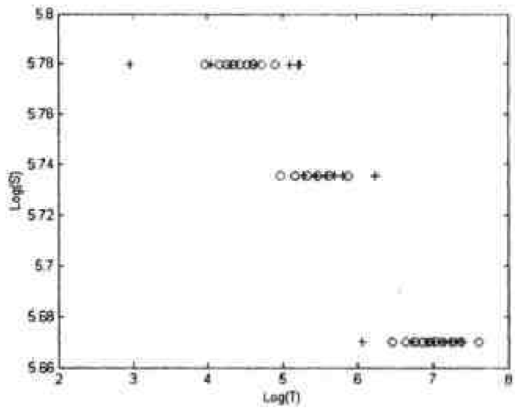
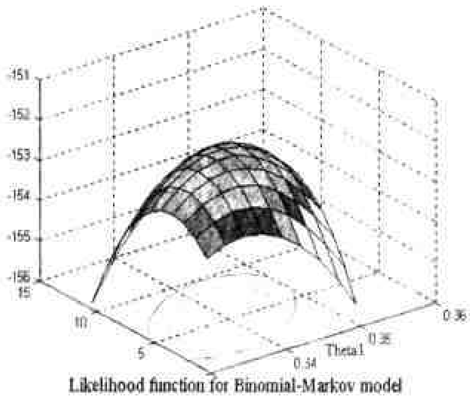


Fig. 11

For the BRM-version of the model the logarithm of this likelihood function and the corresponding contour plot of projection of lines of equal values of this function for the fragment of Kleinhof s dataset are shown in Fig.8 and Fig.9.

Fig. 10 shows that the lognormal approximation of cdf $F_T(t; s, n)$ more appropriate than the normal approximation. In Fig. 11 we see a fragment of Kleinhof's experimental fatigue dataset (+) and the calculated expectation values of the corresponding order statistics (o).

The dataset of Pascual and Meeker (1999) was processed for both the BRM- and BMM-versions of the model. The maximum likelihood function and the fatigue curve (the experiment dataset(.) and calculated expected values of order statistics(o)) for the BMM version are shown in Fig. 12 and 13.



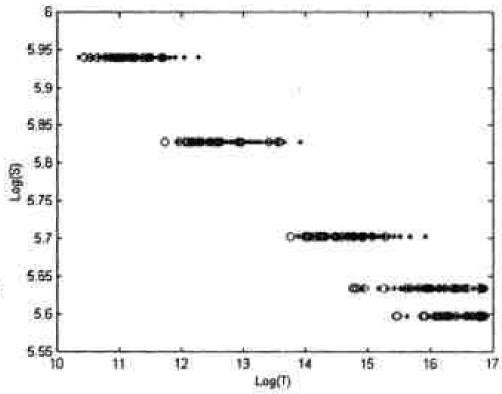


Fig. 12

Fig. 13

Comparing the corresponding fatigue curves visually it is too difficult to make a conclusion about which model is better. We have computed the values of the Akaike information criterion (AIC) statistics for each model to identify which best approximates the true underlying model. It appears that BMM provides the best fit to the data and has the smallest number of parameters. But the difference is not too great. In reality both models can be used in application to specific problems. The models under consideration can be used for calculation of cdf for program loading. Preliminary calculations show that results are very near to the linear hypothesis of the fatigue damage sum. It is known that this hypothesis sometimes is not too far from the experimental data. But it, seemingly, takes place if the fatigue life of a composite does not depend too much on the matrix. The influence of the matrix is taken into account in the model considered in Chapter 6.

Chapter 6 considers a new model in which the yielding of composite matrix is taken into account.

It is shown that this model can be used not only for the approximation of the fatigue curve but also for the description of the fatigue damage accumulation during program loading. It is shown that it gives satisfactory description of the fatigue test data at program loading with one change of loading mode. Now it is assumed that the specimen has two) parts: brittle and plastic, see Fig. 14.

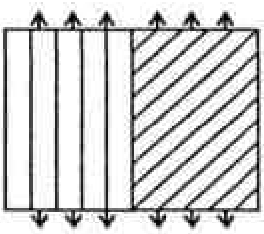


Fig.14

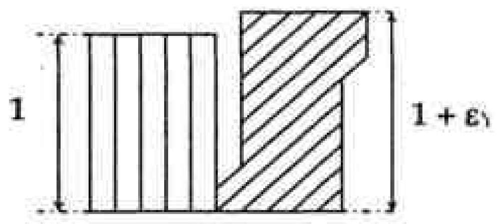


Fig. 15

In Fig. 15 these two parts are shown as independent after some act of yielding. As a consequence of yielding, the length of the plastic part increases by some value e_{y1} . The brittle and plastic parts work together. So after elimination of external loads some internal stress appears: compression in the plastic part and tension in the brittle part. This is the reason for increasing of the probability of failure of some items of the brittle part. After the failure of the critical number of brittle items the failure of specimen takes place. It takes place also after some critical number of acts of yielding. This process is described by MC.

In the model under consideration the transition probability matrix is a complex of blocks. The number of blocks is equal to one plus the critical number of acts of yielding, r_y, corresponding to the yield-type destruction. The number of states inside every block is equal to one plus the critical number of failed rigid items, r_R, corresponding to the brittle-type destruction of the specimen. For example, let destruction of the specimen take place if (1) there is failure of two rigid items (event A), or (2) there are two acts of yielding (event B), or (3) there is coincidence of both events, A and B. A corresponding transition probabilities matrix is described in Table 1.

Table 1 Structure of transition probabilities matrix

		j_Y	*1*			*2*			*3*		
		j_R	*1*	*2*	*3*	*1*	*2*	*3*	*1*	*2*	*3*
i_Y	i_R	i \ j	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	*1*	**1**	$p_{R0}p_{Y0}$	$p_{R1}p_{Y0}$	$p_{R2}p_{Y0}$	$p_{R0}p_{Y1}$	$p_{R1}p_{Y1}$	$p_{R2}p_{Y1}$	$p_{R0}p_{Y2}$	$p_{R1}p_{Y2}$	$p_{R2}p_{Y2}$
	2	**2**	0	$p_{R0}p_{Y0}$	$p_{R1}p_{Y0}$	0	$p_{R0}p_{Y1}$	$p_{R1}p_{Y1}$	0	$p_{R0}p_{Y2}$	$p_{R1}p_{Y2}$
	3	**3**	0	0	1	0	0	0	0	0	0
2	*1*	**4**	0	0	0	$p_{R0}p_{Y0}$	$p_{R1}p_{Y0}$	$p_{R2}p_{Y0}$	$p_{R0}p_{Y1}$	$p_{R1}p_{Y1}$	$p_{R2}p_{Y1}$
	2	**5**	0	0	0	0	$p_{R0}p_{Y0}$	$p_{R1}p_{Y0}$	0	$p_{R0}p_{Y1}$	$p_{R1}p_{Y1}$
	3	**6**	0	0	0	0	0	1	0	0	0
3	*1*	**7**	0	0	0	0	0	0	1	0	0
	2	**8**	0	0	0	0	0	0	0	1	0
	3	**9**	0	0	0	0	0	0	0	0	1

In this table i,j = 1,2,...,9 correspond to 9 states of Markov's chain (three possible values of failure of rigid items (0,1,2) multiplied by three possible values of yielding (0,1,2)). Probabilities p_{R0}, p_{R1}, ... denote probabilities of failure of 0, 1, ... rigid items. Probabilities p_{Y0} , p_{R1} , … denote probabilities of 0,1, ... acts of yielding.

In the above table these notations have only a symbolical sense but in reality these probabilities depend on the state of Markov chain. In the chain we have 5 absorbing states: S_3 is failure of two rigid items (FR) without yielding, S_6 is FR after one yielding, S_7, S_8 are failure after two acts of yielding when the number of failed rigid items can be equal to 0, 1, state S_9 is reached, when there is coincidence of these events.

Probabilities of transitions were calculated for the Binomial version, because in the previous chapter it was shown that it has some advantage over the Bernoulli version.

If there are n_R yet alive rigid items then the probability of the event "number of failures of rigid items is equal to k_R " is defined by the formula

$$p_R(i,j)=\binom{n_R}{k_R}\left(F_R\left(S_R\left(i_R,i_Y\right)\right)\right)^{k_R}\left(1-F_R\left(S_R\left(i_R,i_Y\right)\right)\right)^{n_R-k_R}$$

where $i = (r_R+1)(i_Y - 1) + i_R$; $j = (r_R + 1)(j_Y - 1) + j_R$; $n_R = r_R - i_R$, $k_R=j_R - i_R$ $for\ 0\leq k_R\leq n_R$, $1\leq n_R \leq (r_R-1)$; $F_R(.)$ is the distribution function of the strength of still alive rigid items; S_R (i_R , i_Y) he stress in the rigid part in *ith* state.

Similarly the probability that the number of acts of yielding is equal to k_r is defined by a similar formula:

$$p_Y(i,j) = \binom{n_Y}{k_Y} (F_Y(S_Y(i_R,i_Y)))^{k_Y} (1 - F_Y(S_Y(i_R,i_Y)))^{n_Y - k_Y},$$

where $n_Y = r_Y - i_Y$, $k_Y = j_Y - i_Y$, for $0<k_Y<n_{Yy}$ $l<n_Y<(r_Y -1)$, F_y (.) _ is the distribution function of the local stress of yielding; r_Y is the critical number of acts of yielding ; j_Y - is the number of already made acts of yielding, $S_Y(i_R, i_Y)$ is a stress in the plastic part when the number of yielding is equal to $(i_y - 1)$ but the number of failed rigid items is equal to $(i_R$,-1).

The offered model can be very easily used for calculation of cdf of the fatigue life for the program fatigue test. For any arbitrary stress cycle sequence $\{S_1,S_2,S_3,...\}$ the probability distribution function of time to failure is defined by the formula

$$F_T(t) = a(\prod_{i=1}^{t} P_i)b,$$

where matrix P_i is the matrix of transition probabilities, corresponding to the stress S_i, i = 1,2,3,... ; a - (1,0,...,0) , components of vector b are zeros or units (for absorbing states).

We have at ours disposal the dataset corresponding to the fatigue test in accordance with the program shown in Fig. 16 for both cases: S^S^{11} and $S^{!}$>S^{n} .

For the fatigue test program, which is shown in Fig. 16 the distribution function of time to failure is defined by the formula

$$F_T(t) = \begin{cases} a(P_{S^I}^{t})b, & \text{if } t \le n_1, \\ a(P_{S^I}^{n_1} P_{S^{II}}^{t-n_1})b, & \text{if } t > n_1. \end{cases}$$

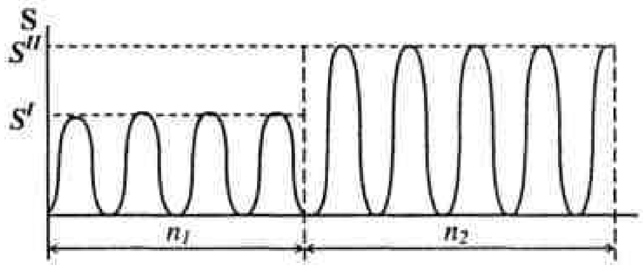


Fig. 16.

where P_S^{i}, P_S^{il} are matrices corresponding to S^1 and S^{11}.

The conditional distribution function of the residual fatigue life, T_2, of the specimen without failure in the first stage test (after n_1 cycles with $S = S'$), is defined by the formula

$$F_{T_2}(t) = \frac{F_T(n_1+t) - F_T(n_1)}{1 - F_T(n_1)}.$$

In order to check the adequacy of the model, the corresponding experimental data were processed. These data were obtained by M. Kleinhof, who has made a test of carbon-fiber specimens to get the fatigue curve and to study the residual fatigue lives in two versions of programs: S^S^{11} un S^S^{11}. The results of processing these data are shown in Fig. 17, 18, 19.

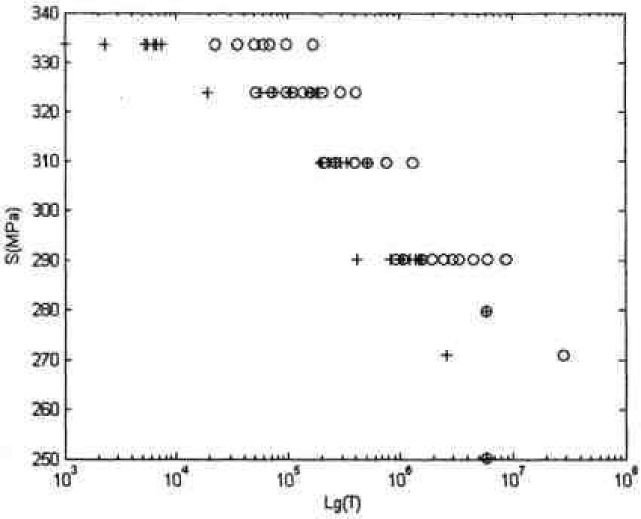


Fig.17.

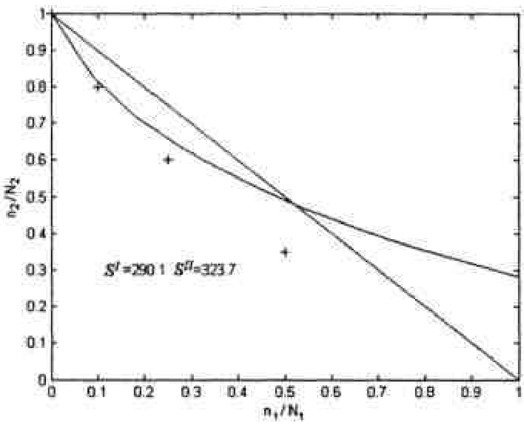


Fig. 18

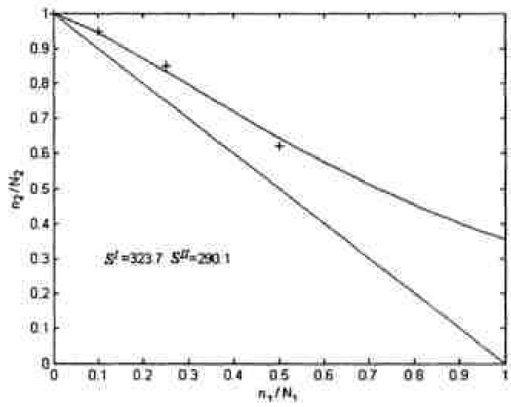


Fig. 19

In Fig. 17 the experimental data for the fatigue curve are shown by "+". The result of calculations of order statistics (corresponding to the size of the experimental sample size) for every stress level for which the test was made are shown by "o". At the lowest stress level the limitation of test time took place.

Dependence of the relative residual fatigue life n_2/N_2 on the relative damage n_1/N_1 of the first stage of program cyclic loading is shown in Fig. 18, 19 . Here the results of calculations are shown by curve lines, but the experimental data are shown again by "+". We see that at chosen parameters, just as in the test, at small n_1/N_1 the calculation gives

$$\frac{n_1}{N_1}+\frac{n_2}{N_2}>1 \text{ if } S^I>S^{II} \text{ and } \frac{n_1}{N_1}+\frac{n_2}{N_2}<1 \text{ if } S^I<S^{II} .$$

In order to understand the behaviour of these curves we should take into account the failure of some specimens already in the first stage of program cyclic loading. Only strongest specimens can be used for loading with stress S^{11}. It is worth to mention, in particular, that usually in publications on this type of figures $n_2 / N_2 = 0$ when $n_1 / N_1 = 1$. But it can be true only if we do not take into account the scatter of fatigue life. In reality at $n_1 / N_1 = 1$ approximately only one half of the total number of specimens are destroyed and in Fig. 18 and 19 at $n_1 /N_1 =1$ the residual fatigue lives in the loading with $S = S''$ of specimens which are not destroyed at $S = S^l$ are shown. It is not equal to zero!

Conclusions

1. The author offers a mathematical model, which as distinguished from other models, presents a description of the fatigue damage accumulation in a unidirectional composite and in the cycling test with constant cycle characteristics (to obtain the fatigue curve) and in program loading. The parameters of the model of the fatigue curve can be "physically" interpreted as parameters of local static stress distribution parameters.
2. This model can be used for the non-linear regression analyses of the fatigue curve and for forecasting the changes having a change in the characteristics of composite components and for forecasting the fatigue life at program loading. In all these cases the model gives not only the mean value of fatigue life but also its distribution function.
3. Having a fixed number of cycles with fixed characteristics of the cycle, the model gives the distribution of the limited fatigue limit.
4. The soundness of the models has been checked by using experimental data.
5. The author has developed a package of the program for modelling static strength and fatigue life as well as estimating model parameters.

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