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POWER WAY OF THE DESCRIPTION OF CLOSED LOOPS MECHANICAL HYSTERESIS

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The differential equation of communication between the amplitude law of scattering of energy and skeletal line of a closed loop of a mechanical hysteresis is injected. It is offered the universal description of the law of scattering of energy permitting to synthesize closed loops of a hysteresis of the various shape.

1. Differential equation of a line of initial loading.

At realization of dynamic calculations of mechanical systems with loopback nonlinearities the precisiouly informations about a skeletal curve, i.e. about relation between a load and movement on a pioneering stage of loading are required. As shows the theoretical analysis of loading and unloading of the most diverse systems with a constructional hysteresis, the outlines of closed loops of a hysteresis are map skeletal curve in the double scale, i.e. the Maising principle (1) is executed. For obtaining a skeletal curve design inappropriate to the known computational schemes, the realization of special experiment requiring large accuracy in measurement of loads and movements is necessary. A new way of obtaining of an equation of a skeletal curve, and consequently, and outline of a closed loop of a hysteresis is offered below. The way installs precisiouly differential ratioes between an equation of a skeletal curve and relation of size of energy dispersed in a design for a cycle of loading, from amplitude of a cycle. Necessary for a realization of a way the data can be simplly and with high accuracy obtained during dynamic tests of a design.

For a conclusion of a ratio, interesting for us, we shall consider quasistatic loading of a system with one degree of freedom. Let's assume, that the system is allocated a set of drains of energy, which with increase of a level of a load sequentially are uncovered. Let's enter the following labels: **x** - movement; **f** - the function is elastic-hysteresis; **A** - area of a closed loop of a hysteresis at a symmetrical cycle, i.e. quantity of energy dispersed for a cycle of loading; $\mathbf{c_i}$ - rigidity of a system on **i** a stage of increase of a load.

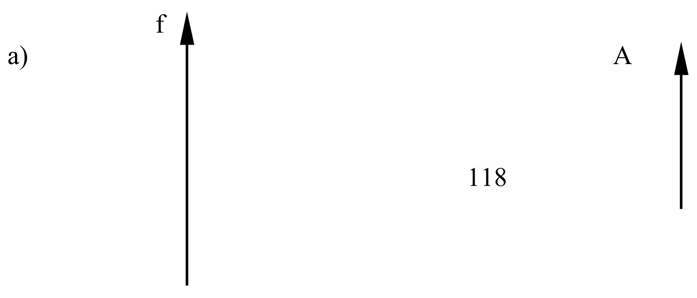
Let's consider the first phase of loading, to which the increasing of a load is corresponded. Right at the beginning of loading, while $\mathbf{0 \le x \le x_0}$, the system remains linear, i.e. the scattering of energy is absent (Figure 1a). With the increasing of a load, when $\mathbf{x \ge x_0}$, in a system the first drain of energy is opened, occur mutual slipping of particles. The cyclical change of a load in this case results in formation of a four-coal closed loop of a hysteresis and scattering of energy. It is possible to show, that thus

$$f = c_1 x + x_0 (c_0 - c_1); \tag{1}$$

$$A = 4x_0(c_0 - c_1)(x - x_0) = k_1(x - x_0), \qquad x_0 \le x \le x_1. \tag{2}$$

From a ratio (2) it is visible, that the area of a four-coal closed loop of a hysteresis linearly depends on increment of movement $\Delta x = x_1 - x_0$.

Let's assume further, that at the first stage of loading there was a deployment not one, bet two drains of energy. Thus, the movement x has exceeded x_1, and on a skeletal line 0_1-0-1-2 in a point 1 one more fracture 1-2 (Figure 1b) has appeared. The cyclical change of a load of the given level results in formation of a six-coal closed loop of a hysteresis, which area develops of the areas of two shaded triangles and



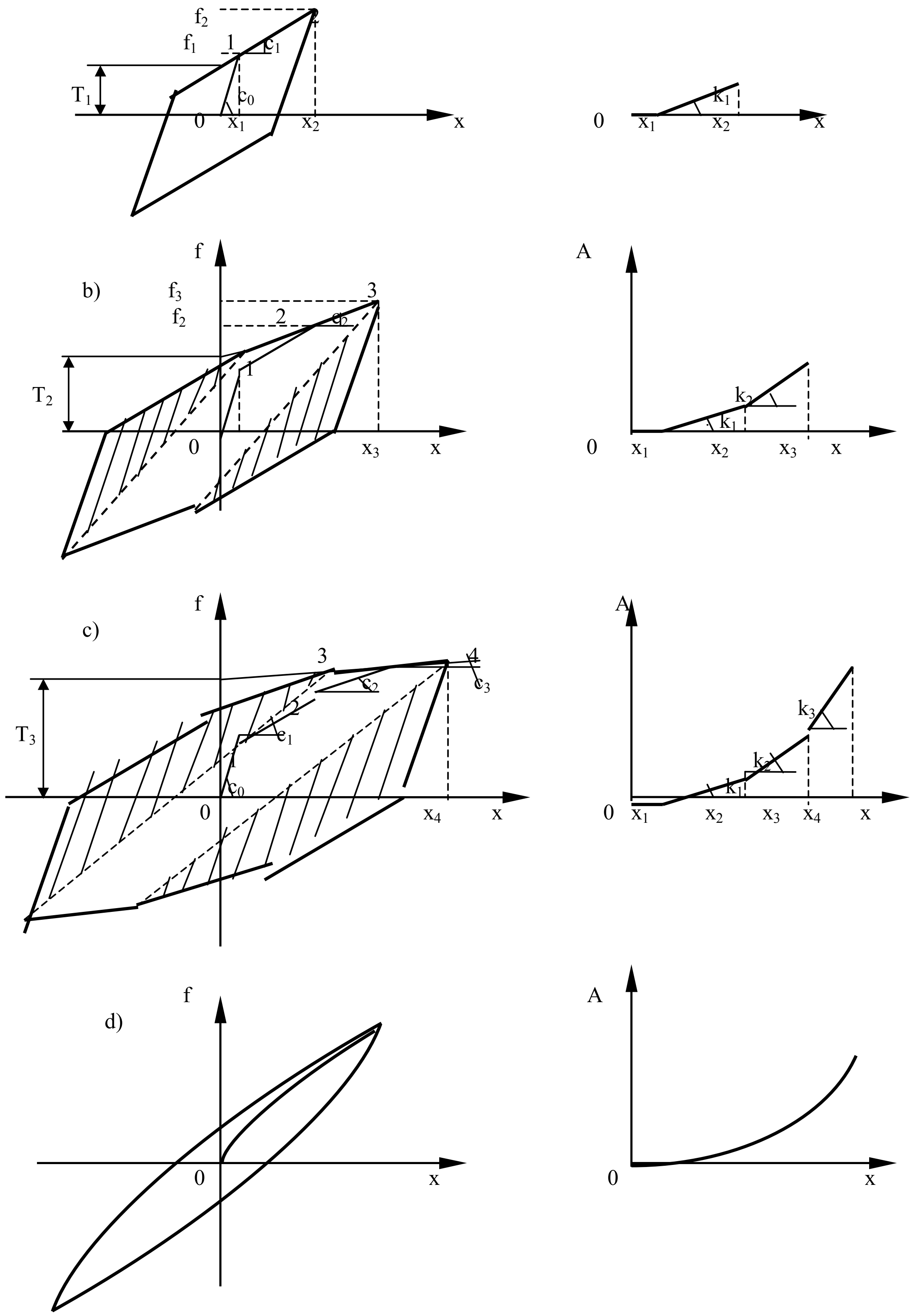


Figure 1. Amplitude transformation of closed loops of a hysteresis and law of scattering of energy

parallelogram (Figure 1a), obtained up to a moment of the second drain of energy deployment at $x=x_1$, $f = f_1$. For a six-coal closed loop of a hysteresis it is possible to record the following ratioes:

$$f = (c_0 - c_1)x_0 + (c_1 - c_2)x_1 + c_2 x; \qquad (3)$$

$$A = A_1 + 4(f_1 - x_1 c_2)(x - x_1) = A_1 + k_2(x - x_1), \qquad (x_1 \le x \le x_2). \tag{4}$$

Thus on the schedule of relation A(x) appear a new linear site with an angular factor k_2 (Figure 1b). Further it is possible to assume, that on a pioneering stage of loading there was a series deployment of three drains of energy, that has resulted in occurrence of a new fracture in a point 2 on a skeletal line (Figure 1.c). The closed loop of a hysteresis obtained at cyclical deformation at this level of a load, represents an octagon. It is uneasy to be convinced, that the hexagon, obtained at the previous stage, is enclosed in outlines of an octagon as two shaded tetragons (Figure 1.c). The area of an octagon can be found as a sum of the areas of a hexagon obtained at loading up to values $f=f_2$, $x=x_2$ and central parallelogram:

$$f = (c_0 - c_1)x_0 + (c_1 - c_2)x_1 + (c_2 - c_3)x_2 + c_3 x; \tag{5}$$

$$A = A_2 + 4(f_2 - x_2 c_3)(x - x_2) = A_2 + k_3(x - x_2), \qquad (x_2 \le x \le x_3) \tag{6}$$

On the schedule of relation A(x) appear one more linear site with an angular factor k_3. It is possible further to present, that the gradual increase of a level of a load of a pioneering stage results in a series deployment of new drains of energy, to occurrence of new fractures on a skeletal line and on relation A(x). Thus number of legs of a polygon presenting a closed loop of a hysteresis at cyclical loading continuously is increased.

Generalizing obtained outcomes for cyclical loading after a deployment **i** of a drain of energy, it is possible to record the following expressions for a skeletal line f(x) and relations A(x):

$$f = \sum_1^i (c_{i-1} - c_i) + c_i x; \tag{7}$$

$$A = A_{i-1} + 4(f_{i-1} + x_{i-1} c_i)(x - x_{i-1}) = A_{i-1} + k_i(x - x_{i-1}), \qquad (x_{i-1} \le x \le x_i) \tag{8}$$

From a ratio (8) it is possible to receive expression for rigidity appropriate to a **i** site of the skeletal broken line:

$$c_i = \frac{f_{i-1}}{x_{i-1}} - \frac{A - A_{i-1}}{4x_{i-1}(x - x_{i-1})}. \tag{9}$$

Taking into account, that with $c_i = \frac{\Delta f_i}{\Delta x_i}$, $\Delta x_i = x - x_{i-1}$, $\Delta A_i = A - A_{i-1}$, expression (9) is rewrited as

$$\frac{\Delta f_i}{\Delta x_i} = \frac{f_{i-1}}{x_{i-1}} - \frac{\Delta A_i}{4x_{i-1}\Delta x_i}. \tag{10}$$

Passing in expression (10) to a limit at $\Delta x_I \to 0$, i.e., supposing, that the deployment of one drain of energy continuously follows other, we obtain a differential equation

$$\frac{df}{dx} = \frac{f}{x} - \frac{dA}{4x dx}. \tag{11}$$

This ratio installs connection between an equation by skeletal curve f(x) and relation dispersed for a cycle of loading of energy from amplitude of a cycle A(x). Knowing relation A(x) and initial rigidity of a system c_0, it is possible by taking advantage ratio (11), to receive an equation f(x) to generate outlines of a closed loop of a hysteresis.

Let's define a decrement of oscillations δ as the relation of energy dispersed for a cycle of loading, to the double energy of elastic deformation W, and we shall accept, that W=fx/2. Then

$$\delta = \frac{A}{2W} = \frac{A}{fx}, \qquad A = \delta f x. \tag{12}$$

We differentiate (12) on x:

$$\frac{dA}{dx} = \delta' f x + f' x \delta + f\delta. \tag{13}$$

Substituting expression (13) in (11), we obtain a differential equation:

$$\frac{f'}{f} = \frac{4 - \delta' x - \delta}{(4 + \delta)x}. \tag{14}$$

2. Equation of a skeletal line and formation of an outline of a closed loop of a hysteresis

The offered above power method of identification of a line of initial loading together with a Masing principle is convenient for a construction of closed loops of a mechanical hysteresis. Amplitude relation of scattering of energy A(x) and initial rigidity c_0 are determined from static and dynamic tests. The integration of an equation (11) with allowance for of relation A(x), obtained from experiment, determines an equation f(x) of a line jf initial deformation. The outlines of a closed loop of a hysteresis, according to a Masing principle, are map of a skeletal line in the double scale. In activities [2,3] is underlined, that for many hysteresis systems the amplitude relation A(x) maybe recorded as a degree function. However, such approximation unsufficiently completely reflects possible versions of scattering of energy in designs, in particular, does not describe the laws of scattering at a polygonal hysteresis. In this connection for A(x) suggest more general format of representation, in which is supposed availability of two sites:

At $0 < x < a$, $A(x) = 0$; at $x > a$, $A(x) = b\,(x-a) + k\,(x-a)^n$, (15)

where b, k, n - factors dependent on a type of a design and its operating time ; and a - the value of generalized movement, after which reaching begins scattering energy.

The solution of an equation (11) for the first stage gives that $f = cx_0$, at $0 < x < a$. Thus initial site of a skeletal line is a section direct, i.e. linear elastic characteristic. At the second stage the equation (11) after a substitution in it of expression dA/dx gains a kind:

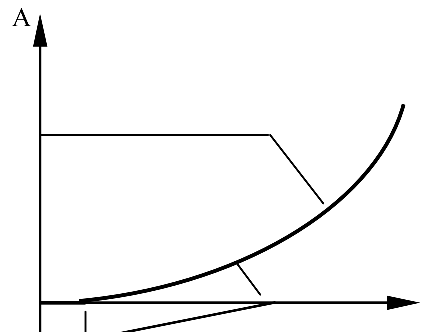
$$\frac{df}{d(x-a)} = \frac{f - b/4}{x-a} - \frac{nk}{4}(x-a)^{n-2}, \qquad (x \geq a). \tag{16}$$

The solution of this differential equation is the following expression

$$f = \frac{b}{4} + c_1(x-a) - \frac{nk}{4(n-2)}(x-a)^{n-1}, \tag{17}$$

circumscribing a skeletal curve at the second stage. Constants of an integration $\mathbf{c_0}$, $\mathbf{c_1}$ on a sense are stiffness factors in the beginning each from sites. Various combinations of factors **a, b, k** in relation A(x) reduce to a realization of closed loops of a hysteresis of the various form. In case, when $a \neq 0$, $b \neq 0$, $k \neq 0$ closed loops have till two curvilinear and straight-line inclined sites (Figure 2.a). If a=0, the inclined lateral sites become vertical (Figure 2.b). At a=0, k=0, but $b \neq 0$ closed loops are transformed in bilinear with vertical lateral segments (Figure 2.c), if $a \neq 0$, $b \neq 0$, but k = 0 - in bilinear with inclined sites (Figure 2.d). At last at a=b=0, but $k \neq 0$ outlines of a closed loop are curvilinear (Figure 2.e). If thus of zero rigidity of a design c_0, c_1 in different directions of initial loading are various, for example, because of availability of a crack, the closed loop becomes asymmetrical, gaining the shape of a banana. Included in expression (15) amplitude laws of scattering of energy parameters a, b, k, n vary in accordance with accumulation with a design in maintenance of not desroying damages. The being available experimental data allow approximately to set ranges of change of these parameters from a moment of a beginning of maintenance of a design before failure.In initial stage the design works as a monolith, it the rigidity is maximum, scattering of energy it is not enough, closed loop of a hysteresis vary narrow. In accordance with a wear of fixed connections reduction rigities of components and joints happens, change the law of scattering of energy. It varies results in change of parameters and transformation of closed loops of a hysteresis. It can call strong influence on dynamic loading of a design.

$$\frac{df}{dx} = \frac{f}{x} - \frac{1}{4x} \cdot \frac{dA}{dx}$$



b (x - a) + k (x - a) ^{n}

$0 \le x \le a, \qquad f = c_0 x$

$a \le x \le x_{max}$

$$f = \frac{b}{4} + c_1 x - \frac{nk}{4(n-2)} \cdot (x-a)^{n-1}$$

b

0 a x

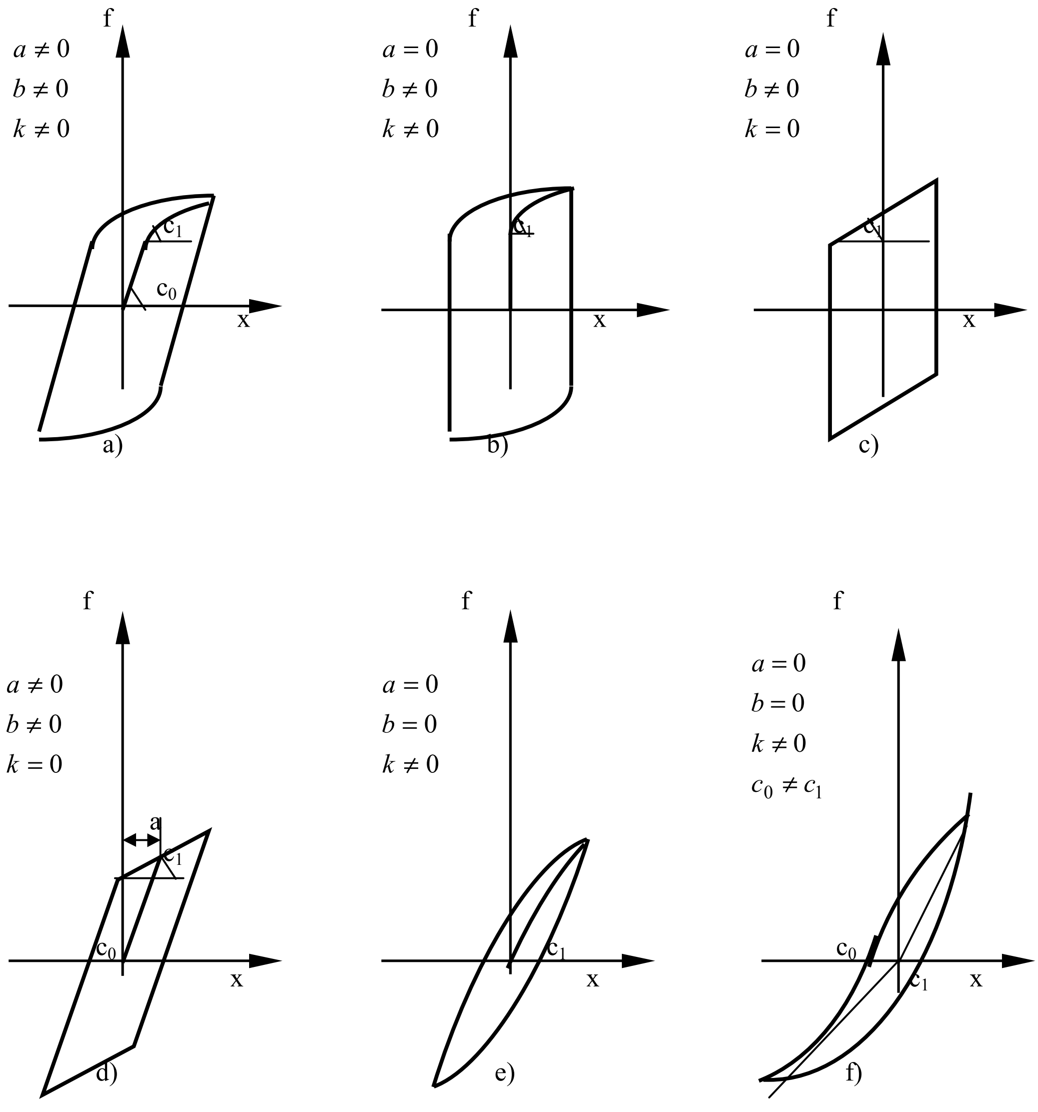


Figure 2. Kinds of closed loops of a hysteresis

3. Synthesis of polygonal closed loops of a mechanical hysteresis

Let's assume, that for some mechanical system is assigned particular the law of scattering of energy A(x). By taking advantage a ratio (11) amplitude-depends, it is possible to determine a kind of a skeletal line f(x), appropriate to it. We shall show, that with introduction in a system of concentrated drains of energy of different power it is possible with a necessary degree of accuracy to supply reproduction f(x) and A(x). Is researched quasistatic loading by a torque of a system from “n” of inertial elements-disks, jointed is elastic-hysteresis by communications. Let's assume, that each communication is a concentrated drain of energy with a closed loop of a hysteresis of a parallelogram kind. Let's enter the following labels: c_i, d_i - maximum and minimum rigidity of a i-site of communication; T_i - width of a closed loop of a hysteresis of i-communication in origin; φ_i^{rel}- angular deformation of i-communication; φ_n - turn angle of an n-element concerning seal; $c_i^{ek}=dM/d\varphi_n$ - equivalent torsional rigidity of a system.

On a pioneering stage of loading of a system, when all drains of energy are closed, the system is deformed as linear (Figure 3.b). Thus

$$\varphi_i^{rel}=\frac{M}{c_i},\quad \varphi_n=M\sum_1^n\frac{1}{c_i},\quad c_0^{ek}=\left(\sum_1^n\frac{1}{c_i}\right)^{-1},\quad (M<M_1). \tag{18}$$

At increase of a load begin to be uncovered drains of energy. Let's assume, that as soon as $M=M_1$, begins to work a drain of energy in the first connection, the remaining sites while remain linear. On the schedule of loading of the first communication at $M=M_1$ occurs a fracture (Figure 3.c). On a skeletal line of all system a fracture called by change of the equivalent rigidity c_i^{ek} also will appear. Up to a moment of a deployment of the second drain the following relations are fair:

$$\varphi_1^{rel}=\frac{M-T_1}{d_1},\quad \varphi_i^{rel}=\frac{M}{c_i},\quad \varphi_n=M\left(\frac{1}{d_1}+\sum_2^n\frac{1}{c_i}\right)-\frac{T_1}{d_1},$$

$$c_1^{ek}=\left(\sum_2^n\left(\frac{1}{c_1}+\frac{1}{d_1}\right)\right)^{-1},\quad (i=2,3,...,n),\quad (M_1\le M<M_2). \tag{19}$$

If at these levels of deformations to conduct cyclical loading of a system, quantity of energy, scattered in the first communication for a cycle, will be defined by the area of its closed loop of a hysteresis

$$A_1=4T_1(\varphi_1^{rel}-\varphi_{11}^{rel})=4\frac{T_1}{d_1}\left(M-\frac{T_1c_1}{c_1-d_1}\right). \tag{20}$$

The external moment M on moving φ_n will make activity, appropriate to the area of a parallelogram in coordinates M - φ_n

$$A_M=4\frac{T_1^{ek}}{c_1^{ek}}\left(M-\frac{T_1^{ek}c_0^{ek}}{c_0^{ek}-c_1^{ek}}\right). \tag{21}$$

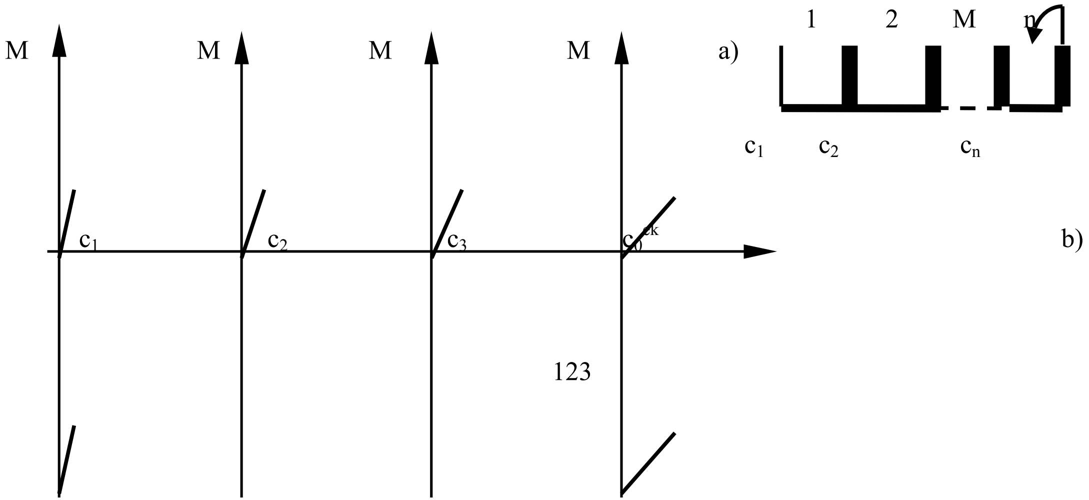
Equating from power reasons of expression (20) and (21), we obtain a ratio

$$\frac{T_1}{d_1}=\frac{T_1^{ek}}{c_1^{ek}}, \tag{22}$$

permitting to select necessary parameters T_1, d_1 for the first drain of energy.

$$\varphi_n=M\left(\sum_1^k\frac{1}{d_i}-\sum_{k+1}^n\frac{1}{c_i}\right)-\sum_1^k\frac{T_i}{d_i}=\frac{M}{c_i^{ek}}-\sum_1^k\frac{T_i}{d_i},\quad \varphi_k^{rel}=\frac{M-T_k}{d_k},\quad c_i^{ek}=\left(\sum_1^k\frac{1}{d_i}+\sum_{k+1}^n\frac{1}{c_i}\right). \tag{23}$$

If at initial loading the operational moment M will exceed value of M_2, the deployment of the second drain of energy (Figure 3.d) will happen. At $M>M_3$ is uncovered the third drain (Figure 3.e), etc. The deployment of each new drain of energy results in increase of number of fractures and sites on a general skeletal line of a system $M(\varphi_n)$ and in a limit $n\to\infty$ will transform a skeletal line and closed loop of a hysteresis to a curvilinear kind (Figure 3.f). Shall consider such level of a load, when in a system from n of drains k are uncovered. Up to this moment the following relations are fair: at $M_k < M \le M_{k+1}$,



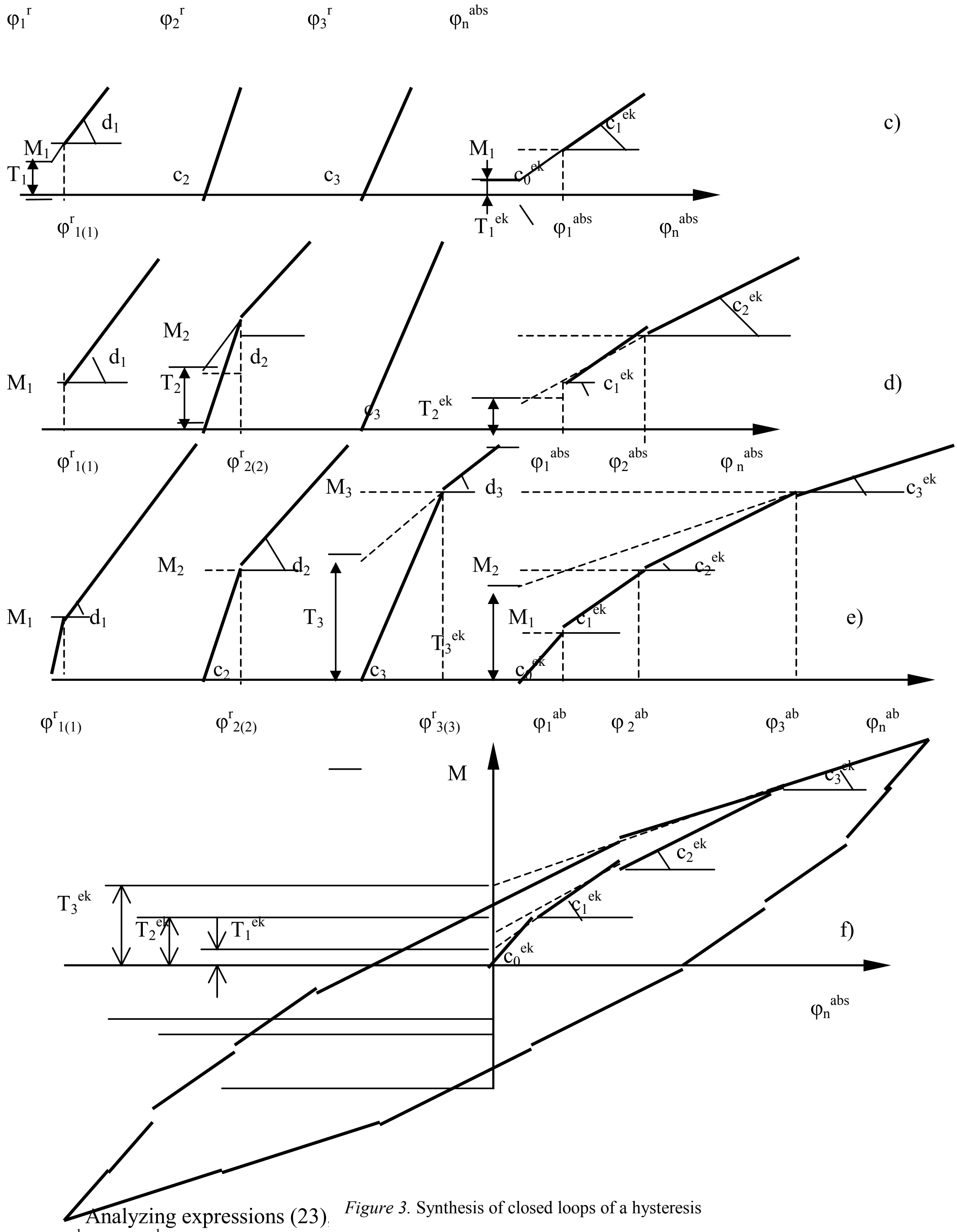


Figure 3. Synthesis of closed loops of a hysteresis

Analyzing expressions (23),

$M = T_k^{ek} + \varphi_n c_k^{ek}$, (24)

obtain the following ratio for definition of parameters of elastic - hysteresis communications

$$\frac{T_K}{d_K} = \frac{T_i^{ek}}{C_i^{ek}} - \sum_{1}^{k-1} \frac{T_i}{d_i}. \qquad (25)$$

Thus, introduction in a system of concentrated drains energy the elementary bilinear kind can achieve anyone beforehand of specific amplitude law of scattering of energy. The selection of parameters of drains of energy can be conducted in the following sequence:

1. Under the specific law of scattering of energy A(x), help of expression (11), determines an equation of a skeletal line f(x);
2. The number "n" of concentrated drains of energy in a mechanical system is assigned;
3. The piecewise linear approximation of relation by f(x) partition on (n+1) site is carried out;
4. The series calculation of parameters of a hysteresis T_i, d_i for each drain on established from of linear approximation to values T_i^{ek} and c_i^{ek} with use of a ratio (25) is carried out.

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