FATIGUE-PRON AIRFRAME ITEM INSPECTION MODELING BY THE USE OF MONTE CARLO METHOD

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Introduction

For some simple cases it is easy enough to get formula for fatigue failure of fatigue-pron airframe item under some inspection program. But for more general cases it is useful to make modelling of inspection process by the use of Monte Carlo method. The special PC program was constructed for this goal. The program can be considered as a project of second phase of the automated system for airframe inspection program development [P.B.]

In the project some limitations of the first phase (all aircraft begin service simultaneously, probability of fatigue crack discover is equal to 1, if the crack is detectable etc.) are omitted and much more wider problem spectrum now can be solved, but there is also and some setback: the time of problem solution increases. So it should be used mainly for solution of the problems, which cannot be solved by the first phase of the system. It can be used also for demonstration purposes in corresponding university courses.

The system is not finished yet and now we'll consider only two main parts of it: fatigue crack growth function parameter and fatigue failure probability estimations.

Determination of Fatigue Crack Growth Function Parameters

The initial information contains data about the size of a crack a_i during the operating time t_i: list of pairs $\{(t_i, a_i)\ i=1,\ldots, n\}$. Later supplementary computed in IBM data's necessary for determination of fatigue crack growth function and construction of illustrative graphics $a(t)$, $\sigma_r(t)$ will be received: $\hat{a}_i, \hat{\sigma}_{ri}$, $i=1,\ldots,n$ - crack size and residual strength, $i=1,\ldots,n$ calculated accordingly to correspondent formulas after estimation of fatigue crack growth function parameters. It is supposed that

$$\frac{da}{dt} = C(\Delta K)^m = Qa^{m/2}, \tag{1}$$

where

$$Q = C(\lambda(\sigma_{\max} - \sigma_{\min})\sqrt{\pi})^m, \tag{2}$$

c, m - parameters of crack growth function,
λ - takes into account width of a pannel, stringers, …
σ_{max} , σ_{min} - maximum and minimum stresses during cycling loading.

Then we have:

$$y = \ln\frac{da}{dt} = \ln Q + \frac{m}{2}\ln a = b_0 + b_1 x\,. \tag{3}$$

The parameters b_0, b_1 – are evaluated by least squares method in accordance with formulas

$$\hat{b}_1 = \frac{(\overline{xy} - \overline{x}\,\overline{y})}{(\overline{x^2} - \overline{x}^2)},\ \hat{b}_0 = \overline{y} - \hat{b}_1\overline{x}\,. \tag{4}$$

By the use of these formulas the calculations are made in the frame of special programme module and the results of calculations are printed together with initial data:

$$Q = e^{\hat{b}_0},\ \hat{m} = 2\hat{b}_1,\ \ \hat{c} = \frac{\hat{Q}}{(\lambda(\sigma_{max} - \sigma_{min})\sqrt{\pi})^{\hat{m}}}\,. \tag{5}$$

For determination of $\hat{a}(t)$, estimation of a(t), first we should make estimation $\hat{a}(0)$ again by the use of least squares method

$$S(a(0)) = \sum_i \left(a(t_i, a(0)) - a_i\right)^2,$$
$$\textit{where}\quad a(t_i, a(0)) = \frac{a(0)}{\left(1 - \mu a(0)^{\mu} Q t_i\right)^{1/\mu}}, \tag{6}$$

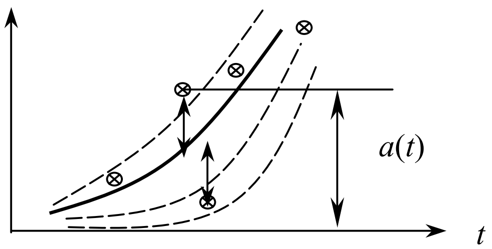


Figure 1

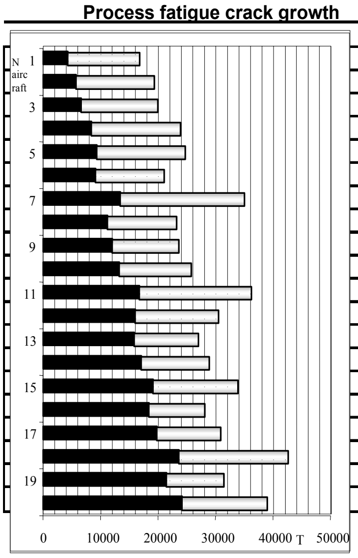
The minimum of a function is searched by the use of a sequence $a(0)_1, a(0)_2, ...$ estimate of the initial size corresponds to min $S(a(0))$.

The calculation of residual strength is made under the formula

$$\sigma_p(t) = \frac{K_c}{\lambda\sqrt{\pi a}}\,. \tag{7}$$

Simulation of a Process of Fatigue Crack Inspection

It is assumed that some inspection technology is characterized by two values: a_{det} the maximum size of a undetectable crack, which further is called also as length of a visible crack, ω is interpreted as probability that the earlier scheduled inspection will be made. Actually, ω is a human factor exerting influence on probability of failure. For an illustration purpose the first experiment consists of simulation of realizations by the use of Monte Carlo method (fig. 2). Results of these calculations are shown on screen.



N 5			ε 112													
Tdet	Tf	Δ	24	48	73	97	121	146	170	195	219	243	268	292	317	341
4289	16777	r	5	3	2	1	1	1	0							
5699	19381	r	5	2	2	1	1	1	1	0						
6590	19957	r	6	3	2	2	1	1	1	1	0					
8339	23885	r	6	3	2	2	1	1	1	1	1	0				
9301	24736	r	7	4	2	2	2	1	1	1	1	1				
9109	21074	r	5	3	1	2	1	1	1	1	0					
13415	35005	r	9	5	3	2	1	2	2	1	1	1	1	1	1	1
11147	23222	r	5	2	2	1	1	1	1	1	1	0				
11996	23630	r	5	2	2	1	1	1	1	1	1	0				
13280	25743	r	5	3	2	1	1	1	1	1	1	1	0			
16702	36218	r	8	4	2	2	1	1	2	1	1	1	1	1	1	1
15991	30522	r	6	3	2	2	1	1	1	1	1	1	1	1	0	
15841	27024	r	5	2	1	1	1	0	1	1	1	1	1			
17065	28903	r	5	2	1	1	1	0	1	1	1	1	1	0		
19097	33937	r	6	3	2	2	1	1	0	1	1	1	1	1	1	0
18351	28109	r	4	2	1	1	1	0	0	1	1	1	1	0		
19809	30901	r	4	2	2	1	1	1	0	0	1	1	1	1	0	
23564	42675	r	8	4	2	2	2	1	1	1	0	1	1	1	1	1
21442	31464	r	4	2	2	1	1	1	0	0	1	1	1	1	0	
24123	39038	r	7	4	2	2	2	1	1	1	0	1	1	1	1	1
ε 12195			1.9	0.00	0.02	0.0	0.0	0.2	0.3	0.	0.	0.4	0.5	0.	0.7	0.
		P(t)	4.2	0.00	0.00	0.00	0.0	0.04	0.10	0.21	0.3	0.54	0.7	0.84	0.92	0.97

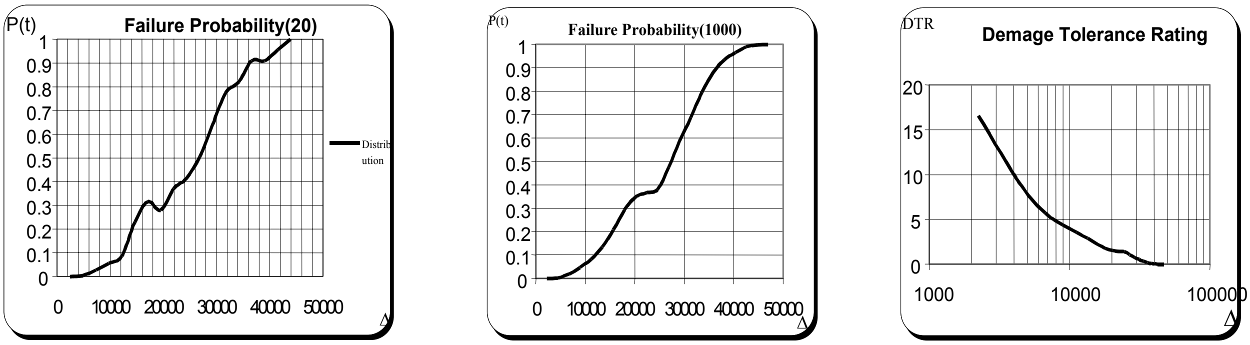


Figure 2.

If we have desire to take into account that the random value of Q is defined not only by the scatter of stress concentration factor, scatter of initial material characteristic, etc. (which define the difference one aircraft from another one) but also by the scatter of load from one flight to another flight, then we have to consider the influence of scatter Q in a random process { ... Q_{j-1}, Q_j, Q_{j+1} ... }, where j - is the order number of flight (or cycle).

To obtain initial couples of random variables (t_{det}, t_f), time when fatigue crack become detectable and time to failure, following calculations are necessary

$$t_{\det i} = \frac{1-\left(\dfrac{a_0}{a_{\det}}\right)^{\mu}}{\mu a_0{}^{\mu} Q_i} = \frac{c_{\det}}{Q_i},$$
$$c_{\det} = \frac{1-\left(\dfrac{a_0}{a_{\det}}\right)^{\mu}}{\mu a_0{}^{\mu}}, \tag{8}$$

$$t_{fi} = \frac{1 - \left(\frac{a_0}{\left(\frac{K_c}{\lambda \sigma_{\max} \sqrt{\pi}} \right)^2} \right)}{\mu a_0^{\ \mu} Q_i} = \frac{c_f}{Q_i},$$

$$c_f = \frac{1 - \left(\frac{a_0}{\left(\frac{K_c}{\lambda \sigma_{\max} \sqrt{\pi}} \right)^2} \right)}{\mu a_0^{\ \mu}}, \qquad (9)$$

$$Q_i = e^{\left(\theta_0 + \theta_1 \Phi^{-1}(Rnd_i)\right)}, \qquad (10)$$

where $a_0 = a(0)$,

$a_{\det}$ - detectable (with p=1) size of fatigue crack,

θ_0, θ_1 are mean and standard deviation of $\ln Q$,

Rnd_i - random variable uniformly distributed in interval [0,1], i=1,…,N; N=20,1000.

For the purpose of Monte Carlo modelling we need to have statistics of estimations of θ_0 and θ_1. In tables 1 and 2 the results of special test data processing are given for two types of specimens [1] for m=2 (table 1) and $m \neq 2$ (table 2).
In accordance with these data during fatigue crack modelling we have used the value $\sigma(\ln Q) = \theta_1 = 0.155 \div 0.2$

TABLE 1

specimen	$Q * 10^{-3}$	coef.of variation.	μ ln$Q$	σ ln$Q$
WPF(33)	0.273	16.86%	-8.219	0.1552
XWPF(37)	0.3725	21.57%	-7.919	0.2197

TABLE 2

Specimen (sanple size)	mean of Q(10^{-3})	coef.of variation 0f Q	mean of m	coef.of variation of m	ρ	μ ln$Q$	σ ln$Q$
WPF(30)	0.182	67.1%	1.745	12.5%	0.921	-8.758	0.499
XWPF(34)	0.3884	92.5%	1.8836	17.63%	0.920	-8.138	0.702

Estimation of Failure Probability

Mean probability of failure (for 20 aircraft), it we have r_j inspections for j-th aircraft j=1,…,20:

$$P_f = \frac{\sum_{i=1}^{20} (1-\omega)^{r_j}}{20}. \qquad (12)$$

The result of calculation can be interpreted both as result of careful continuous inspection (ω=1) and as selective (ω<1) one and the sectional experiment can be done 50 times simultaneously thus the mean probability on 1000 realizations is received, the obtained values are put on the chart $P_f(\Delta)$.

A maximum of allowed interval between inspections should be equal to $\Delta=\Delta^*$ at which $P_f = 1-R$, where $R=1-P_f$ is required reliability level (as example we put R=0.9).
To smooth the results of experiment it is necessary to use the approximation $P_f(\Delta)$, for example, by the normal distribution function. Then the failure probability will be described by formula

$$P_{ftheor} = \Phi\left(\frac{x-a}{b}\right), \tag{13}$$

where

$$a = \frac{x_2\Phi^{-1}(y_1) - x_1\Phi^{-1}(y_2)}{\Phi^{-1}(y_1) - \Phi^{-1}(y_2)},$$
$$b = \frac{x_1 - x_2}{\Phi^{-1}(y_1) - \Phi^{-1}(y_2)}, \tag{14}$$

$(x_i, y_i)=(\Delta_i, P_f(\Delta_i))$, i=1,2 – are results of calculation for two inspection interval Δ_1, Δ_2 .
Boeing uses Damage Tolerance Rating, DTR, instead of P_f. Transition from probability of failure to DTR can be made by the use of formulae:

$$DTR = -\frac{\ln P_f}{\ln 2},$$

because in accordance of DTR definition (15)

$$P_{\det} = 1 - P_f = 1 - \frac{1}{2^{DTR}}.$$

The work of the modules begins after start of automated system. The result appears only after the calculation of failure probability will be finished. As the results of it's work we'll get $P_f(\Delta)$. And then we made calculation of inspection interval by the use of formula

$$\Delta = a + b\Phi^{-1}\left(P_{fa}\right), \tag{16}$$

where P_{fa} – allowable failure probability (P_{fa}=1-R) R is required reliability (for example R=0.9).

References

1. Yang J. N. "Distribution of Equivalent Initial Flow Size", Journal of Aircraft, vol. 22, N 9 1995, pp. 810-817.