

## MODEL OF THE INFLUENCING OF SIZES ON FATIGUE LIFE OF SHEET DETAILS FROM AN ALUMINUM ALLOY

*Vitaly Pavelko, Julia Timoshchenko*

*Aviation Institute of Riga Technical University,*

*Lomonosova 1, Riga, LV 1019, Latvia*

*Ph: (+371) -7089961. Fax: (+371) -7089990, E-mail: aviation.institute@rtu.lv*

**Key words:** fatigue, scale, statistical factor, model

### Introduction

In science about strength of structural materials the fact of influencing of a detail absolute sizes on its fatigue durability is known for a long time. There were numerous attempts to give satisfactory explanation of this phenomenon, moreover as the determining reasons the metallurgical, technological, scale and statistical factors were advanced [1-5]. The influence of the first and second factors connected with the special features of specific technological process and specific equipment for its realization. So it is difficulty to summarize its. The calculation of the influence of metallurgical and technological factors is usually achieved either on the basis of experimental data. It may be to apply the some coefficients (the safety factors), which guarantee against the dangerous errors of overstating the fatigue resistance characteristics of the structure critical elements. Thus, in the existing theories the influence of sizes is limited by statistical factor and factor of the stress state heterogeneity. The fatigue fracture models, constructed on this basis conception, well agree with the experimental results in a number of cases. However, the universal model, applied to different materials and details of them, is not obtained up to now.

This paper completely is constrained with the indisputably established experimental facts about the influence of sizes on the fatigue characteristics of samples and structural elements. There is an attempt to construct a certain new model. The more adequate description of the special features of the fatigue fracture is this model basis. It is known [6-12] that in structural steels and alloys the conception of fatigue fracture occurs in a comparatively thin surface layer that eventually leads to developing of complete destruction of a sample or a detail. In the absence of the surface shaping or other strengthened forms of treatment, the fatigue cracks are conceived from the surface. In this case, the greater gradient (in the absolute value) of the first principal stress in the particular point of surface calls the higher operating time to the origin of crack. The action of gradient is reduced to lowering in the result of effective stresses. In the proposed model it is assumed that the durability to the origin of fatigue crack in the environment of this point of surface is determined by stresses on a certain small depth  $b$ . In this case it is assumed that this parameter is a certain constant of material. The greater this constant, the more sensitive material to the non-uniformity of stresses in the surface layer appears.

This fact is a basic difference in the system of the assumptions of the proposed model in comparison with the known theories.

## 2. Brief Description of Experiment

For purposes of the analysis of governing laws the distribution of fatigue durability to the origin of crack the experiment in the sheet samples with a width of 100 mm and with a thickness of 2 mm from aluminum alloy D16T with the system of circular orifices is made. Data of this experiment have large volume and are good initial information for fulfilling the analysis of the phenomenon in question. The samples of two types are tested. In the first group the samples had two rows of the equidistant orifices (4 in each number), and in the second group – one row of orifices (fig.1). The diameter of orifices for the different groups of samples was taken as the equal to 3, 5, 8, 10 and 20 mm. The system of orifices formed periodic or doubly periodic structure. Tests were conducted in the hydraulic testing machine with a frequency of about 12 Hertz with the cyclic tension. Two regimes of the regular cyclic load have be selected: 1)  $\sigma_{max}=120\text{ MPa}$ ,  $\sigma_{min}=0\text{ MPa}$ ; 2)  $\sigma_{max}=120\text{ MPa}$ ,  $\sigma_{min}=60\text{ MPa}$ .

In the process of tests the operating time to the fatigue crack (a length of 0.5 mm) appearance near each of the stress concentrators was recorded.

In order to prevent further propagation of crack and to reduce to a minimum its influence on the stress-strained state in adjacent concentrators were taken measures for their braking by creation near the vertex of the crack of the local zone of the compressive residual stresses. This goal was achieved by the local plastic deformation of material [13, 14].

The processing of the experimental results was performed on the assumption that the logarithm of fatigue durability was distributed normally. In fig.2 are represented the results, characteristic for the majority of the experimental data: the theoretical straight line, corresponding to the logarithmically normal law of distribution of fatigue durability and obtained by the method of least squares, will agree well with the empirical function.

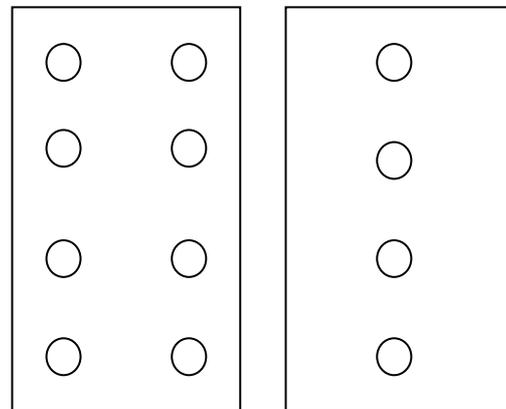


Fig. 1. The samples

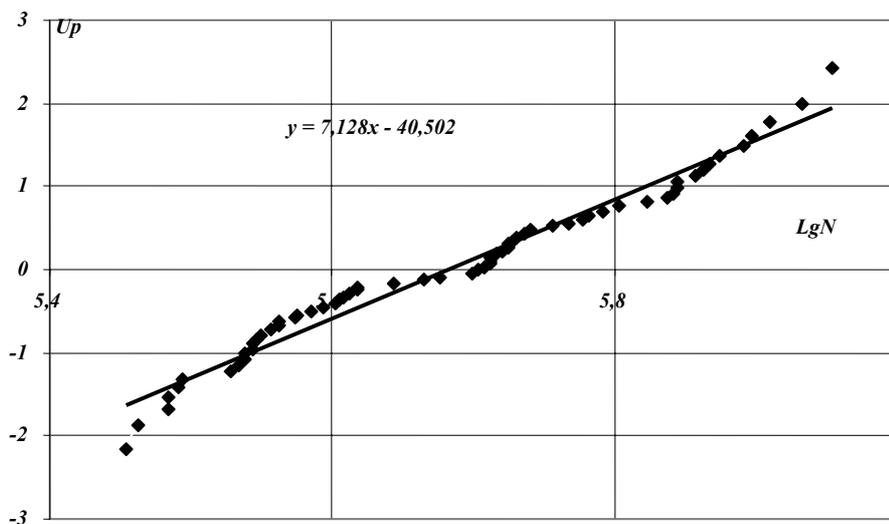


Fig. 2. Empirical and theoretical functions of distribution  
( $d=3\text{ mm}$ ,  $\sigma_{min}=60$ ,  $\sigma_{max}=120\text{ MPa}$ )

The special features of the procedure of statistical processing of information of the type in question in detail were discussed in [15].

Table 1 gives the total results of experimental information about the estimations of the distribution function parameters for the logarithm of the number of cycles. These data were obtained by two different methods. It is evident that the differences between the estimations of the mathematical expectation  $lgN$  and the standard deviations  $S$ , obtained by these methods, differ insignificantly. In the table  $k$  indicates total number of observations, and  $d$  - diameter of orifice. The value  $v=S/lgN$  is the variation coefficient of the logarithm of the cycles number.

Table1

$d, mm$		3	5	8	10	3	20
$k$		80	64	64	80	64	32
1	$lgN$	5,247	5,098	4,8522	4,6684	5,6821	5,2891
	$S$	0,1363	0,1653	0,1114	0,0834	0,1403	0,1602
	$v=S/lgN$	0,02598	0,0324	0,02296	0,01787	0,02469	0,0303
2	$lgN$	5,241	5,064	4,848	4,674	5,705	5,308553
	$S$	0,133	0,153	0,108	0,0814	0,135	0,151
	$v=S/lgN$	0,0254	0,0302	0,0223	0,0174	0,0237	0,0284
		Smin= 0 Smax= 120MPa				Smin= 60MPa Smax= 120MPa	

Fig.3. gives the generalization of the results of tests. Graph demonstrates the degree of the effect of the size of orifice on the fatigue durability in two regimes of the testing

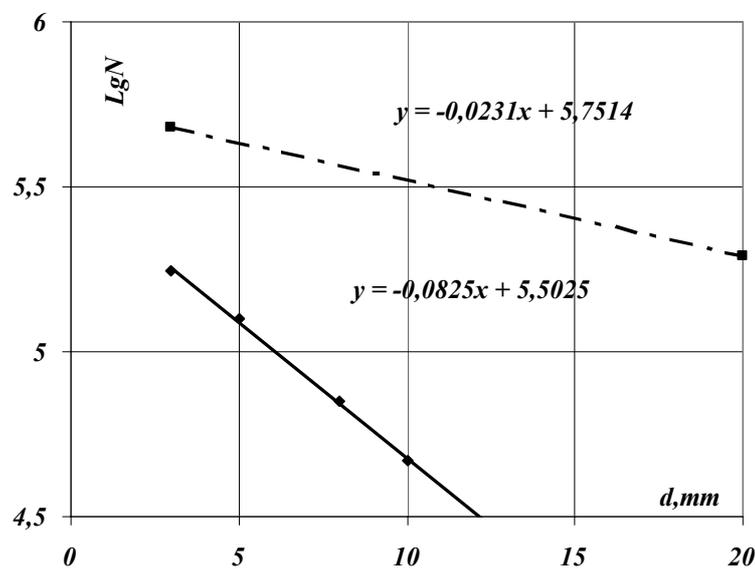


Fig. 3. Effect of the size of orifice on the fatigue durability at two load conditions

Mean-square approximation allows to obtain the equation of dependence between the fatigue durability, which corresponds to the average value of its logarithm, and the size of orifice in the form

$$N = N_0 \cdot e^{ad}$$

where  $N_0$  and  $\alpha$  – experimental constants, and  $\bar{d} = d / d_0$  – the relative diameter of orifice. As the base diameter it is conditionally accepted  $d_0=3$  mm. By the first regime of varying load  $N_0 = 3.18 \cdot 10^5$ ,  $\alpha = -0.57$ , and on second  $N_0 = 5.64 \cdot 10^5$ ,  $\alpha = -0.16$ .

### 3. Model of the Calculation of the Effect of Sizes on the Fatigue Durability

The proposed model of the origin of fatigue fracture in many respects is similar to those existing. The following initial prerequisites are the model basis:

1. For this material, state of semi-finished product, technological methods of preparing of detail, quality of the treatment of its surface there are an elementary section of the surface layer, an appearance of a critical fatigue microscopic crack in which independent of the appearance of analogous microscopic cracks in other sections. Under the critical it is customary to assume the fatigue crack, which uniquely determines further development of destruction in this section. This means that no other crack can grow into the macro-fissure in this section. It is assumed that the significant dimension of this section over the surface is equal to certain constant  $a$ , i.e., the area of the external surface of this section is proportional  $a^2$ . The characteristic of the depth of surface layer accepted value  $b$ . It is assumed that values  $a$  and  $b$  are kept constants under the above-indicated conditions. As consequence, the law of distribution  $F_0(N, \sigma, G)$  fatigue durability  $N$  to the origin of crack on this area depends only on the cyclic parameters of the first principal stress  $\sigma$  and its relative gradient  $G$  in the direction of normal to the surface.

2. The law of distribution  $F(N, \sigma)$  fatigue durability  $N$  to the origin of main crack in a certain critical zone of detail on the assigned level of nominal load is determined on the basis of model of the weakest link.

Thus, the law of distribution  $F(N, \sigma)$  fatigue durability  $N$  to the origin of main crack in a certain critical zone of detail with the assigned load is determined from the formula

$$F(N, \sigma) = 1 - \prod_{i=1}^n [1 - F_0(N, \sigma_i, G_i)], \quad (1)$$

where the parameters  $n$ ,  $\sigma_i$ ,  $G_i$  is set to the results of the analysis of the geometric special features of the critical zone of detail and stressed state in this zone.

It is accepted for purposes of further analysis that in the range in question the fatigue durability  $N$  to the origin of main crack in the elementary section is connected with the characteristic of stresses  $\sigma$  by the power dependency of the following form

$$\sigma^m N = C, \quad (2)$$

where  $m$  and  $C$  the constants of material under given conditions for tests, that depend on the coefficient of the asymmetry  $R$  of varying load.

### 4. Application of a Model for the Evaluation of the Effect of Sizes on the Fatigue Durability of Sheet with the Circular Orifice

Thus, when making these assumptions the procedure of the determination of the function of the distribution of fatigue durability to the zone of detail is reduced to the following:

1. The analysis of the stress-strained state is conducted and define the boundaries themselves of zone with stress level, close to the maximum. The values of maximum stress and relative stress within the limits of this zone are determined.
2. The functional connections between the average values of fatigue durability in the elementary zones of the surface layer of detail are determined in accordance with formula (2). In this case, actual stress, which determines destruction in this elementary section, is determined for the point of surface layer at the depth  $b$  by the formula

$$\sigma_i = \sigma_{i_{max}} (1 + G_i \cdot b), \tag{3}$$

where  $\sigma_{i_{max}}$  and  $G_i$  – the maximum value of the first principal stress and its relative gradient in the direction of internal normal to the surface in the elementary zone  $i$ .

3. By formula (1) is determined the law of distribution  $F(N, \sigma)$  fatigue durability  $N$  to the origin of main crack in a certain critical zone of detail on the assigned level of nominal load.

This procedure is realized for the design diagram of the fatigue fracture of thin sheet from the aluminum alloy D16T with the tensile variable cyclic load. Sheet is weakened by the periodic or doubly periodic system of the circular orifices with a diameter  $d$  with the parameters, which correspond to the carried out experiment.

By the method of finite elements investigated stress-strain state of sheet around the orifices. The obtained values of factors of concentration of stresses well agree with reference data [16]. In fig. 4 they are represented the results of changing the relative gradient of the first principal stress along the normal to the outline of orifice at the point of maximum stresses in the dependence on the diameter of orifice. There the change in the relative stress gradient for the case of the isolated orifice at the same point, obtained with the use of known exact solution by the formula is shown

$$G = \frac{1}{\sigma_{\theta_{max}}} \frac{\partial \sigma_{\theta}}{\partial r} = - \frac{1 + 6 \cos 2\theta}{1 + 2 \cos 2\theta} \frac{1}{R}, \tag{4}$$

where  $\sigma_{\theta_{max}}$  – the circumferential stresses, which coincide in the examined case with the first principal stress at the points of the outline of orifice,  $\theta$  – the coordinate angle of the point of the orifice outline with respect to the transverse axis of symmetry.

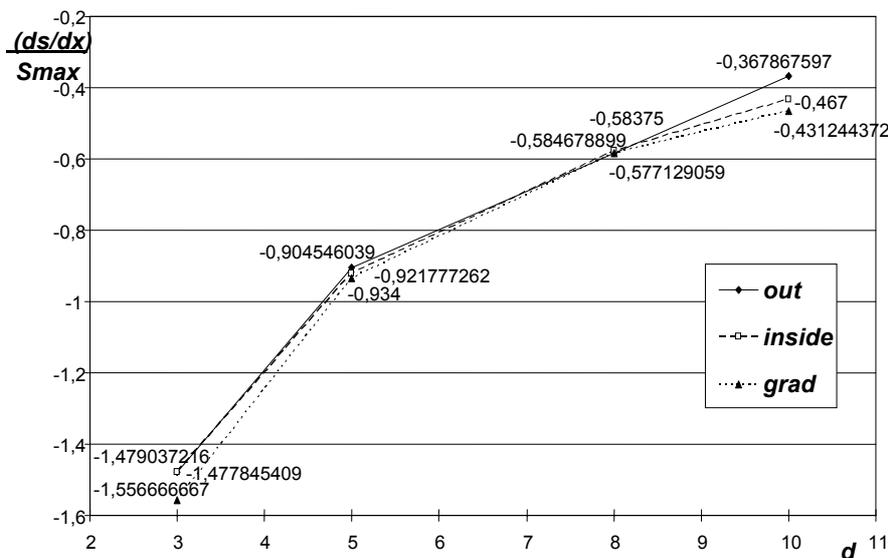


Fig. 4. Dependences of the relative gradient of the first principal stress from the diameter of orifice

The results of the numerical of calculation of relative stress gradient for entire range of diameters it differs little from its value in the case of the isolated orifice. Therefore subsequently for its determination formula (4) is used. Values of circumferential stresses on the internal surface of orifice were determined from the formula

$$\bar{\sigma}_\theta = \frac{\sigma_\theta}{\sigma} = \frac{K_\theta(1 + 2\cos 2\theta)}{3}, \quad (5)$$

where  $K_\theta$  – the factor of load of circumferential stresses for design data diagram of sheet with the periodic or doubly periodic system of orifices.

This coefficient depends on the diameter of orifice. It is reflected with the following formula, obtained by the mean-square approximation of the numerical calculations data

$$K_\theta = 3 - 0.0753\bar{d} + 0.0106\bar{d}^2 - 0.0351\bar{d}^3 + 0.00486\bar{d}^4, \quad (6)$$

where  $\bar{d} = \frac{d}{d_0}$ , moreover as the base value of the diameter of orifice is accepted its smallest value for the experimental samples  $d_0 = 3$  mm.

The calculation of the dependence of the parameters of fatigue durability on the orifice diameter is carried out on the assumption that the logarithm of fatigue longevity has logarithmically normal distribution. Let for the elementary section of surface layer with a certain base stress  $\sigma_0$  and zero gradient the mathematical expectation of the logarithm of the fatigue durability is equal  $\lg N_0$ , and standard deviation –  $S_0$ . It is obvious that base and actual stresses coincide in the uniform stress state. Then in accordance with formula (2) for the section  $i$  with actual stress  $\sigma_i$  the mathematical expectation of the logarithm of fatigue durability to the origin of crack is determined from the formula

$$\lg N_i = \lg N_0 + m \lg \frac{\sigma_0}{\sigma_i}, \quad (7)$$

If as the base to choose the maximal stress on a surface of an orifice, then the difference between  $\lg N_0$  and  $\lg N_i$  will be relatively small. Under these conditions the standard deviation of the logarithm of fatigue longevity for the elementary section  $i$  can be taken as equal  $S_0$ .

Thus, if we divide the surface of the critical zone of detail in the elementary sections, then for each of them with the aid of formulas (3) - (6) it is possible to estimate the ratio of stresses in (7) and then to determine the mathematical expectation  $\lg N_i$  for each elementary section. After this, according to formula (1) can be determined the law of distribution of fatigue durability to the origin of crack, its mathematical expectation and standard deviation.

In the calculations it is accepted that the critical zone of the surface of orifice is limited by coordinate angle  $\theta = \pm 6^\circ$ . Estimations show that in this zone a change of the first principal stress and its gradient do not exceed 1.5%. The purpose of calculations was the selection of the dimensional parameters  $a$  and  $b$ , and also the numerical characteristics  $\lg N_0$  and  $S_0$  such, at which is reached the best correspondence to the theory also of experiment.

The Fig. 5 and 6 represent the graphs of the dependences of average logarithm and standard deviation of fatigue durability for the results of the carried out tests in the comparison with the theoretical curves, obtained as a result of calculations on the basis of the proposed theory.

These results correspond to the values of the parameters  $a = 0.075$  mm and  $b = 0.28$  mm,  $\lg N_0 = 5.225$  and  $S_0 = 0.25$ .

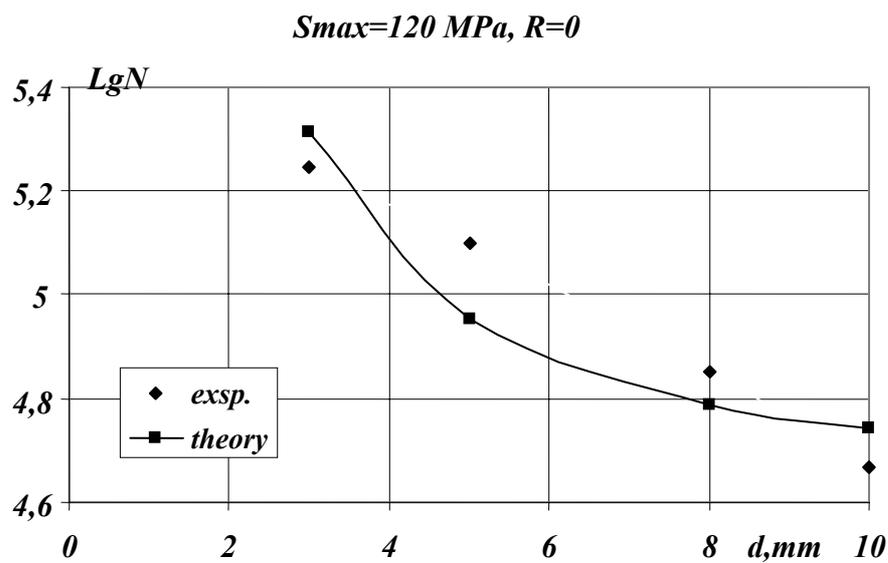


Fig. 5. Dependences between the average logarithm of fatigue durability and the diameter of orifice.

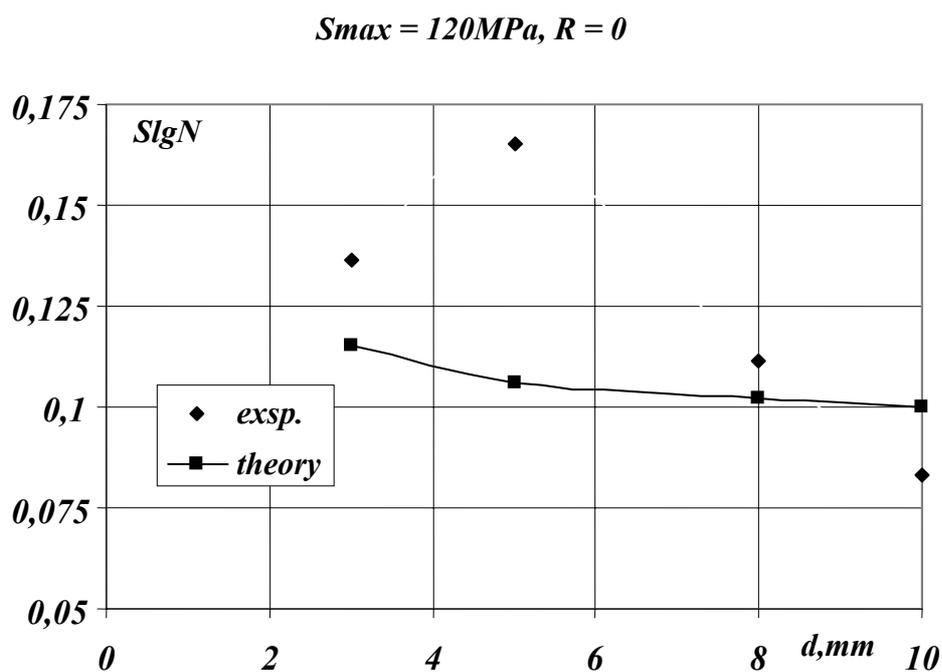


Fig. 6. Dependences between the standard deviation of fatigue durability and the diameter of orifice

## References

1. Серенсен С.В., Когаев В.П., Шнейдерович Р.М. Несущая способность и расчеты деталей машин на прочность. – М.: Машиностроение, 1975. – 448 с.

2. Сосновский Л.А. Статистическая механика усталостного разрушения. – Минск: Наука и техника, 1987. – 288 с.
3. Сосновский Л.А. Уравнения подобия усталостного разрушения деталей с опасным объемом и их экспериментальная проверка // Проблемы прочности, 1977, №4, С.40-50.
4. Турко В.П. Простейшая модель масштабного эффекта на основе гипотезы локальных объемов // Проблемы прочности, 1986, №4, С.34-38.
5. Бужинский В.В., Павелко В.П. О расчете кривой усталости с использованием критериев подобия усталостного разрушения // В сб.: Проблемы эксплуатационной прочности авиаконструкций. – Рига, РАУ, 1992, С.8-11
6. Прокопенко А.В., Торгов В.Н. Поверхностные свойства и предел выносливости металла. Сообщение 1: Зависимость предела выносливости от глубины слоя // Проблемы прочности, 1986, №4, С.28-34.
7. Прокопенко А.В., Маковецкая И.А., Штукатурова А.С. Поверхностные свойства и предел выносливости металла. Сообщение 2: Неравномерность свойств на поверхности // Проблемы прочности, 1986, №6, С.41-44.
8. Прокопенко А.В., Торгов В.Н. Поверхностные свойства и предел выносливости металла. Сообщение 3: Модель усталостного разрушения металла с учетом аномальных свойств поверхностного слоя. Масштабный эффект. Остаточные напряжения // Проблемы прочности, 1986, №6, С.45-51.
9. Прокопенко А.В., Торгов В.Н. Поверхностные свойства и предел выносливости металла. Сообщение 4: Расчет предела выносливости при концентрации напряжений и асимметричном циклическом нагружении // Проблемы прочности, 1986, №10, С.18-26.
10. Radhakrishnan V.M., Prasad C.R. Relaxation of residual stress with fatigue loading // Engineering Fracture Mechanics, 1976, -8, N.4, p.593-597.
11. Pangborn R.N., Weissmann S., Kramer J.R. Work hardening in the surface layer and in bulk during fatigue// Scr. Met., 1978, -12, N.2, p.129-131.
12. Determination of prefracture damage in fatigued and stress-corroded materials by X-ray double crystal diffractometry // R.N Pangborn., R. Yazici, T. Tsakalakos et al.//Proc. Symp. Accuracy powder diffraction, Gaithersburg, Md., June 11-15, 1979. – 1980, p.433-450 (U.S. Dep. Commer. Nat. Bur. Stand., Spec.Publ. N.567).
13. А.с.СССР., № 456003. Павелко В.П., Савинаев И.А. Способ задержки роста усталостных трещин в листовом материале. – 1974.
14. Павелко В.П. О повышении эффективности усталостных испытаний листовых образцов с концентраторами напряжений: Заводская лаборатория, 1981, том 47, N10, С.69-71.
15. Павелко В.П., Тимошенко Ю.В. Проблемы многоочаговых усталостных повреждений в тонкостенных элементах авиационных конструкций // Тезисы докладов международной научно-технической конференции «Современные проблемы машиностроения». – Гомель: ГГТУ, С.21.
16. R.E.Peterson. Stress Concentration Factors // Charts and Relations Useful in Making Strength Calculations for Machine Parts and Structural Elements. – New York: 1974. – 423 pp.