INSPECTION DATA USE FOR INSPECTION PROGRAM DEVELOPMENT

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1. Introduction

Since the second half of the twentieth century problem of aircraft damage due to a fatigue of its body metal elements has been solving using various methods, from discarding of aircraft after the specified number of flight hours (safe-life approach) to developing of periodic inspection programs, which allows to find a fatigue damage and repair or discard an aircraft from service before the damage exceeds regulatory mandated value. The main approach used today is developing of an inspection program using information about one (seldom two) full-scale aircraft fatigue test.

But inspection program is very expensive and the problem is to limit airframe fatigue failure probability by the regulatory mandated level with the smallest possible number of inspections.

A method of this problem solution and numerical example are given below.

2. Probability of Fatigue Failure

As it is shown in 1 and 3, the following simple exponential model of fatigue crack could be used:

$$a(t) = a_0 e^{Q t} \qquad (1)$$

where $a(t)$ is a fatigue crack size at time t (the number of flight hours), a_0 is so called equivalent initial crack size and Q is a parameter which depends on the loading mode. Model parameter estimates are derived from the aircraft full-scale fatigue test using regress analysis as follows:

$$a(t) = a_0 e^{Qt} \qquad (2)$$

$$\log a(t) = \log a_0 + Qt \qquad (3)$$

thus $$\begin{bmatrix} \log a(t_1) \\ \log a(t_2) \\ \ldots \\ \log a(t_n) \end{bmatrix} = \begin{bmatrix} 1 & t_1 \\ 1 & t_2 \\ \ldots & \\ 1 & t_n \end{bmatrix} \times \begin{bmatrix} \log a_0 \\ Q \end{bmatrix}, \qquad (4)$$

where pairs of $\{(t, a)_i\ ;\ i = 1,\ldots,n\}$ are results of factory tests (real crack observations). As soon as we have model parameters a_0 and Q, we could perform crack modelling. On the Fig. 1 below you can see typical result of such kind modelling.

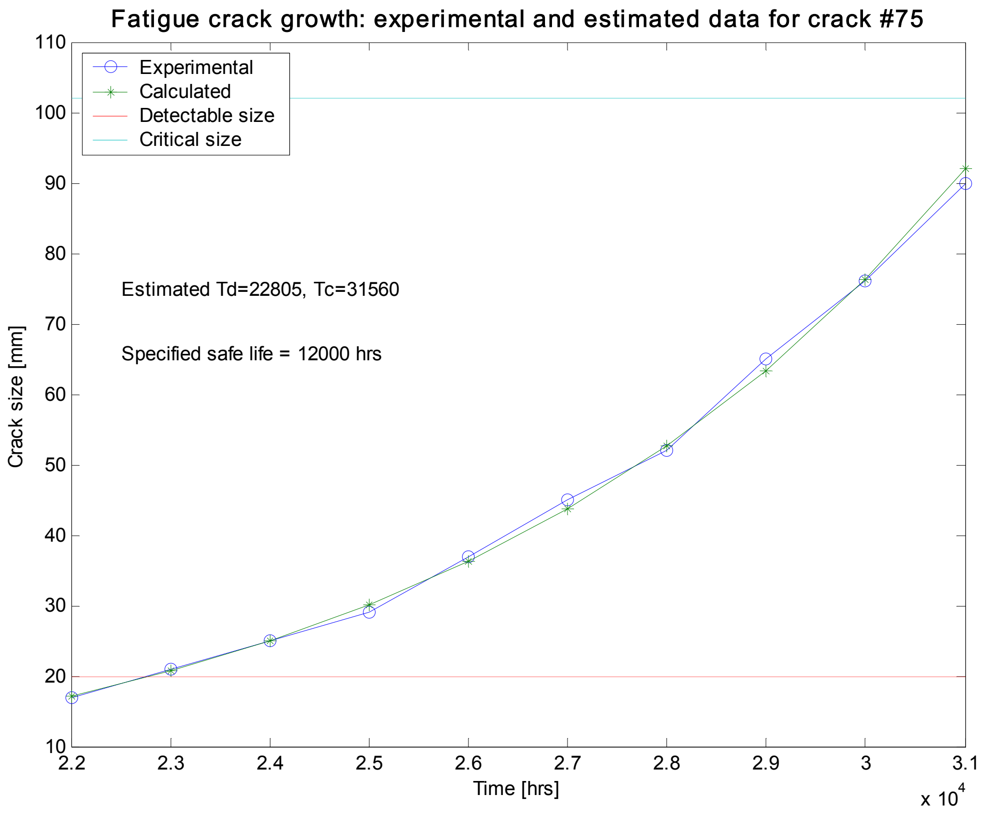

Fig. 1. *Real and modelled crack growth pattern*

Having a pair of model parameters a_0 and Q, we could perform Monte-Carlo modelling for a set of cracks. In this article, we perform modelling in assumption, that initial crack size for all cracks is the same a_0, while the crack growth speed, represented by parameter Q, changes from crack to crack. As it comes from (3),

$$\log a(t) = \log a_0 + Qt \tag{5}$$

$$t = \frac{\log a(t) - \log a_0}{Q} \tag{6}$$

$$T_d = \frac{\log a_{detectable} - \log a_0}{Q} = C_d / Q \tag{7}$$

$$T_c = \frac{\log a_{crit} - \log a_0}{Q} = C_c / Q \tag{8}$$

where $a_{\text{detectable}}$ is a crack size, below which the chances to discover it are negligible, a_{crit} is a crack size, which corresponds to the maximum residual strength of an aircraft component allowed by regulation, T_{d} is a time for crack to growth by its detectable size and T_{c} is a time for crack to growth by its critical size. Usually it is assumed, that time to growth by the critical size has a log-normal distribution

$$\log T_c \sim N(\mu, \sigma^2) \quad (9)$$

Referring to (8),

$$\log T_c = \log C_c - \log Q \quad (10)$$

Since $C_c = const$, we could assume that

$$\log Q \sim N(\theta_0, \sigma^2) \quad (11)$$

To perform Monte-Carlo modelling, we use the following parameter estimate values:

$$\hat{\theta}_0 = \log \hat{Q}_0 \quad (12)$$

$$\sigma = const = 0.346 \quad (13)$$

where $\hat{Q}_0$ is a parameter value, derived from particular crack using regress analysis. Thus generating a set of Q for the given parameter estimates $\hat{\theta}_0$ and σ, we get a set of pairs [T_d; T_c] in accordance with (7) and (8). Having a big number of cracks modelled and defining a fatigue failure situation as a case, when crack has not been discovered before its size exceeded a_{crit} (i.e. we were unable to discover a crack with the size in interval [$a_{detectable}$; a_{crit}]), we could calculate failure probability as a number of "missed" cracks among the total number of cracks (see picture at the right: d is a time between inspections, t_{sl} is a specified safe life):

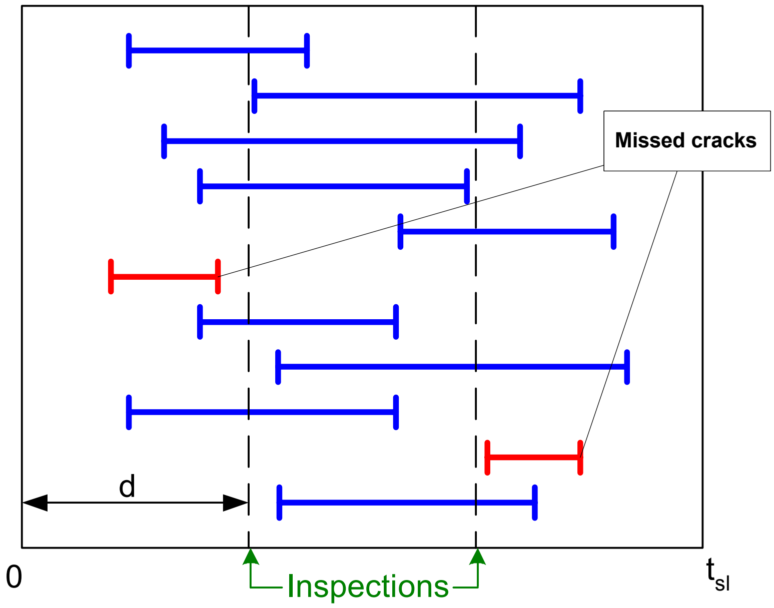

Fig. 2. *Example of "missed" cracks*

$$P_f(n_{inspections}) = \frac{n_{missed}}{n_{discovered} + n_{missed}} \quad (14)$$

Really $P_f(n_{inspections})$ is estimate of failure probability, but we suppose that

$$n = n_{discovered} + n_{missed} \to \infty \quad (15)$$

and this estimate is close enough to exact probability value. Calculating value of $P_f(n_{inspections})$ for a number of various $n_{inspections}$ we can draw failure probability curve (see Fig. 3). Then, using failure probability curve it's easy to determine required number of inspections.

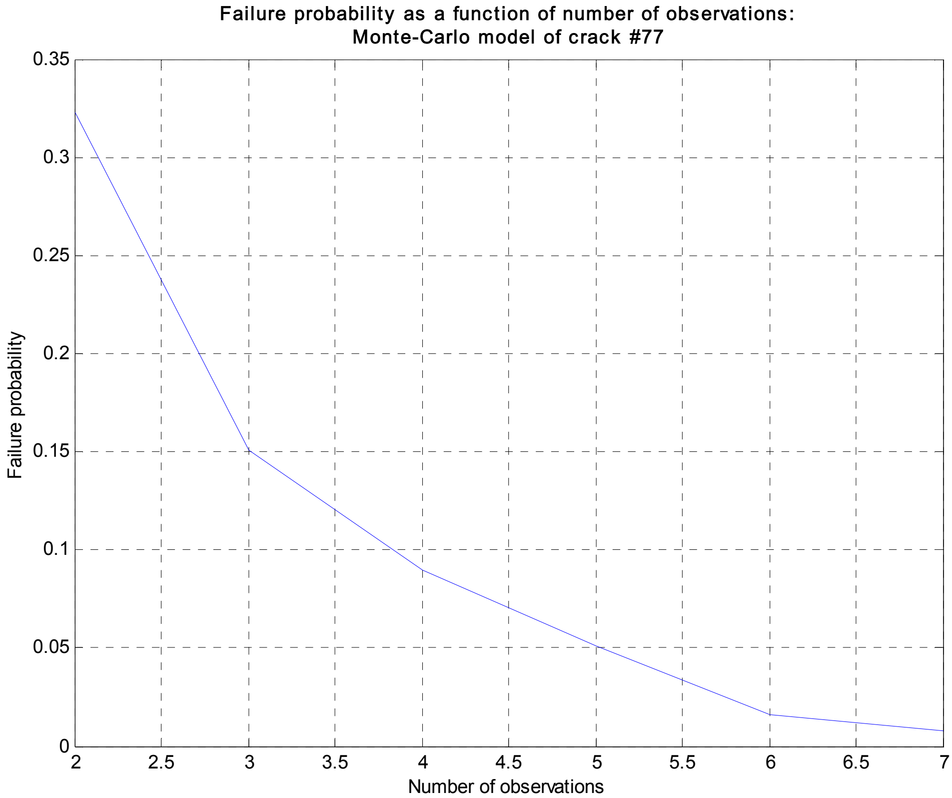

Fig. 3. *Failure probability curve -* $P_f(n_{inspection})$

1. Use of Adjusted Failure Probability Method to Eliminate Lack of Prior Information

As you see from the previous chapter, the required number of inspections could easily be found using Monte-Carlo method to draw a failure probability curve. Nevertheless, the results achieved are not very precise, because our modelling of $\hat{\theta}_0$ has been based on the single crack data only. The real value of θ_0 could be bigger or smaller than $\hat{\theta}_0$, thus the real required number of inspections could differ from the calculated above. To eliminate the problem the following method is offered.

First of all, we don't know the real value of θ_0, just a single estimate $\hat{\theta}_0$. So, with the probability of 0.999, the real value of $\theta_0 \in \left[\hat{\theta}_0 - 3\sigma; \hat{\theta}_0 + 3\sigma\right]$, thus we need to compute the number of inspections for all values in this interval and take the biggest one (which corresponds to the worst case).

Then, let's define the following rule: if required number of inspections (computed as stated above) exceeds certain threshold $n_{\max}$, the aircraft is returned back to the factory as too bad, and number of required inspections is set to infinity. So, if initial calculated number of inspections is $n_0(\varepsilon, \hat{\theta}_{0i})$, then:

$$n(\varepsilon, \hat{\theta}_{0i}, n_{\max}) = \begin{cases} n_0(\varepsilon, \hat{\theta}_{0i}) & , n_0(\varepsilon, \hat{\theta}_{0i}) \le n_{\max} \\ \infty & , n_0(\varepsilon, \hat{\theta}_{0i}) > n_{\max} \end{cases} \qquad (16)$$

Obviously, $P_f(\infty) \to 0$, because this corresponds to the case of constant uninterruptible inspectioning of the aircraft.

So, if we take a number of values $\hat{\theta}_{0i}$ around θ_0, and for each of them perform Monte-Carlo modelling of all aircraft in the fleet, then calculate required number of inspections for each aircraft using (16) and then, backwards, failure probabilities for those numbers of inspections, we could compute mean fleet failure probability for each value of $\hat{\theta}_{0i}$ (see Fig. 4 below).

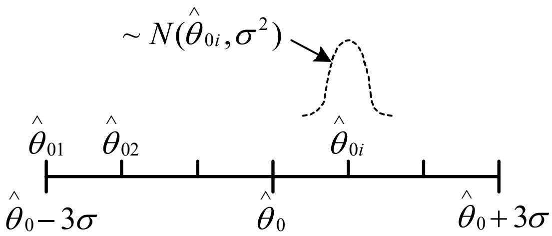

Fig. 4.

This dependence of mean failure probability from $\hat{\theta}_{0i}$ has an extreme (maximum). Indeed, $P_f(\hat{\theta}_{0i} = 0) \to 0$: this is a case when all aircraft are "ideal" and do not require inspections at all (time between inspections tends to infinity). From other side, $P_f(\hat{\theta}_{0i} \to \infty) \to 0$, because number of inspections will tend to infinity in accordance with (16): this is a case when all aircraft are so bad, that require uninterruptible inspection, and are to be returned to the factory.

The extreme of this function corresponds to the "worst" case from the failure probability point of view, and is located at the value of real θ, which could not match our initial estimate θ_0. Thus we could see, that our attempt to calculate number of inspections to support required level of failure probability $P_f \le \varepsilon$ fails: there could be bigger values of P_f at other values of θ.

Let's use the following approach to ensure $P_f \le \varepsilon$: choose another working value of required failure probability ε^*, which minimizes required number of inspections

$$\underset{\theta}{Min}\, E\left(n\left(\varepsilon^*, \hat{\theta}, n_{\max}\right)\right) \tag{17}$$

under condition

$$\underset{\theta}{Max}\, E\left(P_f\left(n\left(\varepsilon^*, \hat{\theta}, n_{\max}\right), \theta\right)\right) \le \varepsilon \tag{18}$$

The Fig. 5 below illustrates procedure, described above. As you see, there is a set of various values of ε_i which provide various maximum values of failure probability. On the picture value of ε^* corresponds to the ε_3. If we continue decrease working value of $\varepsilon < \varepsilon_3$, i.e. take ε_4, ε_5 etc., required number of inspections will grow and (17) will not be achieved.

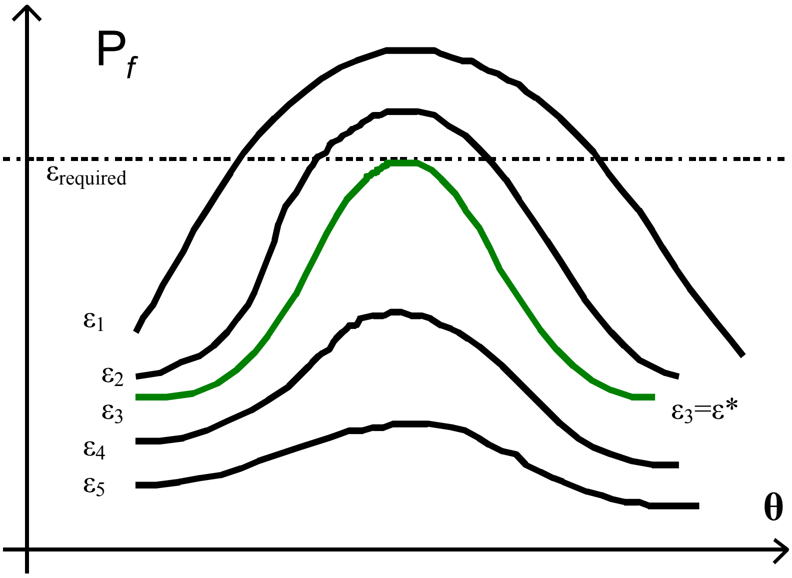

Fig. 5. *Choosing optimal* ε^*

2. Numerical Example

For the next example a real crack observation data has been taken. The required failure probability $\varepsilon_{required} = 0.07$. To achieve this required failure probability we must use $\varepsilon^* = 0.055$.

In such a case $\underset{\theta}{Max}\, E\left(P_f\left(n\left(\varepsilon^*, \hat{\theta}, n_{\max}\right), \theta\right)\right) = 0.0692 \le \varepsilon_{required}$, so condition (18) is met. Optimal number of inspections for this example n(ε^*)=5, time between inspections – 8000 hours.

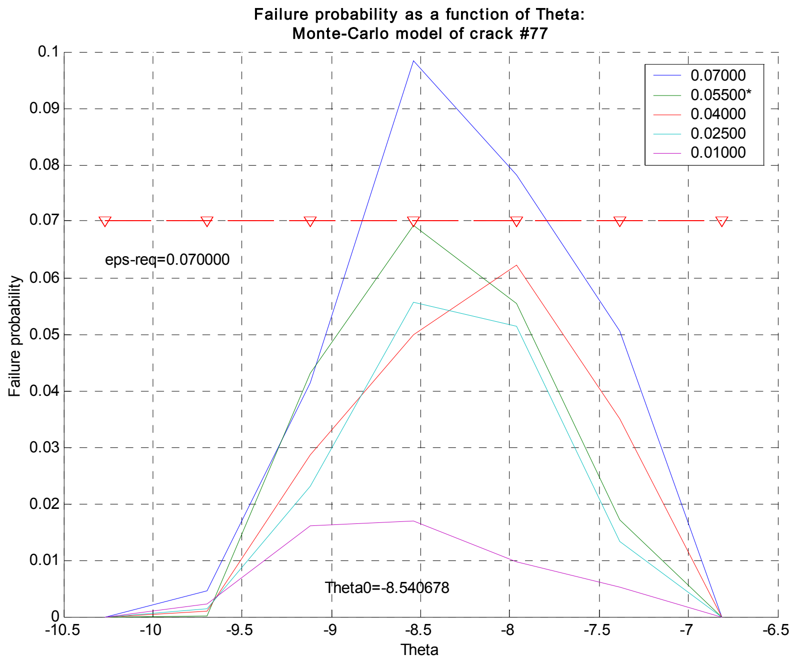

Fig. 6. *Example of choosing* ε^* *which ensures* $P_f \le \varepsilon_{required}$

It's worth to mention, that the difference between ε and ε^* significantly increases with the decrease of absolute value of ε. Table1 and Fig. 7 below demonstrate this.

Table 1. Dependence of $\varepsilon/\varepsilon^*$ on absolute value of ε.

ε	ε^*	$(\varepsilon-\varepsilon^*)/\varepsilon\times100\%$	$\varepsilon/\varepsilon^*$
0.08	0.0780	2.5%	1.026
0.07	0.0675	3.6%	1.037
0.06	0.0575	4.2%	1.043
0.05	0.0475	5.00%	1.053
0.04	0.0320	20.00%	1.250
0.03	0.0230	23.30%	1.304
0.02	0.0135	32.50%	1.481
0.01	0.0040	60.00%	2.500

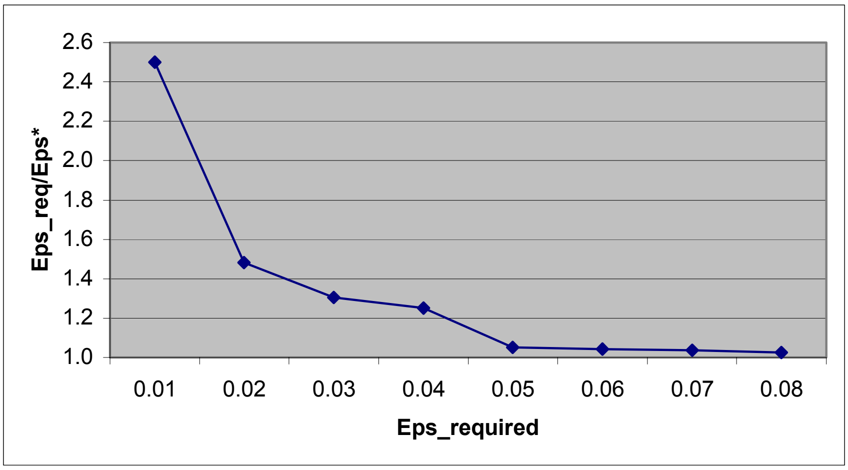

Fig. 7. *Dependence of* $\varepsilon/\varepsilon^*$ *on absolute value of* ε

4. Conclusion

A method of limitation of airframe fatigue failure probability in case of small number of initial fatigue test data is offered. The core idea of the method is the use of another, adjusted designed value of failure probability ε^*, instead of required one. This ε^* is designed to limit airframe fatigue failure probability with the smallest possible number of inspections.

5. References

1. Y.Paramonov, A.Kuznetsov. Fatigue crack growth parameter estimation by processing inspection results // In: Scientific proceedings of Riga Technical University – Transport and Engineering – Issue #6 – Riga, 2001.
2. Y.Paramonov, A.Kuznetsov. Fatigue crack growth parameter estimation by processing inspection results // In: Third International conference on Mathematical Methods in Reliability MMR2002 – Trondheim, Norway, 2002.
3. Y.Paramonov, R.DeSilva. Exponential model of fatigue crack // In: 6th International Conference Aircraft and Helicopters Diagnostics "*AIRDIAG*'97" – Warsaw, 1997 – pp. 173-178.