MODEL OF THE MULTI-SITE FATIGUE DAMAGE IN THE THIN-WALLED STRUCTURE

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Some completed model of the multi-site fatigue damage, constructed for a structure containing a series of the same type concentrators of stresses, located periodically along one straight line in more detail is considered. The model is constructed at the assumptions that the residual structural strength at presence of every possible damages uniquely is defined by the longest series from *k* the injured concentrators following one after another.
The considered calculated scheme well approaches for exposition of process of a modification of residual strength transversal rivet-joint of the hermetic fuselage periodically loaded with interior pressure.

Keywords: *multi-site fatigue damage, rivet-joint*

1. INTRODUCTION

Multiple Site Damage (MSD) is a common phenomenon for ageing aircrafts and its main property is the existence of interacting fatigue cracks at different sites of a structural component (Figure 1). As a rule it occurs along the rows of fastener holes in aircraft wings and fuselage and can call the unallowable decrease of the residual strength of the structure and as result the catastrophic failure [1, 7-8].

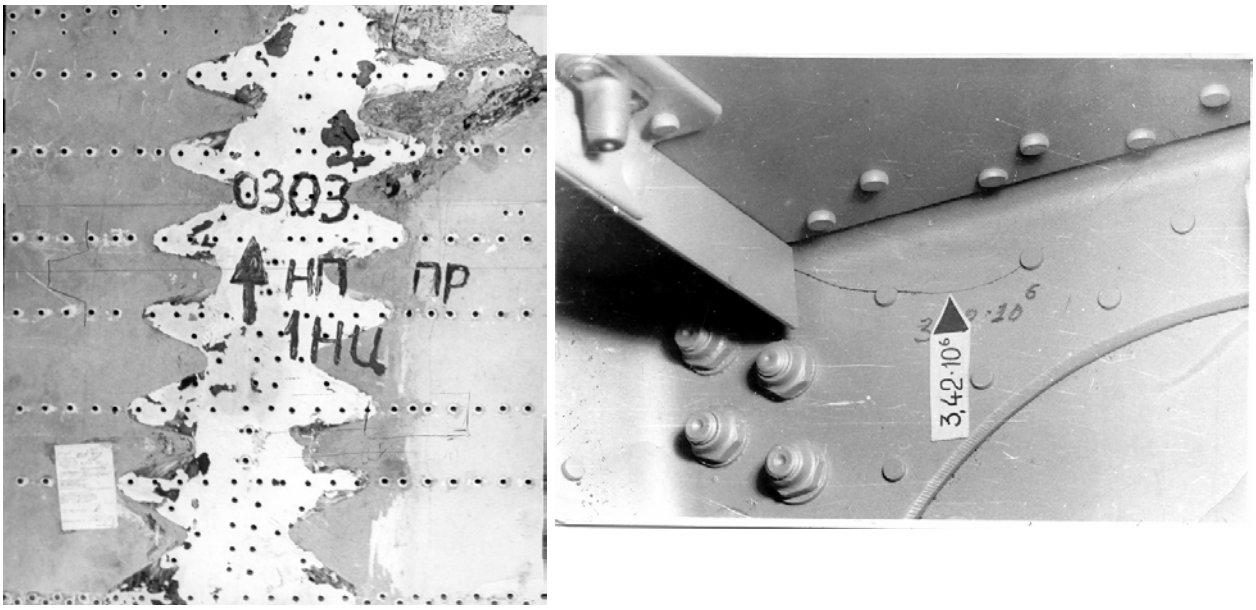

Figure 1. *Examples of Multiple Site Damage:*
a) Fatigue crack and corrosion in the wing panel at the joint points,
b) Fatigue crack in the row of rivets

Most completely the problem of the multi-site of the structural fatigue damages of aircraft structures is possible to be traced, by analysing the work [9]. Methodological questions of the calculation of constructions with the multi-site fatigue damages were also considered in a number of other papers [10-14]. Structure with many stress concentrators represents here the concept of the natural expansion of a single concentrator. On the basis of the probabilistic model of longevity of structure the model of the longevity of a single concentrator, which is represented by the random variable N with a density of distribution of $f(N)$. However, upon the transfer to the collection of single concentrators appears the need for describing probabilistically form for the entire collection, i.e., structure as a whole. The simplest and natural description of this collection is its examination as a combination within the framework of the structure of the set of longevity in the form of independent random quantities. In this case the function of the distribution of the longevity of structure as a whole, is determined by the longevity of the "weakest component", i.e., least durable concentrator, will be expressed through the functions of distribution of longevity $F_i(N)$ of single concentrators in the form

$$\psi(N) = 1 - \prod_{i=1}^{n} [1 - F_i(N)], \tag{1}$$

where n – number of critical places (concentrators). In a particular case, when the structure can be divided into m zones in terms of n_i of identical concentrators in each

$$\psi(N) = 1 - \prod_{i=1}^{m} [1 - F_i(N)]^{n_i}, \tag{2}$$

The use of these relationships is connected with the concrete properties of the structure, which is characterized by the following main parameters:

The relationship of type (1) is written for j^{th} part of the structure in the form

$$\psi_j(N) = 1 - \exp\left(\sum_S \ln[1 - F_j(N, \sigma_S)]^{n_j} \right), \tag{3}$$

by which can be written in the integral form

$$\psi_j(N) = 1 - \exp\left(n_j \int \ln[1 - F_j(N, \sigma)] \ \varphi(\sigma) d\sigma \right), \tag{4}$$

or, keeping in mind the smallest $F_j(N, \sigma)$ in the required range of longevity, then in even simpler form

$$\psi_j(N) \approx n_j \int F_j(N, \sigma) \ \varphi(\sigma) d\sigma . \tag{5}$$

Here n_j – total number of single concentrators in j^{-th} part of the structure. For the entire structure as a whole the longevity to the first failure is determined by the relationship of type (1.1), where role F_j will be played by the function $\psi_j(N)$.

The results of the tests of full-scale wing and tests of separate panels (section along the wing chord is formed with the approximately six elements, equivalent to one panel) are compared. The conversion of "panel to the wing" according to relationship is given in (2). In a number of cases the obvious differences between the theoretical calculations and the experimental data are observed – the probability of failure of the wing is higher as compared to the calculation.

The improvement of the model of multi-site fatigue damages in aviation materials and structures is main purpose of this work.

2. APPROXIMATE MODEL OF THE DEVELOPMENT OF MULTI-SITE FATIGUE DAMAGE

From an applied point of view residual structural strength R is the most important parameter of the multi-site fatigue damage. It is, obviously, the random function of time and depends on many random variables:

1) quantity k of emerging cracks in n potential sources of fatigue damage;
2) size of cracks ℓ_i ($i = 1, \ldots, k$) and their distribution on the sources;
3) the degree of influence of cracks on the stressed state in the sources of fatigue damage;
4) the degree of reciprocal effect of cracks to the rate of their increase.

As a result residual structural strength with operating time N (duration parameter for load of construction) is the random function of many variables

$$R\left(N, \ell_i, k, n, ...\right).$$

If R_{adm} – is the permissible residual strength on the conditions of airworthiness, then equation

$$R(N^*, \ell_i, k, n, ...) = R_{adm}, \qquad (6)$$

determines the random maximum operating time N^*, by reaching it will make the construction inefficient.

Analysing the equation (3.18), it is easy to see that the residual strength is the implicit function of operating time. This means that its level directly depends on the dimensions of crack, their quantity, mutual arrangement and etc. In this situation the task for determining N^* can be divided into two independent tasks:

1) the definition of residual strength as a certain determined function of quantity k, dimensions of crack ℓ_i and of their mutual arrangement;
2) the determination of the random configuration of multi-site fatigue damage depending on operating time N.

Thus, the task of determining the maximum operating time before the reaching the lower permissible boundary of residual strength consists of determined and random component. The requirement of the first task is to use methods of mechanics of destruction. Here primary attention is paid to the second task.

The determination of the random configuration of multi-site fatigue damage is to offer the following stages:

1) determination with the given time N of the distribution of damages between the separate sources $p_i(N)$;
2) determination of the standard probable configurations of multi-site fatigue damage;
3) determination of the residual strength R_k for each standard configuration with the given time.

Diagram designing diagram of the origin of multi-site fatigue damage drawn above is based on the use of a model of fatigue failure in the environment of one isolated source and the assumption of the mutual independence of separate sources in the composition of multi-site fatigue damage. It is possible to use for determining the damages of distribution ($k << n$), and also with the condition of the relative low speed of growth in the fatigue cracks. The mentioned model makes it possible to determine the probability of the appearance of k defects, but it does not distinguish their relative position, on which depends the residual strength. Therefore in the second step of solving the problem it is necessary to determine the probability of the appearance of the standard probable configurations of multi-site fatigue damage.

For the standard probable configurations of multi-site fatigue damage is to appoint such mutual arrangements of the sources of damage, for which with this number of damages and fixed length of cracks ℓ_i residual strength has one and only value. These configurations for the given construction are determined by the results of the analysis.

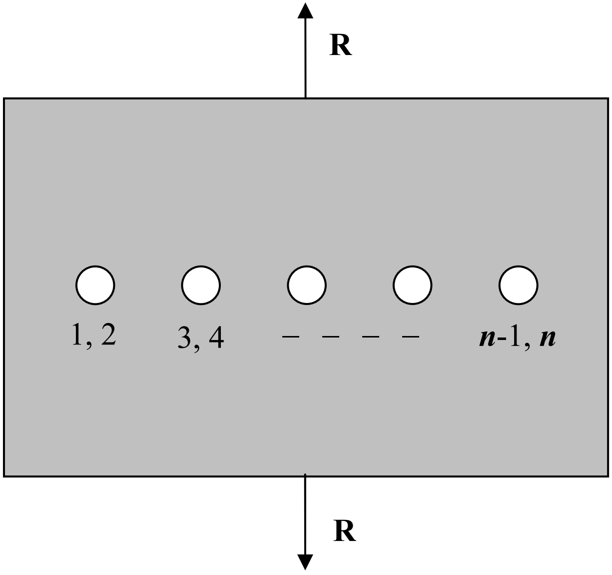

Figure 2. *Scheme of structure with a series of the same type concentrators of stresses*

Let the construction be the uniform stressed sheet with the periodic system of uniform stress concentrators (Figure 2). This can be the riveted seam, a number of the orifices of same sizes etc. It is assumed that each orifice has two concentrators with the total number *n*.

Standard configurations can be isolated as follows. The quantity of site of damage *k* is the first determining parameter. It is obvious that the total quantity of such configurations composes of C_n^k, and the probability of their appearance is p_{nk}. Residual strength *R* with the recorded *k* is determined by the mutual arrangement of defects. For example, the first version: all the *k* damages are located close to each other (Figure 3).

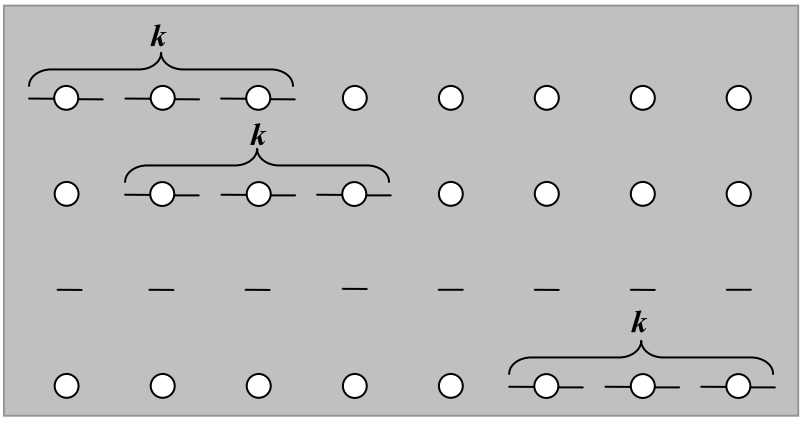

Figure 3. *Scheme of critical MSD*

Then the number of versions of this configuration is equal to

$$L_{nk} = (n-k).$$

Let us assume that the differences in the residual strength are determined only by the presence of the adjacent site. But if in between the site there are undamaged concentrators, then the maximum length of the crack determines residual strength

$$\ell_{\max} = \max_{i \in k}\{\ell_i\}. \tag{7}$$

Let the residual strength of the typical configuration be determined by the critical value of a certain parameter $\ell_s(\ell_1, \ell_2, ..., \ell_k)$. This can be the parameter of type (7), the sum of the crack length, the sum of the two adjacent cracks lengths, etc. The residual strength R_k of the configuration *k* can be determined in the presence of these criteria.

If for each standard configuration *k* the probability of its appearance is known at the given operating time p_k numerical characteristics of residual strength may be defined:

- mathematical expectation

$$\overline{R} = \sum_{k=0}^{n} p_k R_k, \tag{8}$$

- dispersion

$$D(R) = \sum_{k=0}^{n} p_k \left(R_k - \overline{R}\right)^2. \tag{9}$$

Realizing the procedure of successive approximations, it is possible to determine *N**.

The probability of formation of a standard configuration with critical damage may be approximately defined under the formula obtained by double application of binomial distribution. The essence is that the maximum number m_1 simultaneously realized critical damages by a size k in the beginning are defined. It is obvious, that

$$m_1 = fix\left(\frac{n}{k+1}\right),$$

and then the amount of remaining positions n_j in case in a standard configuration arises $j \le n$ critical damages $n_j = n - j(k+1)$.

In outcome the probability of occurrence of standard configurations with critical damage of a size k may be appreciated under the formula

$$p_k = \sum_{j=1}^{m_1} C^j_{m_1+n_j} p^j_m (1-p_m)^{m_1+n_j-j} \cdot \left(\sum_{i=1}^{r} C^i_{n_j} p^i (1-p)^{n_j-i} \right), \tag{10}$$

where $p_m = p^k(1-p)$, and $r = \min(n_j, k-1)$.

It is obvious, that this estimation not quite defines a real situation of formation of a configuration with critical damage. More exact outcomes may be obtained by a method of statistical trials. Outcomes of comparison of these two methods of definition of probability for a configuration with ten of the same type concentrators of stresses are submitted in the table 1.

Here 1 is under the formula (10); 2 by results of statistical trials.

Comparison of result of statistical trials with calculation under the formula (10) shows that in tendencies of a modification of probability are qualitatively saved depending on sizes of damage and probability of unit destruction. However quantitative distinctions at some combinations of parameters are significant. Especially it concerns mesh sizes of damage. At high probabilities of unit destruction the estimation under the formula (10) is a little bit more as contrasted to statistical simulation.

3. RESULTS OF SIMULATION

Below some completed model of the multi-site fatigue damage, constructed for a structure containing a series of the same type concentrators of stresses, located periodically along one straight line in more detail is considered. The model is constructed at the assumptions following:

1) the residual structural strength at presence of every possible damages uniquely is defined by the longest series from k the injured concentrators following one after another. Such damage refers to as critical for the given standard configuration.
2) because the big velocity of confluence of the next unit damages occurrence in a construction of critical damage starts as formation of one main crack with the length proportional k.

The considered calculated scheme well approaches for exposition of process of a modification of residual strength transversal rivet-joint of the hermetic fuselage periodically loaded with interior pressure.

On the basis of the accepted suppositions critical damage is equivalent to a crack by length

$$2l = 2l_0 + d + kt,$$

where d and t are diameter and a pitch of concentrators, and l_0 is some constant size of a crack near the unit concentrator (it may be safely discovered size of a crack).

For such damage the estimation of residual strength can be received, using methods of a linear fracture mechanics under condition of realization of requirements of small-scale yield. In this case the limiting carrying capacity of a construction is connected to a size of damage by a condition of a limiting equilibrium in top of a crack

$$K_I = K_{Ic}, \tag{11}$$

where K_I and K_{Ic} are a stress intensity factor and its critical value (a constant crack resistance of a material).

According to the accepted assumptions about smallness of influence of small partial damages (smaller critical) the stress intensity factor may be defined under the formula

$$K_I = \sigma\sqrt{\pi l} = \sigma\sqrt{\pi\frac{2l_0 + d + kt}{2}}\,. \tag{12}$$

Finally from a condition of a limiting equilibrium the following simple formula for definition of critical stress (residual strength) in long of a yield point of a material σ_0 may be obtained

$$\frac{\sigma}{\sigma_0} = \frac{A}{\sqrt{\left(1+\frac{d}{2l_0}\right)\left[\frac{t}{d}\frac{(k-1)/2+1}{l_0/d+0.5}\right]}}\,. \tag{13}$$

In this formula is the dimensionless constant depending on mechanical properties of a material and a characteristic size of a crack l_0.

$$A = \frac{K_{Ic}}{\sigma_0\sqrt{\pi l_0}}\,.$$

Calculations of a residual structural strength are carried out at $t/d = 4$ depending on an amount of cycles cyclical loading at values of diameter of orifices of 3, 4, 5 and 6 *mm*. It is accepted, that a material of a covering is aluminium alloy D16T (2024-T4) with performances fatigue durability obtained in the experiment circumscribed above. It is accepted that under an operation of variable interior pressure in a pressurized cabin in a covering cyclic tension $\sigma_{max} = 120\ MPa$, $\sigma_{min} = 0\ MPa$ operate. For each value of diameter of an orifice there are parameters of normal distribution of log fatigue durability before origin of a crack of the given size at the once isolated orifice. Then correspondence between numbers of cycles, the probability of formation of probable damage configurations and, finally, with average residual durability is established. In calculations the following basic performances of yield and crack resistance, appropriate to a selected material are taken: $K_{Ic} = 30\ MPa \cdot m^{0.5}$; $\sigma_0 = 300\ MPa$. As a safely discovered size fatigue crack on an orifice surface it is accepted $l_0 = 2\ mm$. On Figure 4 outcomes of calculations of an average value of residual durability are submitted. Influence of a scaling factor is noticeable.

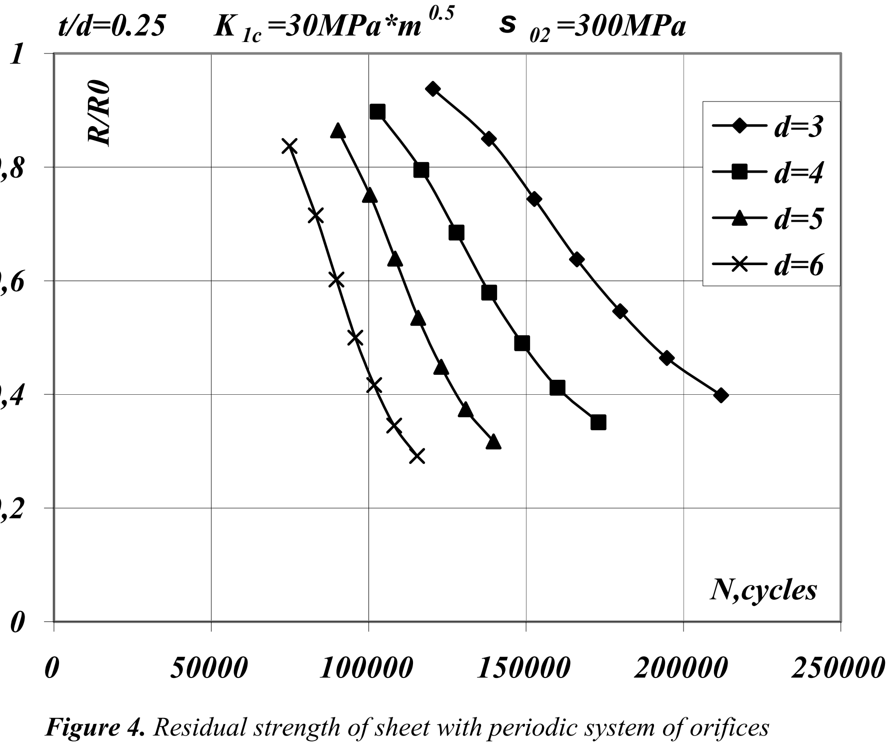

Figure 4. *Residual strength of sheet with periodic system of orifices at multi-site damage*

The scheme of the analysis used here multi-site fatigue damage may be generalized on more complicated variants of a structure and its conditions loading for deriving the justified estimations of residual strength and fatigue durability of airframe primary parts.

4. CONCLUSION

The considered calculated scheme well approaches for exposition of process of a modification of residual strength transversal rivet-joint of the hermetic fuselage periodically loaded with interior pressure.

The scheme of the analysis used here multi-site fatigue damage may be generalized on more complicated variants of a structure and its conditions loading for deriving the justified estimations of residual strength and fatigue durability of airframe primary parts.

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