

# THE TECHNIQUE FOR PROCESSING THE RESULTS OF MATHEMATICAL SIMULATION OF THE MAGNETIC FIELD, AND THE ALGORITHM FOR DETERMINATION OF SPECIAL MODE PERFORMANCES OF SYNCHRONOUS MACHINES

## MAGNĒTISKĀ LAUKA MATEMĀTISKĀS MODELĒŠANAS REZULTĀTU APSTRĀDES METODIKA UN ALGORITMS SINHRONO MAŠĪNU SPECIĀLO REŽĪMU RAKSTURLIELUMU NOTEIKŠANAI AR SKAITLISKAJĀM METODĒM

A.Podgornovs, A.Zviedris

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### Introduction

In this work are given motors load modes analysis method for synchronous motors, which are arranged with self-excitation system. This self-excitation system in the case with variable load can automatically change an excitation current, for example, to vary or provide constant  $\cos\varphi$ . For such excitation system is stipulated to use energy of the magnetic field third harmonic. To use third harmonic energy with high efficiency is necessary in the motor slots to build in auxiliary winding, which have  $2p_3=6p$  number of poles,  $y_3=Z/6p$  winding step and number of phase  $m_3$  [1]. Taking into consideration these conditions EMF of triple frequency is induced in auxiliary winding, but EMF of the fundamental harmonic not induced.

Investigate synchronous motors third harmonic self-excitation analysis and synthesize tasks must be solved. An analysis task includes magnetic fields mathematical simulation to define magnetic field allocation influence to the different parameters and this allocation influence to the motor characteristics (flux linkage, EMF,  $\cos\varphi$ ). It signifies correctly to take into account geometrical shape of the magnetic system and some elements of the magnetic system non-linear characteristics. At the same time, synthesize tasks are to define auxiliary winding parameters, which ensure third harmonic energy high efficiency utilization, to obtain excitation systems necessary voltage.

Partial differential equations are solved in magnetic field calculation relatively to the vector magnetic potential  $\bar{A}$  by finite element method. Taking into account the nature of a magnetic field it could be accepted that field is plane-parallel, i.e. field characteristics in rectangular coordinate system  $xyz$  change only depending on  $x$  and  $y$  and they do not depend on  $z$  coordinate. Consequently the vector magnetic potential  $\bar{A}$  has only one component  $A = A_z$ .

As the result of magnetic field simulation the distribution of the vector potential  $\bar{A}$  in fixed points  $x_i, y_i$  of calculation range (cross section of a machine or its part) is obtained as the discrete argument function  $A_i(x_i, y_i)$ .

Used software provides mathematical simulation of stationary magnetic field, i.e. such a field when  $\partial/\partial z = 0$  and rotor rotational speed is  $n = 0$ . However carrying out the calculations of stationary magnetic field for different rotor positions concerning the stator it is possible to get the change in time of the vector potential as the discrete argument function  $A_{i,j}(x_i, y_i, t_j)$  from its space

distribution  $A_i(x_i, y_i)$ . In this case any  $A_{i,j}$  value is the vector magnetic potential in fixed rotor position which is calculated in point  $i$  in time moment  $t_j$ . This value is equal to the vector potential in point  $i+1$  in time moment  $t_{j+1} = t_j + \Delta t$ , where  $\Delta t$  - time step [2].

Synchronous motors usually designed as silent pole synchronous machines. Silent pole synchronous machines calculations and analysis widely use the theory of two reactions, which based in superposition principle. Theory of two reactions is based on longitudinal and transversal synchronous induction resistances  $X_d$  and  $X_q$ . To tell the truth speaking superposition principle correctly can be used just for linear system, this is, synchronous motors with non-saturated magnetic system.

Using the theory of two reactions, precisely to take into account saturation influence from  $X_d$  and  $X_q$  parameters values are not possible. In practice, some approximated methods are used, but not always this methods gives satisfactory results. By and large the theory of two reactions using for saturated machines magnetic systems don't give this advantages, for which it was created.

Resistances  $X_d$  and  $X_q$  in this situation lose obviousness and physical sense, because it wasn't variable parameters, but its dependent from each other and from operation mode. To take into account this reasons to determination of the synchronous machines characteristics recommended using fig. 1. phasor diagram, where  $U$  – network voltage ;  $E_\delta$  – resulting induced flux EMF;  $F_f, F_a, F_\delta$  – accordingly, an excitation winding, armature winding and resulting magnetic force.. In this phasor diagram resistances  $X_d$  and  $X_q$  in a direct view do not occur. In this phasor diagram axes  $d$  and  $q$  are shown, but others parameters are spotted from the magnetic field numerical calculations, to take into account geometrical shape of the magnetic system and magnetic material non-linear characteristics.

In this work are shown, that synchronous motor rotating magnetic field simulation can be realized with FEM stationary magnetic field simulation methods. As the result of magnetic field simulation the distribution of the vector magnetic potential is obtained. The vector magnetic potential can be submitted as a discrete space distribution argument function and time function -  $A_{i,j} = f(x_i, y_i, t_j)$  [2].

Generally synchronous motors working mode characterize four base values: voltage  $U$ , armature current  $I$ , excitation current  $I_f$  and  $\varphi$  angle.

Analysing motor working mode and it characteristics usually are necessary to determinate excitation current  $I_f$ , which to the given voltage  $U$  and armature current  $I$  will ensure necessary  $\varphi$  angle. Is necessary to determinate functional equation  $I_f = f(U, I, \varphi)$ .

This phasor diagram corresponds to synchronous machines main equations, which follow from electric and magnetic circuit analysis using electromagnetic field an integral equation form [5]. At the same time, solving electromagnetic field equations in differential form, coherency  $I_f = f(U, I, \varphi)$  can't obtain directly, but just with iteration and/or using data processing mathematic methods, which are based on synthesise of equations [3]. Determinate functional coherency  $I_f = f(U, I, \varphi)$  can be used the following algorithm.

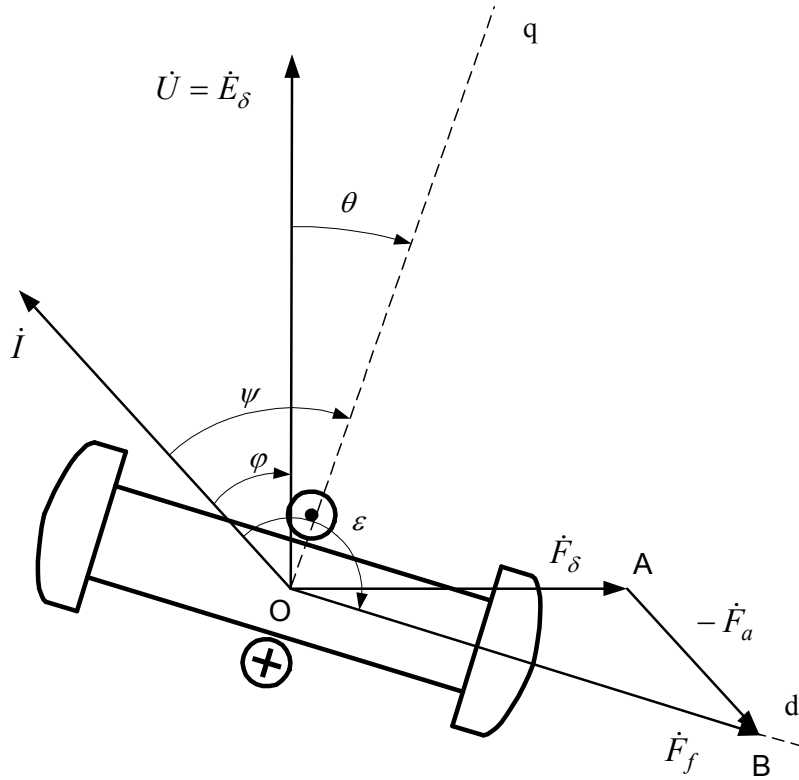


Fig. 1. Synchronous motors phasor diagram.

To the chosen armature current  $I$  are choose three values of excitation currents  $I_{f1}=I_{fmin}$ ,  $I_{f3}=I_{fmax}$ ,  $I_{f2}=(I_{fmin}+I_{fmax})/2$  and three values of  $\varepsilon$  angle  $\varepsilon_1=\varepsilon_{min}$ ,  $\varepsilon_3=\varepsilon_{max}$ ,  $\varepsilon_2=(\varepsilon_{min}+\varepsilon_{max})/2$ . Necessary to choose this values in the really probable spectrum, and for  $3 \times 3 = 9$  this values combination to do magnetic field simulation using the finite element method. From the obtained results and using phasor diagram Fig. 1., table range are obtain functional dependence  $U_{i,k}=f_1(I_{fi}, \varepsilon_k)$  and  $\varphi_{i,k}=f_2(I_{fi}, \varepsilon_k)$   $i=1,2,3$  and  $k=1,2,3$  values.

These dependencies can be approximated by analytical methods – second-degree polynomial:

$$U = a_1 + a_2 I_f + a_3 \varepsilon + a_4 I_f \varepsilon + a_5 I_f^2 + a_6 \varepsilon^2 + a_7 I_f^2 \varepsilon + a_8 I_f \varepsilon^2 + a_9 I_f^2 \varepsilon^2; \quad (1)$$

$$\varphi = b_1 + b_2 I_f + b_3 \varepsilon + b_4 I_f \varepsilon + b_5 I_f^2 + b_6 \varepsilon^2 + b_7 I_f^2 \varepsilon + b_8 I_f \varepsilon^2 + b_9 I_f^2 \varepsilon^2, \quad (2)$$

there are values  $a_1, a_2, \dots, a_9, b_1, b_2, \dots, b_9$  can be determinate to solve two nine equations systems, inserting to equations (1) and (2)  $U_{i,k}, \varphi_{i,k}, I_{fi}$  and  $\varepsilon_k$  values.

Polynomials factors  $a$  and  $b$  can be spotted solving for equations (1) and (2) in matrix form system and can be determinates as

$$\begin{aligned} ZA &= U, \\ ZB &= \varphi, \end{aligned}$$

where

$$Z = \begin{pmatrix} 1 & \varepsilon_1 & I_{f1} & \varepsilon_1 I_{f1} & \varepsilon_1^2 & I_{f1}^2 & \varepsilon_1^2 I_{f1} & \varepsilon_1 I_{f1}^2 & \varepsilon_1^2 I_{f1}^2 \\ 1 & \varepsilon_2 & I_{f1} & \varepsilon_2 I_{f1} & \varepsilon_2^2 & I_{f1}^2 & \varepsilon_2^2 I_{f1} & \varepsilon_2 I_{f1}^2 & \varepsilon_2^2 I_{f1}^2 \\ 1 & \varepsilon_3 & I_{f1} & \varepsilon_3 I_{f1} & \varepsilon_3^2 & I_{f1}^2 & \varepsilon_3^2 I_{f1} & \varepsilon_3 I_{f1}^2 & \varepsilon_3^2 I_{f1}^2 \\ 1 & \varepsilon_1 & I_{f2} & \varepsilon_1 I_{f2} & \varepsilon_1^2 & I_{f2}^2 & \varepsilon_1^2 I_{f2} & \varepsilon_1 I_{f2}^2 & \varepsilon_1^2 I_{f2}^2 \\ 1 & \varepsilon_2 & I_{f2} & \varepsilon_2 I_{f2} & \varepsilon_2^2 & I_{f2}^2 & \varepsilon_2^2 I_{f2} & \varepsilon_2 I_{f2}^2 & \varepsilon_2^2 I_{f2}^2 \\ 1 & \varepsilon_3 & I_{f2} & \varepsilon_3 I_{f2} & \varepsilon_3^2 & I_{f2}^2 & \varepsilon_3^2 I_{f2} & \varepsilon_3 I_{f2}^2 & \varepsilon_3^2 I_{f2}^2 \\ 1 & \varepsilon_1 & I_{f3} & \varepsilon_1 I_{f3} & \varepsilon_1^2 & I_{f3}^2 & \varepsilon_1^2 I_{f3} & \varepsilon_1 I_{f3}^2 & \varepsilon_1^2 I_{f3}^2 \\ 1 & \varepsilon_2 & I_{f3} & \varepsilon_2 I_{f3} & \varepsilon_2^2 & I_{f3}^2 & \varepsilon_2^2 I_{f3} & \varepsilon_2 I_{f3}^2 & \varepsilon_2^2 I_{f3}^2 \\ 1 & \varepsilon_3 & I_{f3} & \varepsilon_3 I_{f3} & \varepsilon_3^2 & I_{f3}^2 & \varepsilon_3^2 I_{f3} & \varepsilon_3 I_{f3}^2 & \varepsilon_3^2 I_{f3}^2 \end{pmatrix},$$

$$A = \begin{pmatrix} a_1 \\ a_2 \\ \dots \\ a_9 \end{pmatrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_9 \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} \\ U_{12} \\ U_{13} \\ U_{21} \\ U_{22} \\ U_{23} \\ U_{31} \\ U_{32} \\ U_{33} \end{pmatrix}, \quad \varphi = \begin{pmatrix} \varphi_{11} \\ \varphi_{12} \\ \varphi_{13} \\ \varphi_{21} \\ \varphi_{22} \\ \varphi_{23} \\ \varphi_{31} \\ \varphi_{32} \\ \varphi_{33} \end{pmatrix}.$$

By and large are obtained analytical equations, which present two equations non-linear system

$$\begin{aligned} U &= f_1(I_f, \varepsilon), \\ \varphi &= f_2(I_f, \varepsilon), \end{aligned}$$

haven solved which for given mode parameters (are given  $U=const$  and  $\varphi=const$  values) obtain for this mode corresponding  $I_f$  and  $\varepsilon$  values. Solving equation system useful to use optimization theory [4] known bisection method with exception of intervals, which are tuned two-argument function and which algorithm is realized in FORTRAN software (fig. 2).

Synchronous motors characteristics determination can be used integrated programs complex, where are provided results exchange with separate program of this complex:

- 1) QuickField – magnetic field mathematical simulation with obtained results initial handling (determination of flux linkage, harmonic analysis);
- 2) EXCEL – QuickField software obtained results utilization to produce equation synthesize;
- 3) FORTRAN – non-linear equation calculation program to obtain synchronous motor characteristics in load modes.

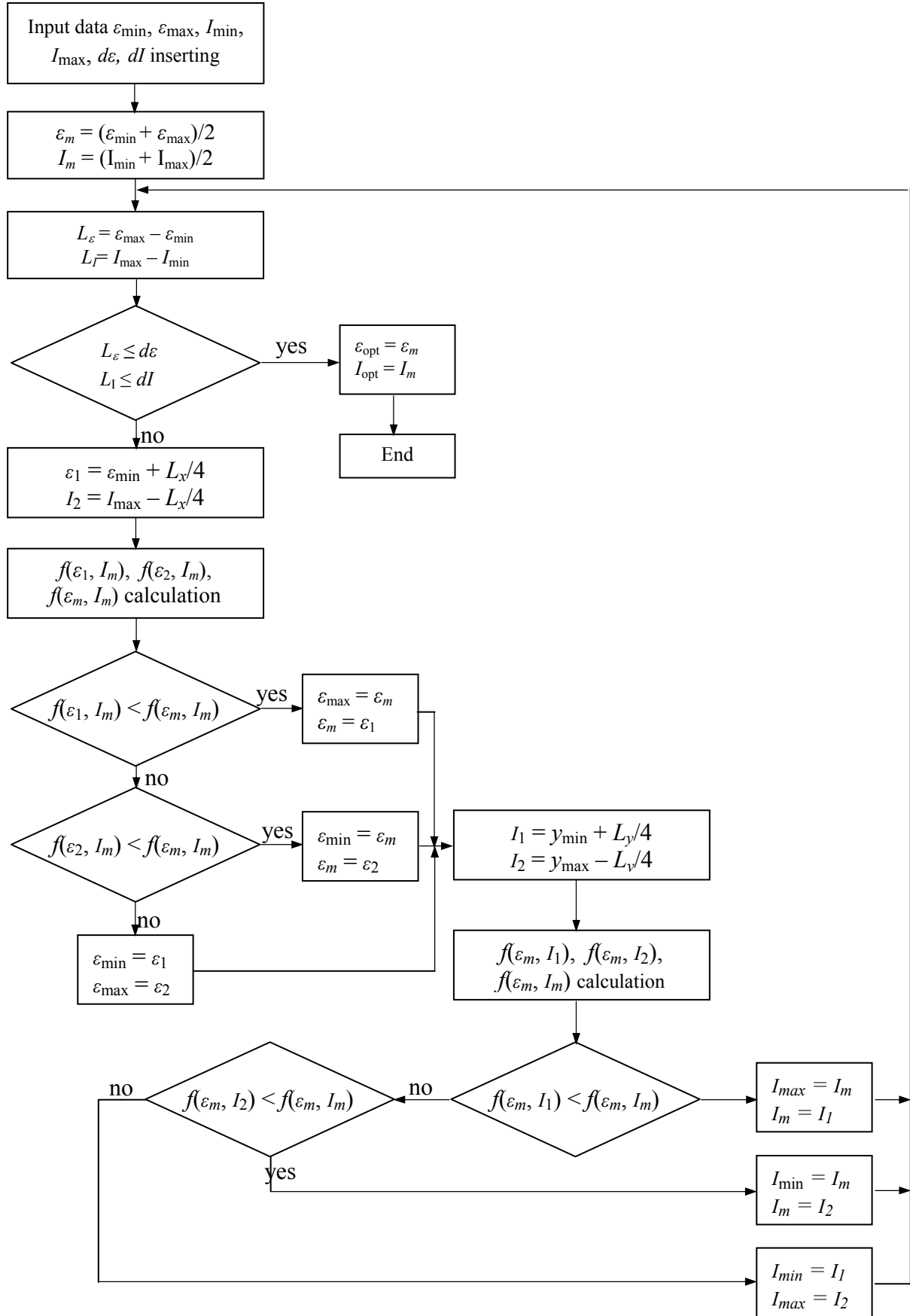


Fig. 2. Two-argument objective function minimization algorithm blocks scheme using bisection method with exception of intervals.

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## References

1. A. Zviedris, A. Podgornovs. Sinhronās mašīnas papildtinuma parametru izvēle pašierosmes sistēmai ar magnētiskā lauka trešās harmonikas izmantošanu. RTU zinātniskie raksti "Enerģētika un elektrotehnika", 4. sērija, 13. sējums, R.:RTU, 2004, 79.-86. lpp.
2. A. Podgornovs, N. Roldugina, A. Zviedris. Sinhronā dzinēja pašierosmes raksturlielumu noteikšana ierosmes sistēmai ar magnētiskā lauka trešo harmoniku. RTU zinātniskie raksti "Enerģētika un elektrotehnika", 4. sērija, 15. sējums, R.:RTU, 2005, 7.-13. lpp.
3. A. Zviedris. Datorrealizācijas matemātiskās metodes. R.:RTU, 2004, 80 lpp.
4. Г. Реклейтис, А. Рейвиндран, К. Рэгсдел. Оптимизация в технике. Кн. 1. Москва, "Мир", 1986, 352 с.
5. A. Zviedris. Elektrisko mašīnu elektromagnētiskie aprēķini. R.:RTU, 2001, 72 lpp.

**Andrejs Zviedris**, Associate Professor, Dr.Sc.Ing.  
Riga Technical University, Institute of Power Engineering  
Address: Kronvalda bulv. 1, LV 1010 Riga, Latvia  
Phone: 371+7089929  
e-mail: [aaazzz@eef.rtu.lv](mailto:aaazzz@eef.rtu.lv)

**Andrejs Podgornovs**, Ph. D. student, Mg.Sc.Ing.  
Riga Technical University, Institute of Power Engineering  
Address: Kronvalda bulv. 1, LV 1010 Riga, Latvia  
Phone: 371+7089928  
e-mail: [andrejsp@eef.rtu.lv](mailto:andrejsp@eef.rtu.lv)

### ***Podgornovs A., Zviedris A. Magnētiskā lauka matemātiskās modelēšanas rezultātu apstrādes metodika un algoritms sinhrono mašīnu speciālo režīmu raksturlielumu noteikšanai ar skaitliskajām metodēm.***

*Darbā aplūkota sinhronā dzinēja darba režīmu analīzes metodika dzinējam, kas aprīkots ar pašierosmes sistēmu, kura mainīgas slodzes režīmā nodrošina ierosmes strāvas automātisku maiņu (piem., lai regulētu vai uzturētu noteiktu  $\cos\varphi$ ). Šādā ierosmes sistēmā paredzēts izmantot magnētiskā lauka trešās harmonikas enerģiju. Pētot trešās harmonikas pašierosmes sistēmas sinhronajos dzinējos, jārisina sintēzes un analīzes uzdevumi. Analizējot dzinēja darbības režīmus, parasti ir jānosaka ierosmes strāva  $I_f$ , kas dotajam tīkla spriegumam  $U$ , slodzes strāvai  $I$  nodrošina uzdoto leņķi  $\varphi$ , t.i., jāatrod funkcionāla sakarība  $I_f=f(U,I,\varphi)$ . Savukārt, risinot elektromagnētiskā lauka vienādojumus diferenciālā formā, sakarību  $I_f=f(U,I,\varphi)$  nevar iegūt tiešā veidā, bet tikai ar iteratīvu procedūru palīdzību un/vai izmantojot datu apstrādes matemātiskās metodes ar empīrisko formulu sintēzi. Nelineāru vienādojumu sistēmas atrisināšanai par pamatu ņemta optimizācijas teorijā pazīstamā intervālu izslēgšanas bisekciju metode, kura pielāgota divu argumentu funkcijai un kuras algoritms realizēts ar FORTRAN programmu.*

**Podgornovs A., Zviedris A. The technique for processing the results of mathematical simulation of the magnetic field, and the algorithm for determination of special mode performances of synchronous machines.**

The work is devoted to the methods for analysing the load modes of synchronous motors with a self-excitation system which at load variations automatically changes the excitation current (e.g. in order to vary  $\cos\varphi$  or keep it constant). In such a system, provision is made for using the energy of the third harmonic of magnetic field. When studying the self-excitation of a synchronous motor from the third harmonic the synthesis and analysis issues should be involved. Usually, at analysing the motor operation it is necessary to determine the excitation current  $I_f$ , which for a given grid voltage  $U$  and armature current  $I$  will ensure the predefined angle  $\varphi$ , i.e. we should define functional equation  $I_f=f(U,I,\varphi)$ . However, when solving the electromagnetic field equations in the differential form, function  $I_f=f(U,I,\varphi)$  cannot be obtained directly but only using iteration procedure and/or mathematic methods of data processing based on the synthesis of empirical formulae. For solving the set of non-linear equations a method from the optimisation theory should be invoked which consists in the elimination of intervals by bisection and is fitted for finding a two-argument function. The algorithm of the method is realised in FORTRAN software.

**Подгорнов А., Звиедрис А., Методика обработки результатов математического моделирования магнитного поля и алгоритм определения характеристик специальных режимов синхронных машин средствами численных методов.**

В работе рассмотрена методика анализа синхронного двигателя в режиме нагрузки, который оборудован системой самовозбуждения, которая при изменении режима нагрузки обеспечивает автоматическое изменение тока возбуждения (например, чтобы регулировать или обеспечить определенный  $\cos\varphi$ ). В этой системе возбуждения предусмотрено использовать энергию третьей гармоники магнитного поля. Исследуя систему самовозбуждения двигателя от третьей гармоники поля необходимо решать вопросы синтеза и анализа. Анализируя режимы работы двигателя, обычно надо определить ток возбуждения  $I_f$ , который для данного напряжения сети  $U$ , тока нагрузки  $I$  обеспечит заданный угол  $\varphi$ , т.е. надо найти функциональную зависимость  $I_f=f(U,I,\varphi)$ . Однако, при решении уравнений электромагнитного поля в дифференциальной форме, зависимость  $I_f=f(U,I,\varphi)$  нельзя получить напрямую, а только с помощью процедуры приближений и/или используя математические методы обработки данных с синтезом эмпирических формул. Для решения системы нелинейных уравнений за основу взят метод в теории оптимизации известный, как метод исключения интервалов методом бисекции, который приспособлен для решения двухаргументной функции и алгоритм которого реализован в программе FORTRAN.