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ENSURING AND OPTIMISING THE SAFETY OF THE COMPLEX SYSTEMS

Author's abstract of doctoral thesis

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I confirm that I have independently worked out this doctoral thesis presented for defence at Riga Technical University for being conferred the degree of Doctor of engineering sciences. This doctoral thesis is not presented to any other university for obtaining a scientific degree.

Andrey Kuznetsov ....

Date: ....

The doctoral thesis is written in English. It contains 4 chapters, conclusion, bibliography, 7 appendices, 86 illustrations, 191 pages, and 38 reference titles.

1. Topicality of the research

Fatigue failures are one of the major factors influencing overall airframe reliability, and the consequences of the cumulative fatigue damages take a significant part of refusals and collapses of planes, irrespective of their type and purpose.

Existing methods of prevention of negative consequences of the given kind of damages are based on two main principles: restriction of airframe service duration by the specified life based on results of fatigue tests, and carrying out some inspection programs with the purpose of detection of fatigue cracks before the residual strength of an airframe will lower below some minimally admissible level. The present work is devoted to creation of the inspection program development technique and investigation of opportunities to correct them during the fleet service period.

2. Objectives of the research

The main objectives of the research are:

- development of the mathematical model of the fatigue prone airframe structural significant items' inspection program development and realization process, estimations of its efficiency and ways of its optimization for a case, when the initial information includes only a limited number of fatigue cracks' growth trajectory observations during fatigue full-scale (bench) tests.
- development of the corresponding software to realize the model and to investigate the ways of its practical applications.

3. Methods of the research

Theoretical methods:

- theory of probabilities and theory of Markov Chains;
- mathematical statistics;
- mathematical modelling.

Experimental methods:

- mechanical tests on fatigue durability;
- carrying out of periodic observations and use of service data for updating the program of inspections;
- mathematical modelling using modern software packages.

4. Scientific novelty

4.1. The technique of development of the fleet inspection program based on small number of preliminary (bench) observations of the fatigue cracks growth trajectories is offered with the following features:

- an exponential model of a fatigue crack growth (Paris' model for the power value equal to two) with two random parameters – the equivalent initial size of a crack and growth rate of a crack (in a logarithmic scale) – is taken as a basis; the correlation of estimates of these parameters is also considered in the model;
- to choose the inspection program for the operating fleet, based on preliminary fatigue test data, the minimax statistical decision function that guarantees the required limitation on the probability of failure in the fleet irrespective of unknown crack growth parameters, is used.

4.2. The mathematical model that realises the technique of the fleet inspection program development is created; the following results were acquired using this model:

- the behaviour of probability of failure as a function of various model parameters (like an interval between consequent inspections, the time moment of the first inspection, etc.), is investigated; some features of its behaviour are received and described;
- a new technique to set up the time of the first inspection, which lets to find the minimal possible number of inspections for the given type of the inspection program, is offered;
- the model of switching on the alternative inspection program is offered at occurrence of the certain event in operation, which allows prolonging the term of service of the fleet keeping the probability of failure at the required level and reducing total average number of inspections.

4.3. The software is developed for the fatigue cracks' data analysis and calculation of the inspection program characteristics, including:

- estimating fatigue cracks' growth parameters;
- clear modelling of fatigue cracks' development process and efficiency of the considered inspection program (which is characterised by the time moment of the first inspection, an interval between inspections, minimal detectable size of a crack, etc.) by visualization of results of the Monte Carlo modelling;
- calculation of efficiency of the inspection programs using analytical dependences of probabilities of failures from the parameters of the inspection program;
- use of the Markov Chains theory for calculation of consequences of switching on the alternative inspection program (in particular – doubling of the inspection frequency after detection of the first fatigue crack in service);
- use of a modular principle of the developed virtual test bench, which lets flexibly adjust system of modelling under current requirements of the researcher, adding new or changing existing modules if necessary.

4.4. Numerical examples are provided.

4.5. The software package with detailed descriptions and manuals is provided.

5. Practical relevance

The work has the following practical relevance:

- the developed software package represents a significant expansion and a deepening of the basic variant of the system, developed earlier by the order of Ilyushin design office and then provided also to the Yakovlev design office;
- use of the given model allows to improve inspection's program economical characteristics, still ensuring the required limitation on the probability of failure for the fleet;
- the toolkit for modelling of the decision-making process and analysis of the consequences of these decisions for various inspection program techniques and inspection program update strategies (using collected service data) is developed;
- the developed system can be applied in educational process of the Aviation institute of RTU.

6. Chapter one

In this chapter the brief history of a problem is presented, the conceptual basis is provided, as well as earlier offered and currently used techniques of restriction of probability of failure are provided highlighting theirs main advantages and weaknesses.

The importance of the problem is confirmed by the statistics presented in work "The review of the serious air catastrophes caused by fatigue failures" written by G.S.Kempbell and R.Lahi: for the period from 1927 until 1981 there were 1885 accidents which took lives of 2240 persons; the latest statistical data shows that there are approximately 69 accidents caused by fatigue failures annually; some of them have caused catastrophes.

There is a set of works that are devoted to the development of the airframe inspection programs with the goal to discover a fatigue crack before its size reaches some critical size. Examples can be found in works of B. Lundberg, F. H. Hook, D. N. Young, V. Y. Sennik, V. V. Nikonov, N. N. Smirnov, V. S. Streljaev, I. Nesterenko and other authors. A big contribution to the development of this direction was brought also by works, supervised by H. B. Kordonsky and J. A. Martynov.

However, in all those works it was not taken into consideration, that the probability of failure depends not only on the random nature of fatigue crack growth model parameters and inspection techniques, but also on the final decision making procedure and, in particular, that in certain cases the airframe project itself may be re-designed.

The beginning of development of mathematical models in the given direction can be found in works of Y. M. Paramonov and P. M. Sobolev, and then the work was continued by N. M. Kimlik.

The present work is a continuation of these works as a modernization of the model by including of two random variables (the initial equivalent fatigue size of crack and the rate of its growth) and taking into account the correlation between estimates of parameters of these random variables.

7. Chapter two

The given chapter contains the description of ideology of the airframe inspection program development approach, which allows to optimise its cost (to minimize an average number of inspection per fleet), maintaining the required level of failure probability. The chapter also includes description of the offered mathematical model and investigation of its behaviour using probabilistic and statistical approaches.

Developing an airframe inspection program that would ensure aircraft failure probability at or below the certain required level we face the lack of statistical data on parameters of fatigue cracks' growth. The task of ensuring the required failure probability of the aircraft in mathematical terms may be formulated as follows: we need to create a procedure (or formulate a statistical decision function) for inspection program development that ensures required limitation of failure probability in circumstances when fatigue cracks growth parameters are unknown, but estimated by processing results of full-scale fatigue tests.

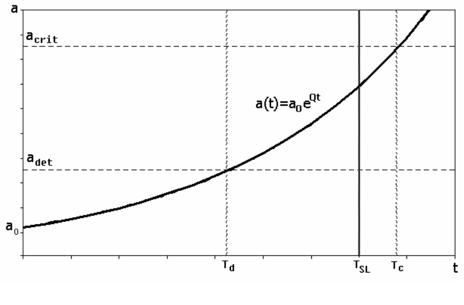


Figure 1. Visual representation of the crack growth model.

To perform further calculations, we need a mathematical model of the process of fatigue crack's growth.

After analysing experimental data of the TU-134 aircraft (wings), we have decided to choose a two-parametric exponential model (Fig.1), i.e. we approximate real crack's growth curve with some exponential function that depends on two parameters (similar approximation is used by Yang): $a(t) = a_0 \cdot e^{Qt} = \alpha \cdot e^{Qt}$.

Crack growth rate in exponential model is represented by parameter Q, its so called equivalent initial size is represented by parameter α. Despite of all simplicity, this formula in the range of observation $[T_d; T_c]$ where T_d is a time when the crack becomes detectable and T_c is the time when the crack reaches its critical size, shows us rather comprehensible results.

First of all, let us define *failure* as the situation, when we were unable to discover cracks with size $a_{\det} \le a < a_{crit}$, or, in other words, if there were no inspections performed in [T_d, T_c] time interval. For example, there are three such missed cracks in Figure 2 and Figure 3 below – no inspections were performed between time moments T_d and T_c for those cracks.

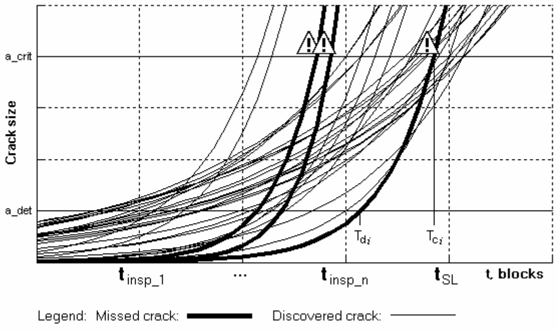


Figure 2. Demonstration of missed cracks.

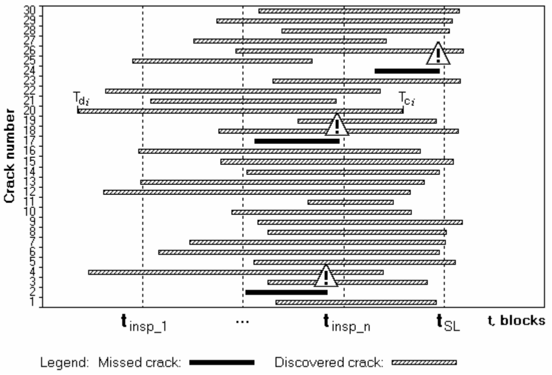


Figure 3. Defining the failure probability.

If the number of cracks observed is sufficiently big we can accept the share of missed cracks among all cracks in the series as the estimate of probability of failure for particular inspection program: $\hat{P}_f \xrightarrow[n_{total}\to\infty]{} P_f = \lim\limits_{n_{total}\to\infty} \left(n_{missed}/n_{total}\right)$.

It is clear that by changing the number of inspections $n_{inspections}$ in the service interval $\left[0, t_{SL}\right]$ we will also change the number of discovered cracks; therefore, the estimate of failure probability will vary as well, as presented in Figure 4. Unfortunately, we don't know the real values of the parameters used to draw the curve in Figure 4. We used the estimates from a small number (one, seldom two) of available observations (fatigue cracks during fatigue test) instead. The real curve could go either higher or lower than the drawn one. That also means that the real failure probability value for particular number of inspections will vary as well. This case is represented in Figure 5.

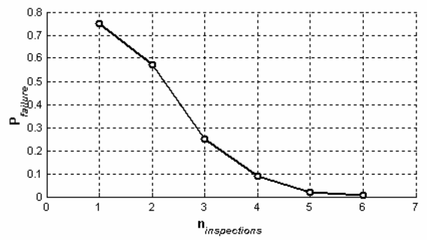


Figure 4. Failure probability dependence on $n_{inspections}$.

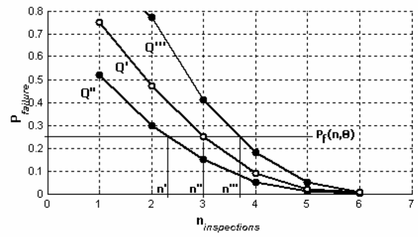


Figure 5. The dependence of failure probability on $n_{inspections}$ and θ.

We can never know how a certain fatigue crack curve will look like. Thus, performing approximation of that fatigue crack curve with a certain model, the fatigue crack growth model parameters (FCGMP) will vary as well, so they are random values with their specific individual distribution parameters. To perform Monte Carlo modelling of the fatigue crack growth process we have to know FCGMPs' distribution types and parameters, i.e. c.d.f. of each FCGMP.

The fatigue crack growth model parameters (FCGMP) of a certain realisation of fatigue crack growth curve a and Q are random variables; let us denote $X = \ln Q$ and $Y = \ln C_c = \ln\left(\ln\left(a_{crit}/a\right)\right)$, so durability $T_c = C_c/Q$. From the analysis of the fatigue test data it can be assumed, that the logarithm of time required the crack to grow to its critical size (logarithm of durability) is distributed normally: $\ln T_c \sim N(\mu_{\ln T_c}, \sigma^2_{\ln T_c})$.

From additive property of normal distribution comes that $\ln T_c$ could be normally distributed either if both $\ln C_c$ and $\ln Q$ ($C_c = \ln a_{crit} - \ln \alpha$) are normally distributed (i.e. $X = \ln Q \sim N(\mu_X, \sigma_X^2)$, $Y = \ln C_c \sim N(\mu_Y, \sigma_Y^2)$), or if one of them is normally distributed while another one is a constant. In figures below these two cases are called one- and two-parametric models:

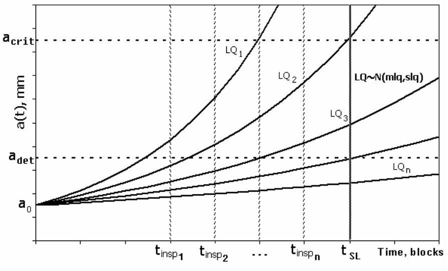


Figure 6. One-parametric crack growth modelling

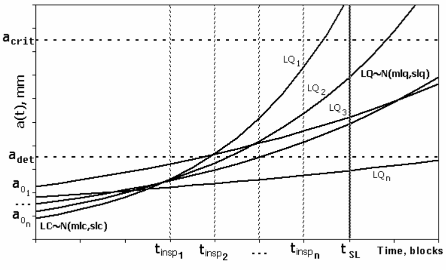


Figure 7. Two-parametric crack growth modelling

To get estimates of FCGMP distribution parameters, we consider statistics of several crack observations. It's worth mentioning here, that the correlation coefficient r between $\ln Q$ and $\ln C_c$ (between X and Y) is significant. For our full-scale fatigue test data processing we have its value r=0.796419 (see Figure 8).

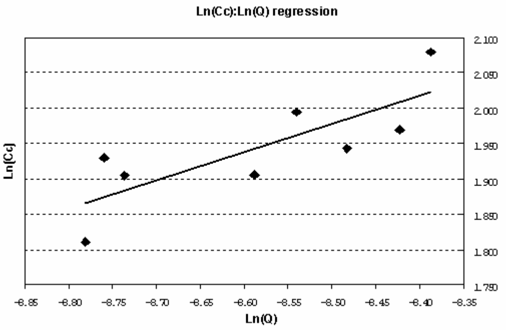


Figure 8. $\ln Q$ — $\ln C_c$ correlation.

In this work three ways to calculate failure probability are presented: using formulas, using Monte Carlo modelling and using Markov Chains theory.

To be able using the Markov Chains the inspection program is presented like the process of several states: first $n+1$ states represent aircraft service in the appropriate interval between two consequent inspections, while three additional states represent aircraft withdrawal from the service due to the successful end of service when the specified life period is over, due to fatigue failure and due to discovery of a crack:

E_i – aircraft is in service in the i^{th} inspection interval; $i = 1, 2, \ldots (n+1)$;
E_{n+2} – aircraft has successfully reached t_{SL} without cracks (SL - state);
E_{n+3} – fatigue failure, i.e. a crack has been missed (FF - state);
E_{n+4} – crack is detected during the inspection (CD - state).

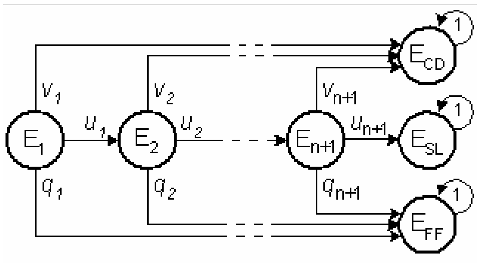


Figure 9. Aircraft service process – graphical representation

Being in the i^{th} inspection interval means that $t \in (t_{i-1}, t_i)$, where $i = 1, 2, \ldots n_{TIP}$, $t_0 = 0$, $t_{n_{TIP}+1} = t_{SL}$.

Then the probability of crack detection during the inspection number i is denoted as v_i; probability of failure as q_i; and probability of successful service continuation as u_i. Since these three cases form a complete set, then $u_i + v_i + q_i = 1$. In our model we also assume that we withdraw an aircraft from service at t_{SL} even if there are no cracks discovered

	E_1	E_2	E_3	$\cdots$	E_n	E_{n+1}	E_{n+2} (SL)	E_{n+3} (FF)	E_{n+4} (CD)
E_1	0	u_1	0	$\cdots$	0	0	0	q_1	v_1
E_2	0	0	u_2	$\cdots$	0	0	0	q_2	v_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
E_{n-1}	0	0	0	$\cdots$	u_{n-1}	0	0	q_{n-1}	v_{n-1}
E_n	0	0	0	$\cdots$	0	u_n	0	q_n	v_n
E_{n+1}	0	0	0	$\cdots$	0	0	u_{n+1}	q_{n+1}	v_{n+1}
E_{n+2} (SL)	0	0	0	$\cdots$	0	0	1	0	0
E_{n+3} (FF)	0	0	0	$\cdots$	0	0	0	1	0
E_{n+4} (CD)	0	0	0	$\cdots$	0	0	0	0	1

Figure 10. Structure of transition probability matrix

by the time moment of t_{SL}. Graphically such a transition process is presented in Figure 9. The transition probability matrix of this process can be composed as it is presented in Figure 10.

As we see, the matrix shown on the Figure 10 could be presented as a composition of four sub-matrices *Q*, *R*, *0* and *I*, and the Markov Chains theory lets us to get the probability to absorb in each state. Markov Chains theory is especially attractive to model various scenarios of switching to the alternative inspection programs when the certain event takes place. For example, we can double the frequency of inspections when the first crack is discovered in the fleet. An example of this approach is shown below for initially two-inspection program: the state graph is presented in Figure 12 and composed transition probability matrix – in Figure 11.

	E_1	E_2	E_3	E_4 (E_{SL})	E_5 (E_{FF})	E_6 (E_{CD})
E_1	0	u_1	0	0	q_1	v_1
E_2	0	0	u_2	0	q_2	v_2
E_3	0	0	0	u_3	q_3	v_3
E_4 (E_{SL})	0	0	0	1	0	0
E_5 (E_{FF})	0	0	0	0	1	0
E_6 (E_{CD})	0	0	0	0	0	1

Figure 11. Transition probability matrix for two-inspection program

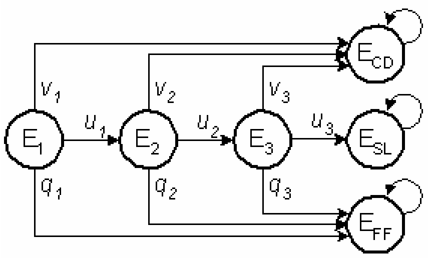


Figure 12. Ordinary two-inspection strategy state graph

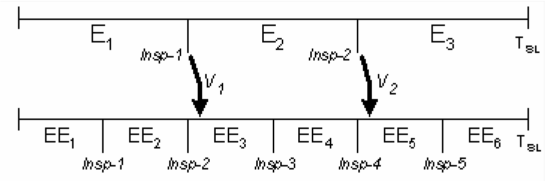


Figure 15. Switching to doubled inspection frequency (2-5-inspection model)

For this example there are two possible time moments of switching to the doubled inspection program – at the time of the first inspection (if a crack is discovered during the first inspection) and at the time of the second inspection. The corresponding graph is presented in Fig. 13. The modified state graph is shown in full form in Figure 13 and in the reduced form in Figure 14.

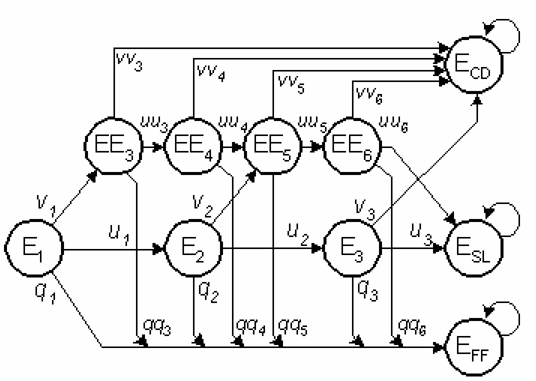


Figure 13. Switching to doubled inspection frequency state graph (2-5-inspection model)

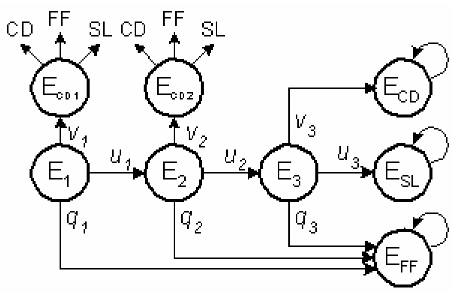


Figure 14. Switching to doubled inspection frequency reduced state graph (2-5-inspection model)

It is obvious that the random inspection program $R_D(\cdot)$ has in fact three possible realizations here; these three realizations (scenarios) are graphically presented in Figure 16.

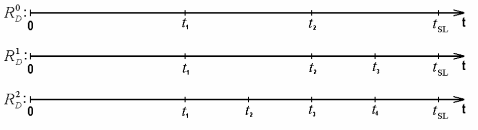


Figure 16. Realizations of the random inspection program (2-5-inspection model)

$$R_D^0: \ \{t_1, t_2, t_{SL}\};$$
$$R_D^1: \ \{t_1, t_2, \tfrac{1}{2}\cdot(t_{SL}+t_2), t_{SL}\};$$
$$R_D^2: \ \{t_1, \tfrac{1}{2}\cdot(t_2+t_1), t_2, \tfrac{1}{2}\cdot(t_{SL}+t_2), t_{SL}\}.$$

The probability of each scenario to realize depends on the probability to discover a crack during the inspections of the basic (R_D^0, i.e. without switching to the doubled inspection frequency) scenario. The modified transition probability matrix for the case of 2-5-inspection program looks like presented in Figure 17. Having these modified transition probability matrices we are able now to calculate failure probability of modified inspection programs.

	E_1	E_2	E_3	EE_3 (E_{CD1})	EE_5 (E_{CD2})	E_4 (E_{SL})	E_5 (E_{FF})	E_6 (E_{CD})
E_1	0	u_1	0	v_1	0	0	q_1	0
E_2	0	0	u_2	0	v_2	0	q_2	0
E_3	0	0	0	0	0	u_3	q_3	v_3
EE_3 (E_{CD1})	0	0	0	0	0	uu_3^*	qq_3^*	vv_3^*
EE_5 (E_{CD2})	0	0	0	0	0	uu_5^*	qq_5^*	vv_5^*
E_4 (E_{SL})	0	0	0	0	0	1	0	0
E_5 (E_{FF})	0	0	0	0	0	0	1	0
E_6 (E_{CD})	0	0	0	0	0	0	0	1

Figure 17. Modified transition probability matrix: 2-5-inspection program

In general case there are n_{TIP} inspections in the inspection program and $n_{TIP}+1$ transient service states. Thus, there are n_{TIP} possibilities to switch to the doubled inspection frequency, generating a set of $n_{TIP}+1$ realizations (or scenarios) of the random inspection program $\vec{\mathbf{R}}_{\mathbf{D}} = \left\{R_D^0, R_D^1, \dots R_D^{n_{TIP}}\right\}$.

Let $\vec{\mathbf{V}}^* = \left\{ \overset{*}{v_0}, \overset{*}{v_1}, \overset{*}{v_2}, \dots \overset{*}{v_{n_{TIP}}} \right\}$ is a vector of probabilities: $\overset{*}{v_i}$ is a probability to discover a crack during t^{th} inspection, $i = 1, \dots, n_{TIP}$, $\overset{*}{v_0} = 1 - P_f^0 - \sum_{i=1}^{n_{TIP}+1} \overset{*}{v_i}$, where P_f^0 is a failure probability for the whole inspection program in accordance with scenario R_D^0, $\overset{*}{v_0}$ has a meaning of non-discovery of any crack and staying with the initial inspection scenario R_D^0 up to t_{SL}.

	E_1	E_2	$\cdots$	$E_{n_{TIP}+1}$	E_{SL}	E_{FF}	E_{CD1}	E_{CD2}	$\cdots$	$E_{CD(n_{tip}+1)}$
E_1	0	u_1	$\cdots$	0	0	q_1	v_1	0	$\cdots$	0
E_2	0	0	$\cdots$	0	0	q_2	0	v_2	$\cdots$	0
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$E_{n_{TIP}+1}$	0	0	$\cdots$	0	$u_{n_{TIP}+1}$	$q_{n_{TIP}+1}$	0	0	$\cdots$	$v_{n_{tip}+1}$
E_{SL}	0	0	$\cdots$	0	1	0	0	0	$\cdots$	0
E_{FF}	0	0	$\cdots$	0	0	1	0	0	$\cdots$	0
E_{CD1}	0	0	$\cdots$	0	0	0	1	0	$\cdots$	0
E_{CD2}	0	0	$\cdots$	0	0	0	0	1	$\cdots$	0
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$E_{CD(n_{tip}+1)}$	0	0	$\cdots$	0	0	0	0	0	$\cdots$	1

Figure 18. Modified transition probability matrix for calculation of $\vec{\mathbf{V}}$

Then another interpretation of the switching to doubled inspection frequency is given, interpreting the random inspection program R_D as an element of the set of inspection scenarios $\vec{\mathbf{R}}_{\mathbf{D}}$. For this case $\vec{\mathbf{P}}_f\left(\vec{\mathbf{R}}_{\mathbf{D}}\right) = \left\{P_f^0\left(R_D^0\right), P_f^1\left(R_D^1\right), \dots, P_f^{n_{TIP}}\left(R_D^{n_{TIP}}\right)\right\}$, and the common failure probability of the random inspection program R_D can be presented as a sum of failure probabilities of all scenarios multiplied by the probabilities of these scenarios to realize: $P_f = P_f^0 + \sum_{i=1}^{n_{TIP}} \left(\overset{*}{v_i} \cdot P_f^i\right)$. The values of elements of vector $\vec{\mathbf{V}}^*$ can be derived by creating modified transition probability matrix and calculating matrix B of absorption probabilities as demonstrated in Figures 18 and 19.

Only the values of our interest are shown in matrix in Figure 19. The matrix $B = N \cdot R$, $N = (I - Q)^{-1}$.

	E_{SL}	E_{FF}	E_{CD1}	E_{CD2}	$\cdots$	$E_{CD(n_{tip}+1)}$
E_1	v_0^*	P_f^0	v_1^*	v_2^*	$\cdots$	$v_{n_{TIP}+1}^*$
E_2	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdots$	$\cdot$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$E_{n_{TIP}+1}$	$\cdot$	$\cdot$	$\cdot$	$\cdot$	$\cdots$	$\cdot$

Figure 19. Matrix of absorption probabilities B

Of course, it is possible to apply the approach of doubling the frequency of inspections (or changing it by any other rule) when some additional information about aging fleet becomes available. This can require, for example, recursive use of the technique described in this section or additional modifications of the transition probability matrices. Such scenarios are subject of additional studies and are not a part of this work.

Then the minimax statistical decision function can be introduced, if we will return the airframe project for redesign in case of the estimate of expectation value of some set of parameters $\vec{d}_L = \vec{d}_L(\hat{\theta})$ does not belong to the set of its allowed values D_L:

$$\hat{P}_{f_{corrected}} = \begin{cases} P_f(\theta, \vec{d}_L) & , \vec{d}_L \in D_L \\ 0 & , \vec{d}_L \notin D_L \end{cases}.$$

The number of elements in $\vec{d}_L$ and, therefore, the dimension of the set D_L, may vary depending on the modelling situation and the specific requirements. The parameter θ which defines the c.d.f. of vector (T_d, T_c), is a vector parameter. In the considered work, if both crack model parameters are random and have normal distribution, it consists of five components:

$$\theta = \left[\theta_{0_{\ln Cc}}, \theta_{1_{\ln Cc}}, \theta_{0_{\ln Q}}, \theta_{1_{\ln Q}}, r\right],$$

$$\theta \in \Theta = \left\{ \; (-\infty, \infty); \;\; [0, \infty); \;\; (-\infty, \infty); \;\; [0, \infty); \;\; [0,1] \; \right\},$$

where θ_0 stands for a location and θ_1 stands for a scale parameter of the appropriate crack growth model parameter $\ln C_c$ or $\ln Q$; r is a coefficient of correlation between $\ln C_c$ and $\ln Q$. It is proved that for considered decision making procedure the random variable $\hat{P}_{f_{corrected}}$ has expectation value, which is a function of θ, and this function has a maximum value. An example, where all parameters are fixed except one $\theta_{0_{\ln Q}}$ (crack growth rate) is presented in Figure 20 and Figure 21:

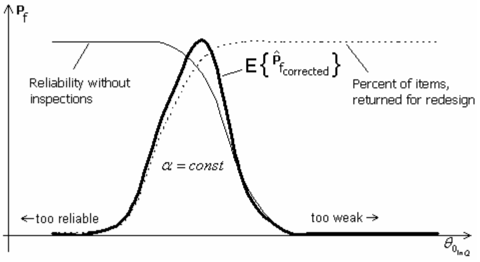


Figure 20. Corrected failure probability example for $\alpha = const$.

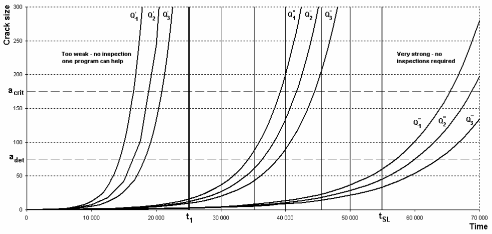


Figure 21. Explanation of the minimax approach.

If we suppose, that parameters $\theta_{1_{\ln Q}}$, $\theta_{1_{\ln Cc}}$ and r depend on technology, which does not change (for a new aircraft), and these parameters can be estimated using information of previous designs, than we have only two unknown parameters $\theta_{0_{\ln Q}}$ and $\theta_{0_{\ln Cc}}$, so we come to the three-dimensional extreme picture $P_f = P_f\left(\theta_{0_{\ln Q}}, \theta_{0_{\ln Cc}}\right)$.

The value of failure probability used for calculations (to choose the number of inspections required, or choosing vector $\vec{t}$) is denoted as $P_{f_{calc}}$. The value of expectation of failure probability as function of $\hat{\theta}$ is denoted as $\mathrm{E}\left\{\hat{P}_{f_{corrected}}\right\}$. We have named it "corrected" to distinguish it from $P_{f_{calc}}$. The main idea is to find such a maximum value of failure probability for calculations $P^*_{f_{calc}}$ for which the corrected value of failure probability $\mathrm{E}\left\{\hat{P}_{f_{corrected}}\right\}$ does not exceed the required limiting value of failure probability $P_{f_{required}} = 1 - R_{required}$, where $R_{required}$ is required reliability:

$$P^*_{f_{calc}} : \widetilde{P}\left(P_{f_{calc}}\right) \le P_{f_{required}}, \text{ where } \widetilde{P}\left(P_{f_{calc}}\right) = \max_{\theta}\left(\mathrm{E}_{\theta}\left\{\hat{P}_{f_{corrected}}\right\}\right).$$

This approach for a case, when one of parameters $\theta_0 = const$, is presented in Figure 22, and for the more complex case in Figure 23:

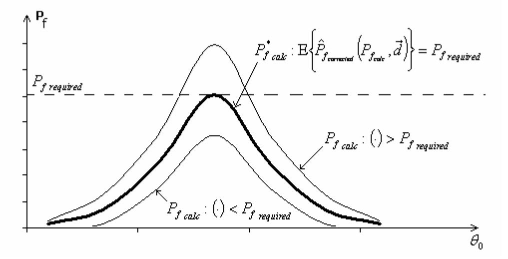


Figure 22. Minimax approach example ($\alpha = const$ **or** $\ln Q = const$)

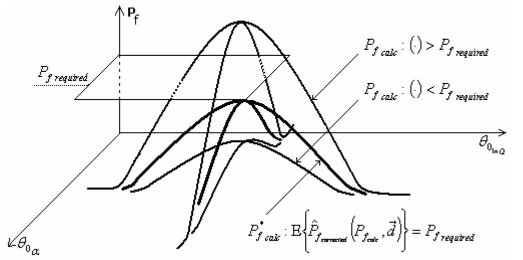


Figure 23. Minimax approach example (general case)

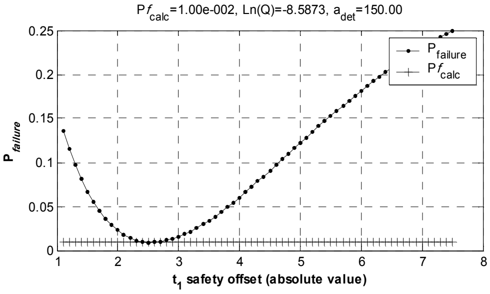
8. Chapter three

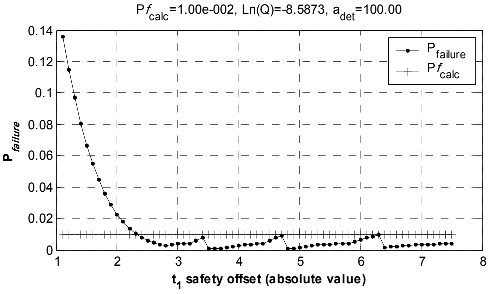
This chapter includes description of Experimenter's Workplace (EW), created for this work. There are two EW created: Excel-based EW and Matlab-based EW. Excel-based version is simpler and is more suitable for students to investigate its work; it lets play with parameters and see results in real-time interactive mode, but has limited functionality. Matlab-based version is much more powerful and has no Excel EW limitations, but requires some basic knowledge of Matlab system if any modification is required. All results derived in this work were received using these EW. Chapter three gives complete description of the internal structure, logic an algorithms of all EW modules.

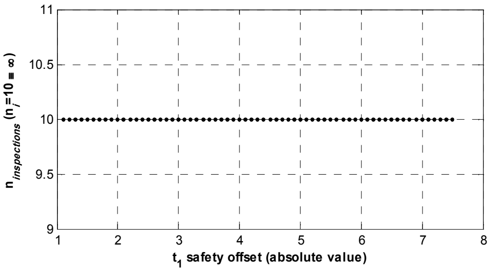
9. Chapter four

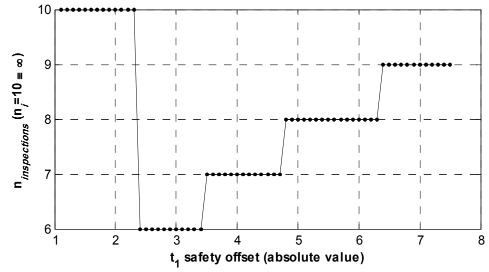
This chapter includes several numerical examples, acquired using developed EW and based on new model. Some examples of results (no numbers, visual representations only) are given below.

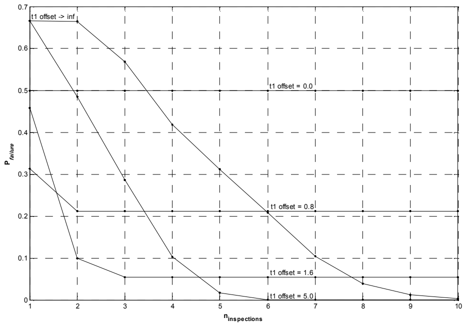
Illustration of choosing optimal offset for t_1, which ensures the minimum possible number of inspections for the given inspection program type. You can see, that optimal offset is in [-3.3; -2.4]:

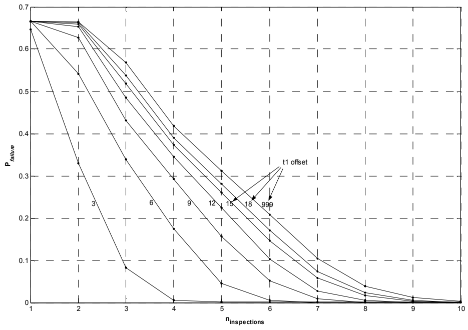




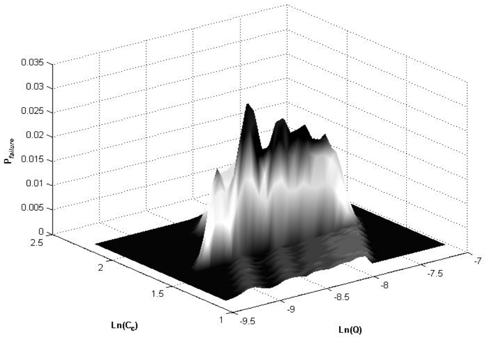


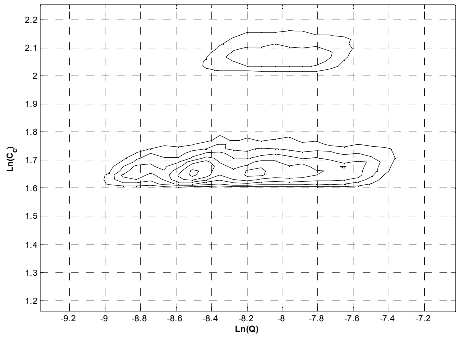




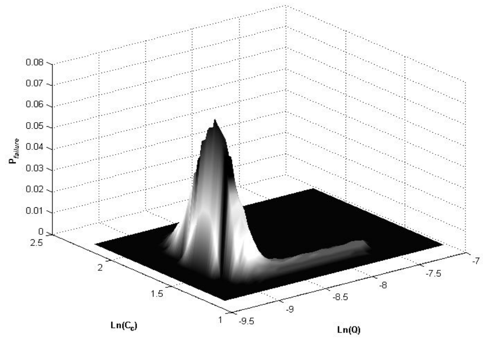


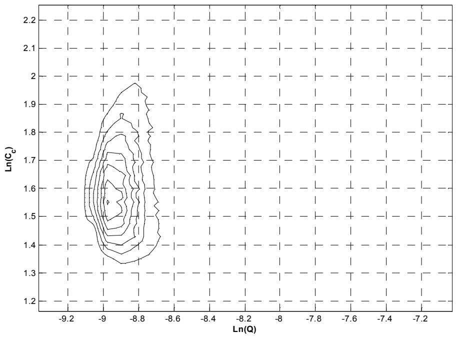
All types of inspection programs, except the program with evenly distributed inspections, can produce “harmonics” — a set of shifted local maximums, usually repeating the form of the main maximum (this effect can mislead the investigator):

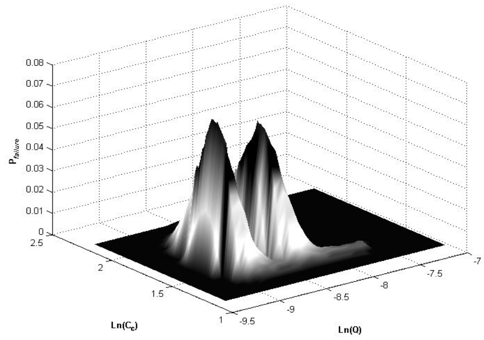


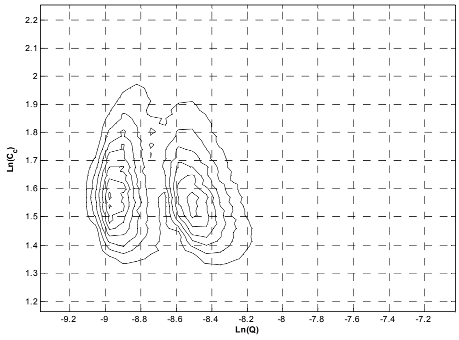


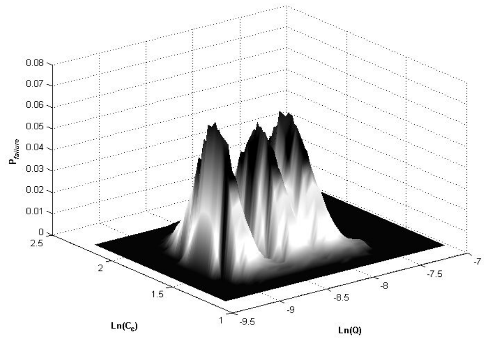
Illustrations of several specific properties of $P_{f_{corrected}}$ surface: there are several extremes (for small $n_{\max}$ it is equal to the $n_{\max}$, for big $n_{\max}$ some of "peaks" can merge):

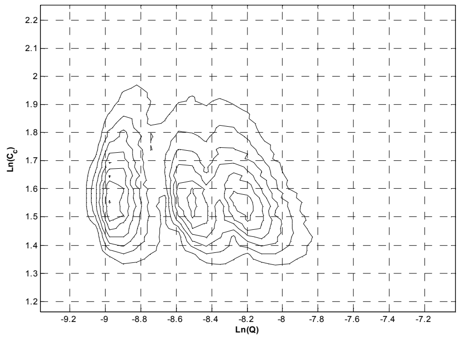


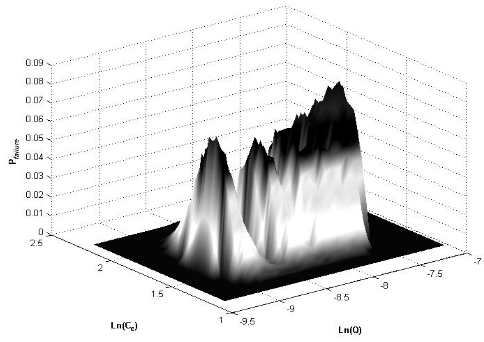


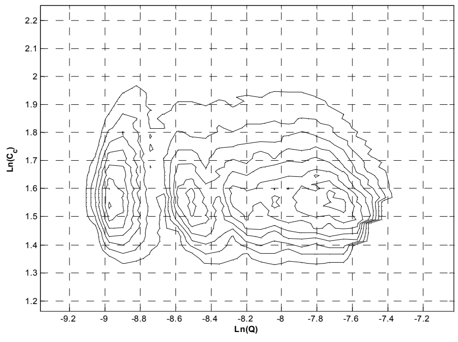






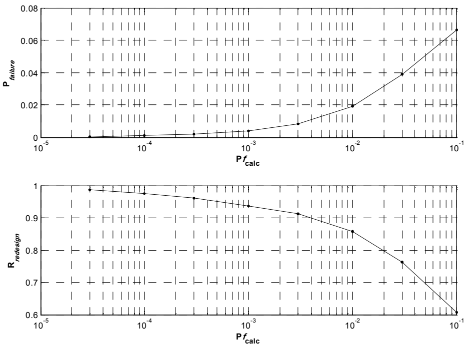


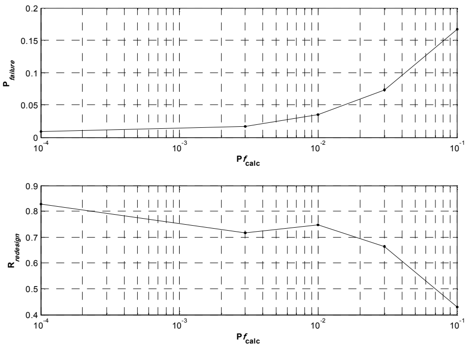




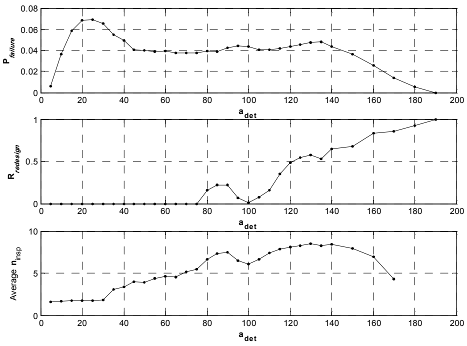
The knowledge of this feature lets optimising the search of the global maximum.

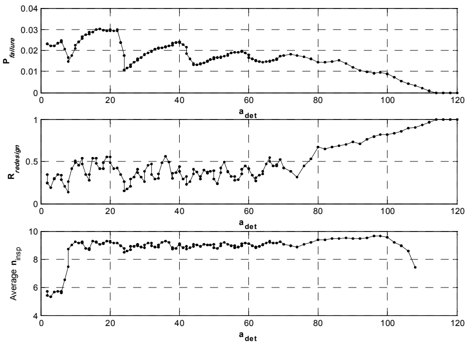
The value of relation $P_{f_{corrected}} \Big/ P_{f_{calc}}$ depends on absolute value of P_{fLcalc} for any type of the inspection program (variant with special t_1 is presented at left, variant with evenly distributed inspections – at right):

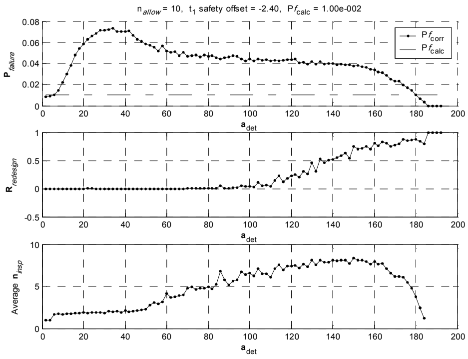


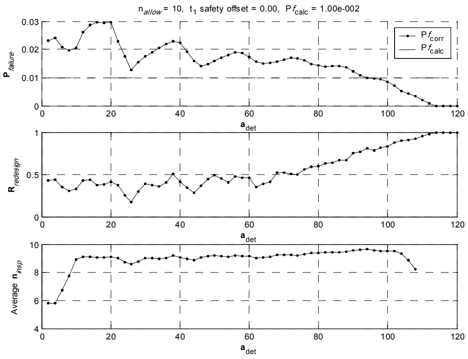


For inspection programs with comparatively small $n_{\max}$ dependencies of $P_{f_{corrected}}$ from $a_{\det}$ is also a non-monotonic function (variant with special t_1 is presented at left, variant with evenly distributed inspections – at right):

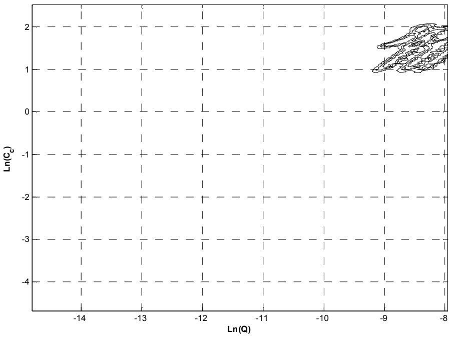


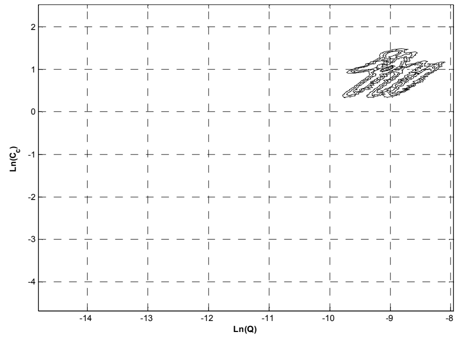


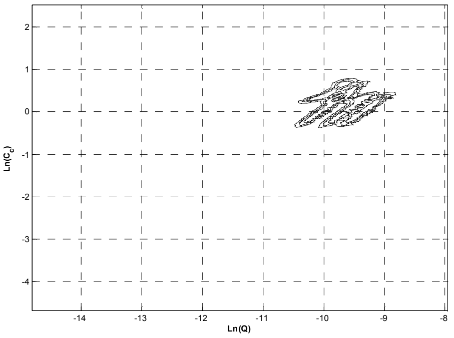


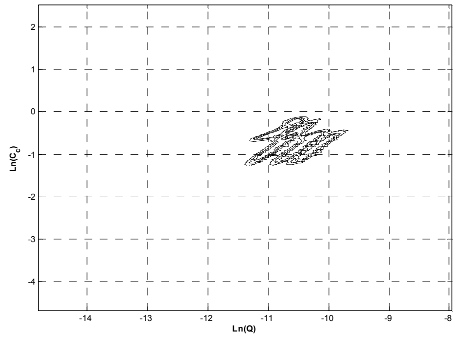


The position of the global maximum depends on the inspection method (it “shifts” when $a_{\det}$ changes):

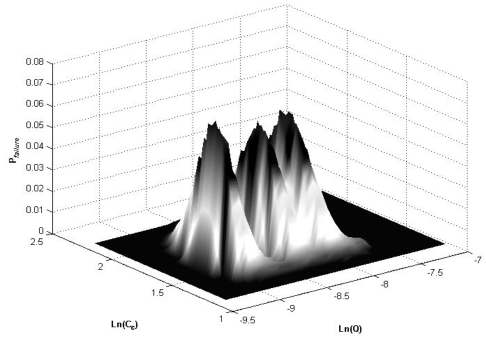


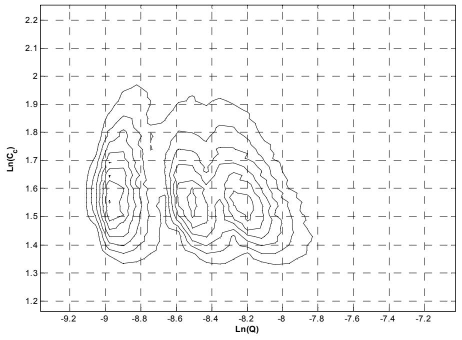


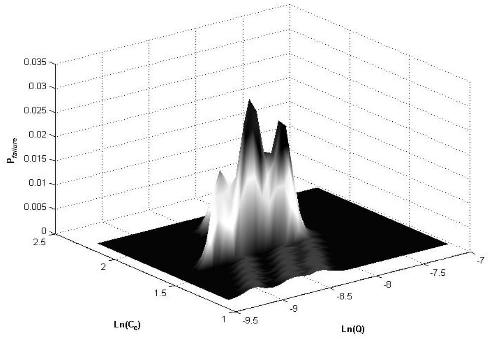


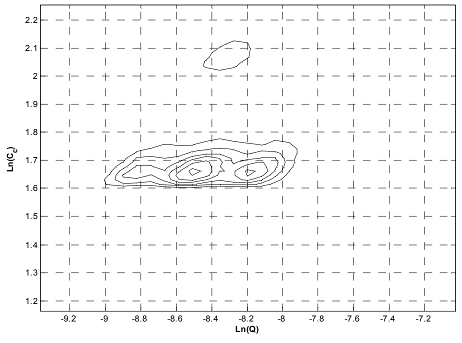


Use of special value of t_1 in comparison with the evenly distributed inspections smoothes out the surface:

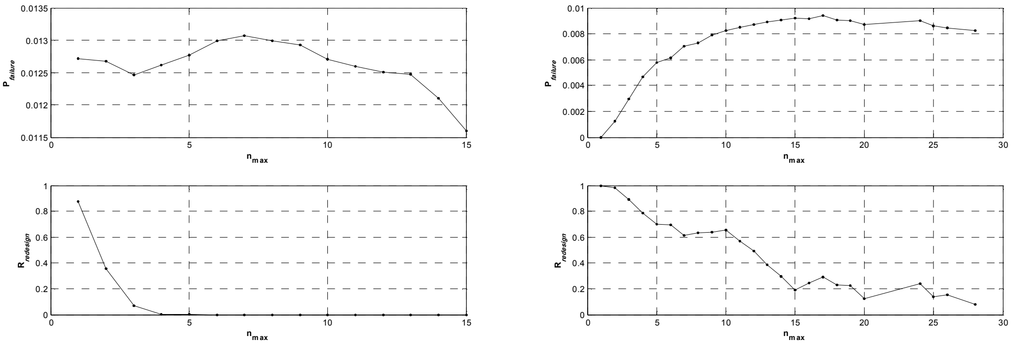








For inspection programs with comparatively small $n_{\max}$ dependencies of $P_{f_{corrected}}$ from $n_{\max}$ itself could be a non-monotonic function (variant with special t_1 is presented at left, variant with evenly distributed inspections – at right):



As you can see in this example, the bigger number of inspections in certain cases may also increase the probability of failure.

10. Conclusions

- As a result of the work done the new mathematical models are created, allowing to optimise the inspection programs of fatigue prone structural significant items of an airframe; unlike others, the offered approach is based on the exponential version of Paris' model (for a curve of growth of a fatigue crack) with two random parameters (the equivalent initial crack size and crack growth rate in a logarithmic scale) with a nonzero correlation between them.
- The modular software package is created to realise the model specified above; this software package is a convenient, flexible and functional working tool of the researcher, the specialised test bench, allowing calculating parameters of inspection for a set of more than thirty variables and parameters. The software package realises five various optional functions of optimisation of the calculations based on knowledge of model specific features. The results of modelling results can be stored for further use in a special database. All the main graphic results can be presented as in the form convenient for interactive modelling, as in the form convenient for printing; the optional function of n-point interpolation of results and an optional tool of averaging of the results calculated with the same values of modelling parameters (arguments) are realized. Furthermore, software supports batch modelling with four various types of formation of argument sequences. The software has the modular structure and can be replenished with the new modules realising additional functions.
- Using the developed software complex it was possible to carry out wide investigation of the offered model, to study some features of its behaviour, which, in its turn, has allowed optimising operation of the software itself. As a result of investigations and modelling the new technique of setting up the time of the first inspection has been offered; this allows finding minimally possible number of inspections for the given type of the inspection program. All the received results are evident and with the use of real data give verisimilar, theoretically expected results.
- Using the developed software complex, investigation of opportunity to switch on the alternative inspection program at occurrence of the certain event in service has been made; it was shown, what even in elementary case of switching to the doubled inspection frequency on the remained part of the fleet after detection of the first fatigue crack allows to reduce the general average number of inspections for the fleet.

11. Appendices

Appendices include all detailed information that was not reasonable to include in the main text of the work to do not overload it. The following mail appendices are included: source fatigue test data used for numerical examples; structure of all databases; brief user manuals for all software; program source codes; several detailed numerical examples, which are too big to be included in Chapter 4, as well as some program screenshots to demonstrate software user interface.

12. Publications

1. Y. Paramonov, A. Kuznetsov. Fatigue crack growth parameter estimation by processing inspection results // In: Scientific proceedings of Riga Technical University – Transport and Engineering – Issue #6/6 – Riga, 2001 – pp. 6-17.
2. Kuznetsov A. S., Paramonov Yu. M. Airframe inspection program development using adjustment of required failure probability // Mašīnzinātne un transports, sērija 6. sējums 8, Transports, Aviācijas transports, izdevniecība RTU, Rīga, 2002 – 110-117 lpp.
3. Paramonov Yu. M., Kuznetsov A. S. Fatigue crack growth parameter estimation by processing inspection results // Proceedings of Third International Conference on Mathematical Methods in Reliability (MMR2002), Methodology and Practice. – Trondheim, Norway, 2002, NTNU – pp. 505-508.
4. Paramonov Yu. M., Kuznetsov A. S. Inspection data use for inspection program development // Transport and Telecommunication Institute.v.4, #2, Riga, 2003 – pp.101-107.
5. Kuznetsov A., Paramonov Yu. Switching to doubled aircraft inspection frequency strategy analysis for exponential fatigue crack growth model // Mašīnzinātne un transports, sērija 6. sējums 13., Transports, Aviācijas transports, izdevniecība RTU, Rīga, 2003 – 23-31 lpp.
6. Paramonov Yu. M., Kuznetsov A.S. Inspection data use for airframe inspection interval correction // Aviation, Issue #6 – Vilnius, Technika, 2002 – pp.109-116.
7. Yuri M. Paramonov, Andrey S. Kuznetsov. Inspection data use for inspection program development // Abstracts of International Conference “Reliability and statistics in transportation and communication (RelStat’02)” – Riga, Latvia, 17–18 October 2002 – p. 25.
8. Kuznetsov A., Paramonov Yu. Switching to doubled aircraft inspection frequency strategy analysis for exponential fatigue crack growth model // In the book “Longevity, Ageing and Degradation Models in Reliability, Medicine and Biology”, Volume 1, pp.143-154 – St. Petersburg, Russia, 2004.
9. A. Kuznetsov, Yu. Paramonov. Investigating doubled aircraft inspection frequency strategy for exponential fatigue crack growth model // In: Fourth International conference “Mathematical Methods in Reliability (MMR2004)” – Santa Fe, New Mexico, USA, 21-25 June 2004.
10. A. Kuznetsov, Yu. Paramonov. Using of P-set function for aircraft inspection program development for exponential fatigue crack growth model. // Book of abstracts of Sixth International Seminar on Recent research and design progress in aeronautical engineering and its influence on education – Riga, Latvia, October 14-16, 2004, – p.26.
11. Paramonov Yu., Kuznetsov A. Using of p-set function for airframe inspection program development // Proceedings of the International Symposium on Stochastic models in reliability, safety, security and logistics. 15-17 February 2005, Beer Sheva, Israel – pp. 280-283.
12. Paramonov Yu., Kuznetsov A. Planning of inspection program of fatigue-prone airframe. // Proceedings of International scientific school “Modelling and analysis of safety and risk in complex systems MASR2005” – Russian Academy of Science, St. Petersburg, Russia, June 28-July 1 2005 – pp. 390-396.

13. Y. Paramonov, A. Kuznetsov. Inspection program development for fatigue crack growth model with two random parameters. // Abstracts of International Conference “Reliability and statistics in transportation and communication (RelStat’05)” – Riga, Latvia, 13–14 October 2005 – p. 97.

13. Participation in conferences, symposiums and presented abstracts

1. The third International conference on Mathematical Methods in Reliability MMR2002 – Trondheim, Norway, 17-20 June 2002.
2. RTU Conference “Production Engineering and Transport” – Riga, Latvia, 10 October 2002.
3. International scientific conference “Reliability and Statistics in Transportation and Communication RelStat’02” – Riga, Latvia, 17-18 October 2002.
4. RTU 44th International scientific conference – Riga, Latvia, 17-18 December 2003.
5. The third International conference on Longevity, Ageing and Degradation models in Reliability, Medicine and Biology LAD2004 – St. Petersburg, Russia, 7-9 June 2004.
6. Fourth International conference on Mathematical Methods in Reliability MMR2004 – Santa Fe, New Mexico, USA, 21-25 June 2004.
7. Conference “Diagnosis of technical systems, numerical and physical non-destructive quality testing – 2004” – Vilnius, Lithuania, 23 April 2004.
8. Starptautiskā zinātniskā konference “Reliability and Statistics in Transportation and Communication RelStat’04” – Riga, Latvia, 14-15 October 2004.
9. Sixth international seminar on Recent research and design progress in aeronautical engineering and its influence on education – Riga, Latvia, 14-16 October 2004.
10. International Symposium on Stochastic models in reliability, safety, security and logistics – Beer Sheva, Israel, 16-18 February 2005.
11. International conference “Modelling and analysis of safety and risk in complex systems MASR2005” – St. Petersburg, Russia, 28 June – 1 July 2005.
12. The fifth International conference “Reliability and statistics in transportation and communication RelStat’05” – Riga, Latvia, 13-14 October 2005.
13. RTU 46th International scientific conference – Riga, Latvia, 13-15 October 2005.