

**RIGA TECHNICAL UNIVERSITY**

**Normunds TIRĀNS**

**STRENGTHENING OF FLEXURAL  
REINFORCED CONCRETE ELEMENTS  
WITH COMPOSITE COVER PLATES**

Civil Engineering, Structural Analysis

Summary of Doctoral Thesis

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## THESIS OVERALL REVIEW

### Topics actuality

Necessity to reconstruct outworn infrastructure is a commonly known issue in recent years. Many findings show, as reinforced concrete elements with bending actions are usually out-of-date according to modern requirements. Therefore a necessity to reinforce old or constructively inadequate building elements has become important, both in field of strength and stiffness. There are many occasions when existing constructions are morally aged. It is especially popular within structures designed according to out-of-date codes with relatively low level of imposed actions. All this together leads to a necessity to reinforce individual building elements with bending actions.

When applying loads, flexural elements deform, furthermore, the process is followed by a continuous generation and development of cracks. During this process a continuous redistribution of stresses between concrete in compression and reinforcement in tension is going on, reaching one of limit states followed by a failure of flexural element. Damage and corrosion can also be a reason for the failure of this element. If the action level is increased, the resistance of high quality building structures can also be insufficient.

Carbon fiber cover plates - strips and sheets are recently used for strengthening concrete structures. The aim of this strengthening is to increase the strength and stiffness of particular elements in determined conditions. A range of different carbon fibers with different binders are commonly used. Best conditions for strengthening flexural reinforced concrete elements can be found working with different fiber and binder features. Extra reinforcement with composite structure materials on the area under tension in flexural elements influences opening of cracks in concrete and increases the strength and stiffness.

The stiffness of bending elements is affected by the development of cracks in the tension part of cross section. As a result of increasing crack height the compression zone decreases and the stresses in the materials increase. Because of particular nonlinear behavior of concrete stress-deformation relationship, when the stresses rise and redistribute, the concrete layer with nonlinear deformations builds up.

Most of constructive building elements are heterogeneous and asymmetric. It is typically inherent in cases of reinforced flexural building elements. Because of the asymmetric structure complicated stress and deformation relationships develop.

Heterogeneous, yet longitudinally oriented reinforced concrete elements have a lamellar structure, therefore for calculations of such elements the calculation models of lamellar structure can be used. Hereby the hypotheses and solutions of classical laminate theory can be used in these calculations.

Usage of lamellar mechanics in creation of structural elements clears the way for influencing the physically-mechanical features of these elements. This method gives an opportunity to fully use the potential features of any layer in creation of new material with adjustable features. The most important adjustable features are strength and stiffness, however, in reality dead weight, corrosion resistance, thermo and damp insulation etc. are likewise very important. The combined constructive elements can be created in the production phase, according to forecasted conditions of exploitation. Traditionally, lamellar structure elements are created symmetrical with respect to their center line (plywood, composites with cement or polymer binders). However, frequently asymmetric structure is more favorable. The principle of non-symmetric cross sections is commonly used in flexural reinforced concrete elements. The different features of concrete in tension and compression are compensated with irregular placing of steel reinforcement. Asymmetric lamellar structures are also created when reinforcing the existing, frequently operating, constructions.

Initially used in bridge building, the method of adding a carbon fiber polymer layer to the tension zone is currently very widespread. To create scientifically verified principles of reinforcing flexural reinforced concrete elements and obtain estimations based on lamellar bar structure mechanics for determining quantitative and structural composition of reinforcement, the research using multi-component lamellar structure bar mechanics approach and methods is conducted in this thesis. The main problems in this research are concerned with internal action distribution interchange and fluctuation regularity in combined multi-component bars with asymmetric structure and changing deformative and strength features during applying of loads.

Lamellar profiled cross section bar and bar system mechanical features were examined. Considering the profile of the cross section, asymmetry of the structure and mechanical features of particular layers, a methodology for determining stresses in a beam is developed. The results obtained theoretically are compared to the experimental values.

There are two fundamentally different approaches to reinforcement of exploited beams. One of them is concerned with removal of loads and reinforcement in such relieved state. In this case most of the deformations accumulated during exploitation are removed, practically all the components of the beam relieved and cracks closed. When the reinforcement is made with sticking of carbon fiber polymer strips, a new calculation of model is created with typical lamellar structure. The main components of this structure are concrete and reinforcement bars in compression, cracked concrete layer and reinforcement bars and carbon fiber strip in tension. If the yield strength of reinforcement bars is not exceeded during the exploitation, the stiffness and strength calculations of reinforced beam can be performed with the previously mentioned method, considering the existing structure of the beam.

The second approach in reinforcement of existing beams is to create a combined beam without removal of loads. The deformations accumulated during exploitation remain and during loading stresses in lamellar structure elements are already present. Thus a beam with initial stresses and strains is being reinforced. One has to consider that the deformations calculated with the proposed method are only a part of deformations that arise in the beam, the same happens with stresses.

Deformative features of reinforced concrete elements in all their loading range till failure are investigated in this thesis. The method can be used for determination of flexural stiffness in all linear, formation of cracks and metal rebar yielding stages. Methodology is materialized in numerical program allowing consideration of elastic features of beam components, nonlinearity of concrete deformative features, development of cracks, steel rebar yielding and redistribution of stresses between components during loading of the beam. Results of the research show that reinforcement of the existing beams with carbon fiber polymer strips significantly reduces the deformations of the beam and reduces stresses in metal reinforcement bars. Yielding of those reinforcement bars begins at much higher loads. That is the reason for increasing of strength of such reinforced beams. The results obtained can be used in reinforcing of flexural beams with extra layers of reinforcement in tension zone and for determination of characteristics of these layers and attachment technology.

Creation and development of shear cracks in flexural composite beams research shows that rational way to increase shear strength and stiffness is to attach layers with high shear strength perpendicularly to the neutral plane of the beam.

Confirmation of the method results is obtained by comparing the theoretically forecasted results with those experimentally determined.

Experiments show that sticking of carbon fiber polymers to the reinforced concrete surface gives significant increase in flexural strength, if the cohesion between the beam and added layers is guaranteed. The failure of such reinforced beams mainly happens due

to the failure of bond in the ends of the added layer. Removal or reduction of such threat gives an opportunity to increase performance of the beam. This is important both for exploitable beams as well as for the ones that are never used before, when creating their reinforcement by adding external layers.

The developed methodology allows, by using lamellar bar mechanics, predict and influence reinforced concrete material deflections and displacements, which are the main restricted positions of building construction quality.

The methodology for calculating strengthening of the existing beams developed in this thesis is used to perform a strengthening of twenty years exploited three span reinforced concrete bridge of Jurmala ring road crossing Varkalu channel, stretching from Lielupe River and Babītes Lake in Riga district, Latvia.

### **Aim of thesis**

To develop a calculation method for determination of strength and stiffness of concrete elements, reinforced with steel rebars and composites, undergoing bending actions, considering the changing structural and deformative features of element during its loading, and use this method for calculation of particular operating structure reinforcement.

### **Scientific novelty of thesis**

A new calculation method for determination of strength and stiffness of concrete elements, reinforced with steel rebars and composites is developed.

A variable calculation model for a combined, lamellar structure during loading and its implementation in numerical algorithm is worked out.

Relationship between qualitative and quantitative features of reinforcing layers and dynamics of formation of cracks is evaluated.

The regularity of redistribution of stresses in pure bending, considering the nonlinear deformative nature of concrete, formation and development of cracks during loading is developed.

A methodology for forecasting deformations in case of changing structure is worked out for flexural reinforced concrete elements that are strengthened with external materials.

### **Thesis in practice**

There is now a possibility to predict exploited flexural element reserve in load bearing capacity, and a method of reinforcing these elements to required level is developed.

The results are used to perform the reinforcement of twenty years exploited three span bridge crossing Varkalu channel, according to characteristic actions.

### **Following is advanced for presentation:**

- The calculation for distribution of stresses in flexural, reinforced with composite material additions, reinforced concrete materials which considers cracking of concrete while loading and nonlinear deformative features of concrete;
- Methodology for calculation of regularity of changes in reinforced concrete element stiffness with composite reinforcement;

- Relevances between reinforcing structure quantitative and qualitative features and strengthened element load bearing capacity in bending;
- The realization of calculation methodology in estimation of twenty years exploited bridge concrete beams strengthening up to 40%.

### **Contents and scope of thesis**

The doctorate thesis consists of introduction, 5 chapters, conclusion and bibliography. The wordage is 130 pages, there are 67 drawings, 10 tables and a reading list with 107 titles.

### **Approbation of thesis and publications**

The doctorate thesis is reported and discussed in international conferences:

- o RTU, Arhitektūra un Būvzinātne, Riga, Latvija, 2002. „Cracking Criteria of Reinforced Concrete Beams Strengthened for Shear”
- o RTU, Arhitektūra un Būvzinātne, Riga, Latvija, 2003. „Method of Prediction of the Deflections of Reinforced Concrete Beams Considering Cracking”
- o SDSMS'03 International Conference, Klaipeda, Lithuania, 2003. „Model of Nonlinearly Deforming Laminated Material”
- o 13. International Conference Mechanics of Composite Materials, Jūrmala, Latvija 2004. „Deformability Prediction for Ferroconcrete Beams Strengthened with Carbon-Filled Plastic Layers”
- o 8. International Conference of Modern Building Materials, Structures and Techniques, Vilnius, Lithuania, 2004. forecasting of Deflections of Reinforced Concrete Beams Strengthened with Carbon Plastic Sheets”
- o RTU 45th International Scientific Conference, Riga, Latvija, 2004. „Recovery and Enhancement of Load Carrying Capacity of Bent Operating Reinforced Concrete Beams Strengthened with Carbon Plastic Sheets”
- o RTU 46th International Scientific Conference, Riga, Latvija, 2005. „Saistes īpašības ar kompozītiem materiāliem pastiprinatos liektos dzelzsbetona elementos”

Main results of the thesis are outlined in 9 publications

The work has been carried out in Riga Technical University Department of Structural Analysis between years 2000 and 2006.

## **CONTENTS OF THESIS ;**

The introduction includes definition of a problem for exploitation of outworn infrastructure and formulation of the aim of thesis, tasks, scientific novelty and practical use.

**The first chapter** contains classification of methods and technologies for strengthening of reinforced concrete structures, review and evaluation of possibilities and disadvantages of the existing calculation methods.

When the actions on a construction element rise and because of corrosion of concrete and reinforcement bars the resistance of building elements is frequently insufficient. The history of strengthening of reinforced concrete constructions is nearly the same age as the concrete constructions themselves.

It is possible to distinguish between the following reinforcing technical solutions.

1) Change in static scheme

This is one of the most effective ways of strengthening of reinforced concrete building elements. It is possible to widen the bearings of a beam thus reducing the span or even impose new bearings.

Turning a simple beam into a suspended construction where beam serves as a top boom of a complex construction. Thus bending stresses are significantly reduced in the beam and the top boom is especially loaded with compression stresses, which are more favorable for reinforced concrete elements. It has to be taken in mind that this solution is possible only for elements that have both tension and compression reinforcement bars which can become tensioned if the bending moment changes its direction.

2) Building up of reinforced concrete cross section

Widely applied solution contains adding an extra height to the cross section by adding more concrete onto the compression zone. Thus the arm of force in tension rebar can be increased thus the ultimate bending moment can be increased.

It is very important to secure bond between the existing and new concrete in this kind of strengthening. This method has little efficiency in cases when shear strength has to be increased.

This solution is safe, with high corrosion resistance and simple, however it increases the size of structure and adds extra self weight loads. However the strengthening can be created to resist both extra self weight and extra imposed loads. It is important to notice the extra loads on beams supports. For this reason this kind of strengthening is not possible for bridge buildings, if the piers can hold only the extra imposed loads but not the increased self weight without any reserves.

3) Increase in cross section of tensioned reinforcement.

It is possible to drill holes in flexural reinforced concrete element, insert extra supports and unify the work of the existing cross section with extra reinforcement bars.

These reinforcement bars have to be covered with concrete. This kind of strengthening technique is also corrosion resistant and safe, if all technically complicated works are done properly. However, this method is not popular due to technological complexity and different technical limitations which can not be considered due to constructive restraints.

There is a more widespread method of attaching metal sheets with epoxy bonds to the tension zone of a concrete cross section. This method of strengthening is similar to the method discussed in this thesis - attaching polymer composites to the tension zone. Corrosion resistance is a concern in this case, however, it can be secured. Technologically this method is harder to realize, yet it is not as complicated as tension rebar attaching to tension zone of cross section.

4) Strengthening with additional composite materials reinforcing.

Composite material addition layers can be bonded on the tension zone similarly to the attaching of metal sheet.

It has to be minded that, despite the easy technology and high corrosion resistance, this method is still relatively expensive. However, recently this kind of reinforcement has been performed for long span bridges in Switzerland, Germany, and USA as well as in a couple of bridges in Latvia.

Multiple methods of reinforcing can be distinguished. The first includes bonding a carbon fiber polymer composite layer or layers to the tension face of the reinforced

concrete cross section. Epoxy resin glues are commonly used to bond carbon fiber reinforced polymer layers to the reinforced concrete surface. Usually carbon fiber polymer layers are realized with constant thickness and constant width strips that are placed parallel on the surface of the beam. Standard width of these strips is usually 50mm and they are placed 5-10 mm apart from one other. Thickness of strips is 1 - 8 mm and they are placed in one or more layers. The other method uses multiple very thin (0,15 – 1 mm) carbon fiber polymer layers attached in whole width of the beam. These layers can be both oriented in one direction carbon fiber polymers as well as textile-plastics. The third option includes carbon fiber polymer angle profiles, which are in turns attached to the corners of the beam.

Possible structural diversity of composite materials allows significantly regulate the mechanical features of these materials according to the requirements of exploitation. The other option in upgrading and improving features of composite materials is the creation of multi-component materials with multiple fibers of different types, i.e. hybrid materials. The contents and structure of hybrid materials is created with an aim of unifying the features of its components in one material. For example, in carbon-organic-epoxy composites the high strength and stiffness features of carbon fibers are unified with viscous features of organic fibers still not losing the high strength and deformative abilities. The main fiber types are glass, high strength carbon, and high modulus carbon and high modulus organic fibers.

Therefore usage of very thin fibers (not only glass) is favorable due to their high strength. It can be marked that similar effect - increase in strength with reducing diameter - is discovered also for metals.

Second half of the last century marked a beginning for widespread experimental research with composites (both carbon fiber polymers and glass fiber polymers) reinforced flexural reinforced concrete elements. Experiments with multiple reinforced concrete structures show lower deflections in case of using carbon fiber reinforced layers. Experiments show that bond between the existing and reinforcement material must be secured in whole length of the construction.

Failure of beams reinforced with composite material additions on the tension zone of the cross section can be divided in two groups:

- Failure due to delamination between reinforcement and beam
- Failure due to both failure of concrete in compression or reinforcement in tension or so called "classical" failure.

The so called "classical" failure happens after exceeding the resistance of any of the materials used. However, the failure due to delamination frequently happens long before reaching the resistance levels, caused by mistakes in inspection, technological faults or other circumstances.

A correctly reinforced with composite material additions one can call a cross section which finds all of the following reinforcing conditions.

- the concrete cover resistance is correctly evaluated, in case of necessity recovery materials are used;
- the technology of preparing concrete surface and bonding the strips to it is appropriate,
- a proper end anchorage of the composite material strip is secured.

With this kind of reinforcing, a classical failure can happen in reinforced concrete element, as a result, yielding of rebar begins, compression concrete or carbon fiber polymer strip fails.

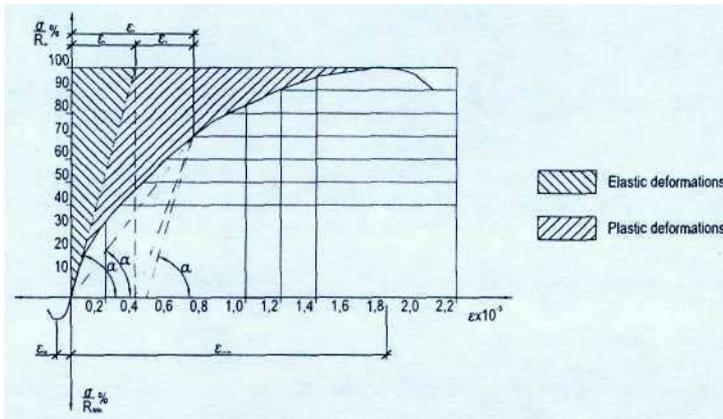
Experiments of deformation and failure differences between reinforced concrete beams with and without additional composite cover plates reinforcement have been

performed widely in last decades. The results of experiments have been compared to the theoretical values. Methods written in building codes and finite element method have been widely used for these theoretical calculations.

Finite element method is used both with linear deforming plates in calculations of stresses in anchoring zone of addition materials where concrete is expected to work in elastic state and with non-linear deforming flat plate finite elements, that take concrete nonlinearity in calculation, and using spatial finite elements. The results obtained clearly show the main features of distribution of stresses in flexural multi component continuous environment.

Theoretical calculations have also been made with lamellar material mechanics, however, these calculations are done only for non cracked concrete and placement of neutral axis is determined with methods obtained from building codes.

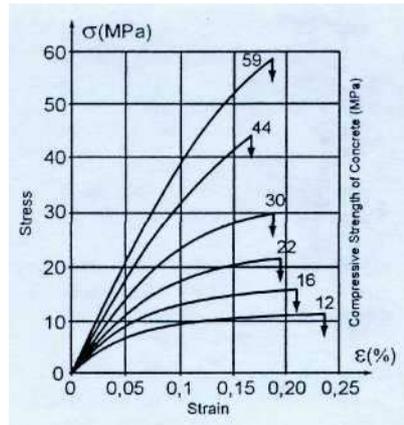
**The second chapter** contains definition of deformative features of concrete and analysis of results obtained by using principles stated in building codes.



1. fig. Relationship between compression stresses and strain in concrete

Concrete is a deformable material with its elastic and plastic features, it features high strength in compression while the tension strength is low. The stress - strain curves obtained in multiple experiments show the character of deforming is mainly dependant on the load imposed to the material, (fig. 1.)

Stress and strain relationship ( $E_c = \sigma/\epsilon$ ) is a changing value and it depends on the stress in the material. Parameter  $E_c$  is frequently called a secant modulus. It must be taken into consideration that concrete class significantly influences the character of nonlinearity; it is displayed in fig. 2.



2. fig. Stress-strain relationship for different concrete classes

Step by step loaded flexural reinforced concrete elements show three typical stages of deformation. The reason for these stages is the characteristic feature of reinforced concrete elements - development of cracks. Given that the tension resistance of concrete is much lower than that of compression, flexural reinforced concrete element deforms nonlinearly. Due to the progressive formation of cracks in tension zone the stresses in tension reinforcement bars increase and redistribution of stresses occurs in the whole cross section.

Flexural reinforced concrete elements work in three principally different stages. The first stage can be described as linear deformation stage. At relatively low loading levels and low bending moments respectively, the deformations in all components of reinforced concrete element are elastic. Relationship between stresses and strain is linear both in compression and tension zone and stress diagram shows triangular shape. When loads increase, the stresses in concrete tension zone reach critical state and the stress diagram in tension zone becomes curved. The compression zone concrete still works in elastic stage and diagram remains triangular. Compression reinforcement bars deform together with concrete and resist some of the compression stresses.

The second stage is concerned with origination and development of cracks in concrete tension zone. The loads increase and crack height increases till the maximal level. In this case, the tension stresses ( $\sigma_s$ ) are found in tension reinforcement bars and compression stresses ( $\sigma_c$ ) are distributed between concrete and reinforcement bars. The stress diagram in compression zone gradually becomes curved; the edge of the cross section is loaded with highest stresses.

The third stage is the failure of the reinforced concrete cross section. The origin of failure can be ultimate stresses ( $\sigma_{sv}$ ) in tension rebar followed by yielding of steel or failure of compression zone concrete. The yielding of reinforcement bars shows as over-deflection, however, the failure of compression concrete can not be observed visually. Therefore failure of concrete should be excluded, because total failure of beam can only be observed as hardly visible cracks in tension zone.

Relationships between stresses and strain in compression for concrete are regulated by Eurocode for concrete constructions (EN 1992-1, Eurocode 2: Design of Concrete

Structures; Part 1: General Rules and Rules for Buildings). Initial modulus of elasticity  $E_b$  values for different concretes is displayed and these equations are to be used:

$$\frac{\sigma_c}{f_{cm}} = \frac{k \cdot \eta + \eta^2}{1 + (k-2) \cdot \eta} \quad (1)$$

where  $\eta = \frac{\epsilon_c}{\epsilon_{c1}}$ , where  $\epsilon_c$  - compressive strain in the concrete,  $\epsilon_{c1}$  - compressive strain

in the concrete at peak stress  $f_c$ , where the deformations still rise without changes in stresses (depending on concrete class, defined in table 3.1 Stress and deformation characteristics for normal concrete, EN1992-1);

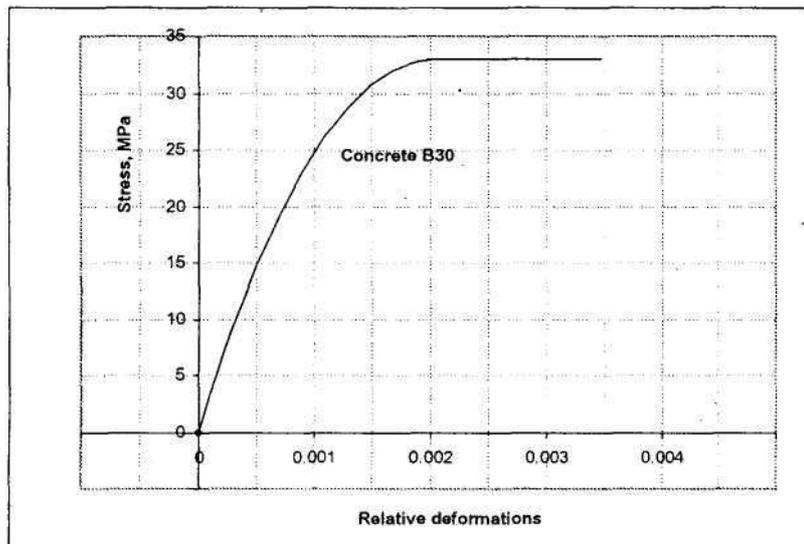
$k = 1,1 \cdot \frac{E_{cm} \cdot \epsilon_{b1}}{f_{cm}}$ , where  $E_{cm}$  - secant modulus of elasticity for normal weight

concrete (usually 27 - 44MPa);

$f_{cm} = f_{ck} + 8$  - mean value of concrete cylinder compressive strength, given in table 3.1. (Stress and deformation characteristics for normal concrete, EN1992-1);

$f_{ck}$  - characteristic compressive cylinder strength of concrete in 28 days;

According to Eurocode, the equation (1) can be used for calculation of deformation ranging from zero to characteristic deformation  $\epsilon_{c2} = 0,002$  accordingly to maximal characteristic value of stresses. The code defines that within limits of 0,002 to 0,0035 deformations continue to rise without rising of stresses. At strain equal to  $\epsilon_c = 0,0035$  the concrete failure occurs.

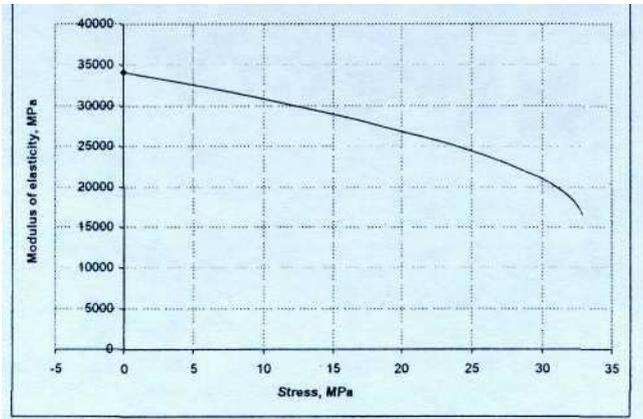


3. fig. Recommended stress-strain diagram for calculations for concrete B30 in compression according to EN 1992-1.

Using equation (1), the deformation parameter  $E_\sigma$ - secant modulus of elasticity can be determined

$$E_\sigma = \frac{2 \cdot f_{cm} \cdot \sigma_c}{\left( -(k \cdot \sigma_c - 2 \cdot \sigma_c - f_{cm} \cdot k) - \sqrt{(k \cdot \sigma_c - 2 \cdot \sigma_c - f_{cm} \cdot k)^2 - 4 \cdot f_{cm} \cdot \sigma_c} \right)} \quad (2)$$

The determined stress-strain equation gives an opportunity to create a diagram showing the relationship between these values for calculations. The relationship between stresses and secant modulus of elasticity is shown in fig. 4.



4. fig. The relationship between compression stresses and secant modulus of elasticity  $E_\sigma$  for calculations according to EN 1992-1

Similar data for deformed features of reinforced concrete constructions are included in latest Russian (year 2004) and USA building codes.

For calculations of reinforced with composites concrete elements Latvian designers have to use methodology recommended by International Concrete Federation -FIB (Federation Internationale du Beton), because Latvian Building codes do not contain methodology for such calculations.

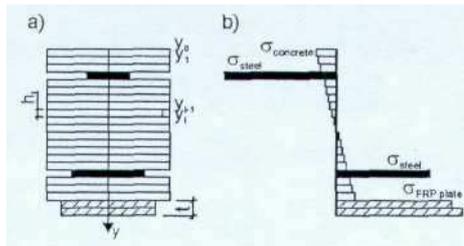
Methodology is based on the limiting relative longitudinal deformations between concrete, reinforcement bars and reinforcing composite material. This method gives an opportunity to calculate using iterations, checking if the longitudinal deformations do not differ too much for each cross section. The iterations provided in these calculations deform the theoretically expected distribution of stresses. The uncertainties in simplified stress distribution calculations are compensated with safety factors.

**In third chapter** a calculation method of reinforced concrete elements, strengthened with additional composites, undergoing bending actions, using classical laminate theory and considering the changing structural and deformed features of element during its loading is developed

Given that many of the building elements used for bending applications are similar to the form of a bar, one can assume that the bending theory and its features for these elements can be reduced to a uniaxial calculation model.

Classical laminate theory practically usable equations can be obtained using multiple hypotheses for deformations of individual layers and the whole bar.

Geometric location of particular layers plays significant role in lamellar structure bar mechanics. To escape from a possibility for the bar to gain unfavorable deformations, for example, twist, the structure of the bars is frequently created symmetric against its central plane. To most efficiently use the potential of determined layers, asymmetric setup is frequently created, i.e. relatively to the central plane layers are placed with different deformative features. This situation can also be originated if the reinforced concrete element is additionally reinforced with bonded composites to the tension plane of the cross section. An important feature of composite reinforced beams is that formation structure is changing during their loading of the element. The origin for this change is cracks formation and development in concrete in tension zone



5. fig. Lamellar structure bar cross section (a) and corresponding bending stresses (b)

Because the crack originated on the tension plane is continuously developing of the tension stresses, into the direction of the compression zone bearing part of cross section is continuously diminishing. As a result of this process, continuous redistribution of stresses occurs both between the components of the bar and in concrete. Due to the load increase the stresses significantly exceed the limit of linear deformations in the greatest part of concrete. Usage of many layers in the lamellar material mechanics bar model the precision of calculation results is magnified.

If equilibrium equations are written for each layer using their physical equations, the diaphragm, mixed and bending stiffness can be determined.

In case of undetermined number of layers  $n$ , the stiffness can be written as follows:

$$A = \sum_{i=1}^n b_i E_i h_i = \sum_{i=1}^n E_i F_i ; \quad (3)$$

$$B = \frac{1}{2} \sum_{i=1}^n E_i F_i (y_i + y_{i-1}) (2y_i - h_i) - y_n A; \quad (4)$$

$$D = \frac{1}{3} \sum_{i=1}^n E_i F_i (3y_i (2y_i - h_i) + h_i^2) - y_n (2B + y_n A) \quad (5)$$

These equations contain conventional signs:  $F_i$  - the cross sectional area of layer  $i$  ;  $h_i$   $b_i$  -  $i$ -the width and depth of layer  $i$ ;  $y_i$  - the maximal coordinate of limiting surface of layer  $i$ ,  $y_n$  - the location of neutral axis of lamellar material set,  $E_i$  - deformative constant of layer  $i$ ,

Using the provision that in case of linear axial loading of the bar, there is no bending, the condition can be obtained that the mixed stiffness equals zero ( $B = 0$ ).

Location of neutral axis of lamellar material bending element can be determined using this condition:

$$y_n = \frac{\sum_{i=1}^n b_i E_i (y_i^2 - y_{i-1}^2)}{2 \cdot \sum_{i=1}^n b_i E_i (y_i - y_{i-1})} \quad (6)$$

The advantage of classical laminate theories calculation is the possibility to define individual deformative features for each layer, thus the nonlinear deformative nature of concrete can be taken into consideration. It is especially important because the distribution of stresses in cross section is changing both because of changing loads and cracking of tension concrete.

If we analyze the stress distribution and deflections of reinforced concrete beams with relatively low level of loads a linear continuous lamellar calculation model is valid, it is frequently used for forecasting of lamellar structure deformative feature determination. Unfortunately, usage of this model for calculations for reinforced concrete elements is restricted with very low limits due to the very early cracking of tension concrete. Cracks originate when the tension stresses rise till the tension resistance of concrete, which, depending on the class of concrete differs from 1/7 to 1/15 of the compression strength.

In combined beam case with bending plane parallel layer structure, created by different materials, the reduced cross section method is frequently used. According to this method, the composite beam cross section is replaced with an equivalent one material cross section. These two cross sections can be called equivalent if the location of neutral axis and the bending resistance are the same. The reduced cross section method usage for reinforced concrete elements strengthened with composites is limited. It can be used only in the elastic deformation stage of all materials, i.e. till the appearance of first cracks. Therefore the calculation model using classical laminate theory was created using the following assumptions — the layers reinforced with steel reinforcement bars are replaced with pure metal layers, concrete located between the bars is divided to both sides of steel layer.

The practical realization of method is carried out using numerical method by creating electronic spreadsheets (Microsoft Excel or Open Office Calc). Necessary results are obtained with step by step iterations repeating the calculations with adjusted features of layers.

Practically the calculation for reinforced concrete beam with lamellar material mechanics model can be carried out in following way:

1. Particular cross section of the beam is replaced by parallel different width layer set. Number and thickness of layers is determined by the user. The stiffness  $E_i F_i$  of the layers depends on the geometry (area)  $F_i$  and deformative constant  $E_i$ .

The layers containing steel reinforcement bars are included in the set of layers and replaced with layers of such width that their area is equal to the total area of reinforcement bars.

2. The algorithm of classical laminate theories calculation is applied to the lamellar structure model of reinforced concrete beam and distribution of stresses in components is obtained.

The obtained result is relevant to the loading situation when none of the tension stresses in the concrete exceed the limits of linear deformation in tension. Results can be displayed in numerical or diagram format.

3. In cases when a stress larger than the limit of linear deformation in tension is reached in any layer, a repeated calculation is performed with adjusted deformation modulus for this layer. This calculation is repeated multiple times, until the required

accuracy is obtained. This calculation is performed for a determined level of bending actions.

4. In cases that in any concrete layer the stress reaches its tension resistance, a crack develops. This situation is modeled with an immediate change in value of deformation module for this layer for the next iteration - the deformation modulus value is equaled to zero (the layer is excluded from the calculation) and a repeated calculation is performed.

5. In cases that stress in compressed concrete reach its resistance, a failure is recognized.

6. In cases that external stresses in reinforcement bars reach their yield strength the reinforced concrete beam is recognized as failed.

7. Stresses exceeding yield strength in reinforcement bars for reinforced concrete elements reinforced with composites loaded with static loads can not be called as a failure phase. If the calculation problem allows yielding of metal, a deformative model permitting yielding for metal can be included in the calculation. Ultimate resistance of beam in this case is reached when either composite material fails in tension or concrete fails in compression.

**Fourth chapter** includes the methodology for determining resistance and compliance of reinforced concrete beams strengthened with composite materials.

When reinforced concrete beams are strengthened, it has to be minded that not always one can remove loads from the element and return it to initial shape. This happens because of formation of cracks in concrete. It means that at the moment of reinforcing the beam has its specific shape and its components - concrete and reinforcement bars are deformed due to particular distribution of stresses. However it is also possible to completely remove loads and prepare the beam for a "second time" usage. It can be assumed that the loads of the beam are practically removed, deformations are removed and stresses in components do not exist. Certainly the cracks in the concrete can not be reckoned to have disappeared. These cracks can be closed but they still exist. Therefore if reinforcement of such beams is carried out the designer needs the information about the state of the beam regarding the depth of the cracks.

The third case of strengthening reinforced concrete beams is to bond extra composite material reinforcement to its external surface (usually on the tension zone).

An equation for determining stiffness for bending element can be determined using classical laminate theories model:

$$D_{pl} = D_k + D_s + D_b, \quad (7)$$

where

$$D_k = E_k F_k h(h - h_k) + \frac{1}{3} h_k^2$$

$$D_s = \frac{1}{3} \sum_{i=1}^{n_s} E_s F_{si} (y_{si}^2 + y_{si} y_{si-1} + y_{si-1}^2)$$

$$D_b = \frac{1}{3} \sum_{i=1}^{n_b} E_{bi} F_{bi} (y_{bi}^2 + y_{bi} y_{bi-1} + y_{bi-1}^2) - \frac{y_n}{2} \sum_{i=1}^{n_b} E_i F_i (y_i + y_{i-1})$$

Distribution of normal stresses in the direction of axis of beam is determined by the equation:

$$\sigma_{y_{iv}} = E_i \frac{(y_{iv} - y_n)}{D_i} M \quad (8)$$

The change in stresses is not smooth on the edges of the layers. The diagram becomes smoother by increasing the number of layers.

The bending moment value at which first cracks originate can be determined with equation:

$$M_{pl} = \frac{d_i}{h - h_c - y_n} f_b^* \quad (9)$$

This equation gives an opportunity to determine the influence of number of layers to the origination of cracks.

Analysis show that for combined and reinforced concrete beams the origination of cracks limit relation changes unsubstancially at low number of composite layers.

The nature of change in stresses in flexural beam components during the loading is shown in fig. 9.

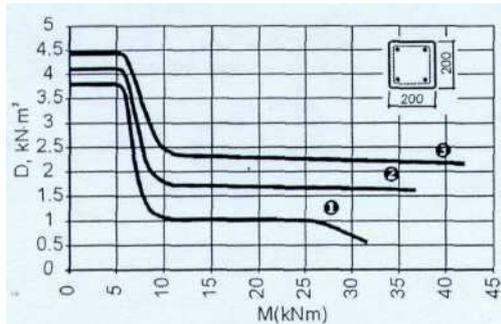
The displayed calculation methodology based on lamellar material mechanics was realized with calculation program and it is used to calculate two type reinforced concrete beams reinforced with carbon fiber polymer layers. The mechanical features of components are displayed in table 1.

Table 1

Mechanical features of components

Component feature	First type beams	Second type beams
$E_b, GPa$	25	25
$R_b^+, MPa$	2,6	1,9
$R_b^-, MPa$	33	30
$E_m, GPa$	200	200
$R_{mT}, MPa$	540	340
$E_k, GPa$	267	400
$R_k^+, MPa$	2900	3000
Thickness of layer	1,3	0,17

The first type, cross section is built up in rectangular form with sizes 200x200mm with four metal reinforcement bars 0 14 mm. For strengthening of these beams 50mm wide and 1,3mm thick carbon fiber polymer strips, bonded to the concrete surface with epoxy resin glue in three layers, are used. The bending stiffness of first type beams is displayed in diagram in fig. 6.

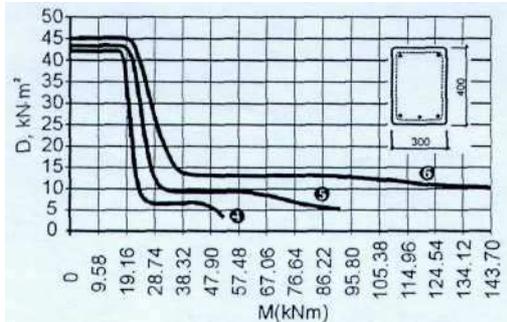


6. fig. First type beam bending stiffness relation with bending moment

Curve 1 refers to first type beams without carbon fiber polymer strengthening. Curve 2 refers to change in stiffness for reinforced concrete beams strengthened with three

strips of carbon fiber polymer, width 50mm, thickness  $h_k = 1,3$  mm, but curve 3 refers to case when the thickness of carbon fiber polymer strip is three times higher,  $h_k = 2,6$  mm.

The second type cross section is built up in rectangular form with sizes 400x300mm with five metal reinforcement bars 0 13 mm, two of which in compression zone, three in tension zone. For strengthening of these beams 1,3mm thick carbon fiber polymer strips in whole width of the beam are used. The bending stiffness of the second type beams is displayed in diagram in picture 7.



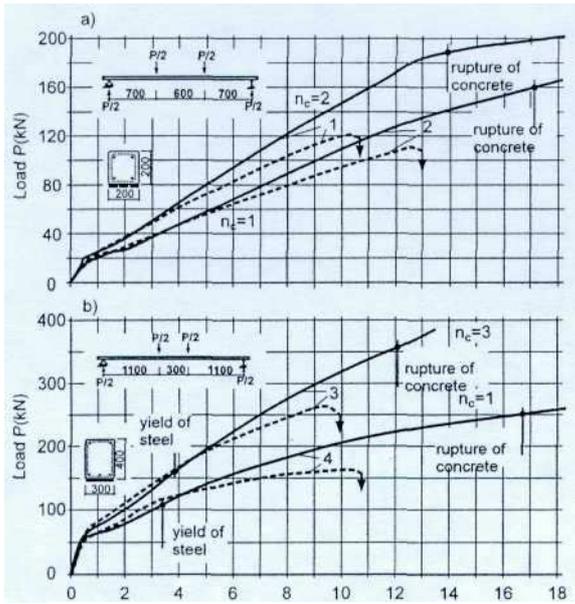
7. fig. Second type beam bending stiffness relation with bending moment

The changes in bending stiffness of second type beams are displayed in curves 4 -6 in picture 7, the width of carbon fiber polymer strengthening is equal to the width of the beam, and thickness of one layer equals 0,17mm. Curve 5 refers to a beam, reinforced with one layer of carbon fiber polymer, but curve 6 refers to three times thicker carbon fiber polymer reinforcement layer.

The obtained results show a direct influence of carbon fiber polymer strengthening on bending stiffness of the beam and consequently its deformability. Curves show three typical loading stages. The first refers to linear deformation stage (practically constant bending stiffness). This stage refers to the loading stage until the appearance of first cracks in the tension concrete. This stage is relatively small and in realistic loading stages is insignificant. The second stage marks appearance and development of cracks, redistribution of stresses between components of cross section. Bending stress changes significantly in this stage. These stiffness changes have to be considered in realistic calculations, because the level of loading is similar to realistic level in practice. The third stage shown increase in stresses without significant redistribution between layers. The end of this stage is failure of any on the components. Failure can occur either by concrete failure in compression, yielding of reinforcement bars or tension failure of carbon fiber polymer layer. In case of yielding of steel reinforcement bars a fourth stage occurs with redistributed tension stresses to the carbon fiber polymer layer and significant increase in deformations. When yielding of reinforcement bars occurs, delamination of carbon fiber polymer strips is often observed and leads to the failure of whole beam.

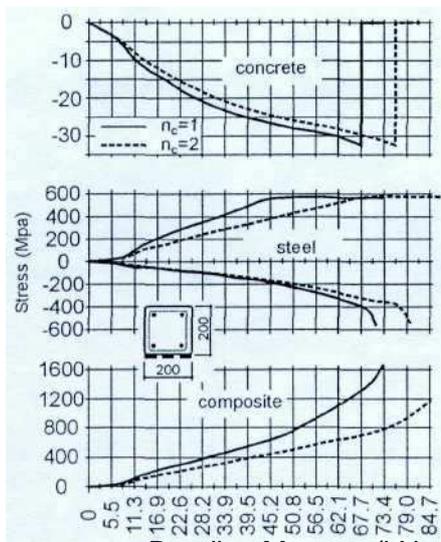
Comparison between literature described reinforced concrete beam deflection curves and experimental deflection curves was performed to evaluate the precision of described methodology. The structure, dimensions and loading scheme is displayed in fig. 8. Maximal deflection curves for four point bending were determined using the above method.

The results obtained are displayed in fig. 8. — diagram with continuous line. These results are compared with experimental results, displayed with dot line. Curves show explicit nonlinear shape and they show typical loading stages.



Mid-span deflection (mm)

8. fig. Experimental and forecasted beam deflections depending on loads



Bending Moment (kNm)

9. fig. Changes in stresses in beam components while loading, continuous line - with 1 composite coverplate; dashed line - with 2 composite coverplates.

Loading the beam statically till the failure of the beam, experimentally maximum beam deflections were measured in the middle of the span. The shape of the curve shows deflection line consists of several different deforming typical stages. The shape of the curve significantly changes at deflection value of 2mm, which equals 0,1% of the span. It can be assumed that at this loading stage all the tension zone of the beam is already cracked. In loading range till 10mm or 0,50% of span the curve is nearly linear. At this stage the cracking has practically ended and stresses are rising in compressed concrete and tensioned reinforcement. At bending moment 72kNm infinite increase in deflections were observed, caused by steel bar yielding.

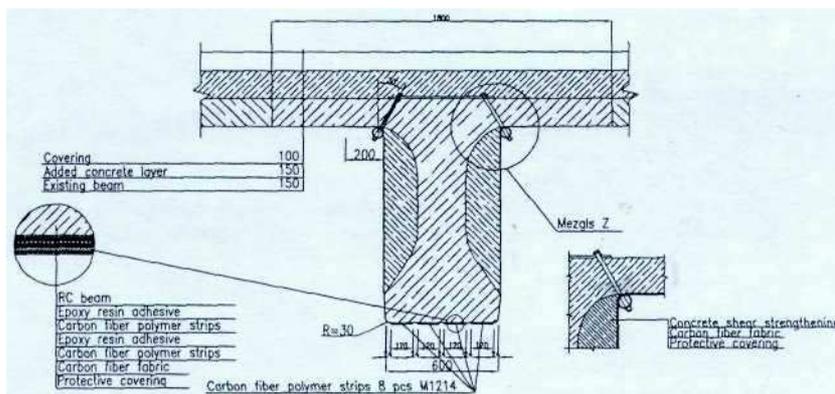
Numerically obtained curve in representative points both according to values and to shape, approximates the experimental results. It allows to state that proposed composite beam deflection calculation method can be successfully used to calculate different cross section elements and in cases of oriented components. The changes in stresses in pure bending are displayed in fig. 9 diagrams.

**Fifth chapter** contains description of classical laminate theories calculation model application to reconstruction of reinforced concrete bridge.

Twenty years exploited three span reinforced concrete bridge of Jurmala ring road crossing Varkalu channel, stretching from Lielupe river and Babites lake in Riga district, Latvia, beam resistance is only 67%-75% of the requirements of Eurocode LVS ENV 1991:2000. Total length of the bridge is 68 meters; three spans of 18 + 24 + 18 meters form this length. Calculations show that there has been a very economic calculation according to codes in force in 80-ies. Resistance reserve for loads does not exceed 5%. However, the piers are designed with significant load bearing reserve, which allows resisting also reinforced building constructions with increased loads.

The decision was made to perform strengthening with bonding carbon fiber polymer strips to the tension plane of the bridge beams. The strengthening is illustrated in fig. 10.

Beam's L=24m cross sections strengthening at the pylon



10. fig. Cross section of reinforced bridge beam of Varkalu channel above support

To secure the load bearing capacity of beam flanges, an extra 150mm layer of concrete has to be added in the compression zone.

The currently exploited beam cross section can withstand bending moment  $M_{\text{exist}}=4,28$  MNm . Because of required load bearing capacity means the bending moment is  $M_{\text{reg}}=7,00$  MNm, the beams are by 63% overloaded. Thus an increase in bending moment bearing capacity of  $M_{\text{reinf}} = M_{\text{req}} - M_{\text{exist}} = 7,00 - 4,28 = 2,72$  MNm must be secured.

Carbon fiber polymer strips M1214 distributed by Sika Latvija Ltd are used in this reconstruction.

Using the lamellar material mechanics calculation model a required amount of additional reinforcement was determined. Eight carbon fiber polymer strips,  $h= 1,4$ mm, width 120mm, have to be bonded in two layers in four parallel rows 5cm distant from one other.

## CONCLUSIONS

Based on classical laminate theory, a united methodology has been created for calculating stress-strain relationships in combined structure flexural reinforced concrete elements and a numerical calculation model for such analysis is created. Methodology allows to forecast the stiffness and load bearing capacity for strengthened reinforced concrete beams depending from cross section and materials mechanical characteristics. Applying the created methodology it is possible to watch the cracks rising and developing process and beams stiffness changes in all the beams loading process.

The methodology let to appraise different mechanical properties of concrete, steel reinforcement and composite strengthening materials. Based on results, which are obtained using developed methodology, there are verified solutions for strengthening of existing reinforced concrete beams with composite cover plates.

### In doctoral thesis:

- a new calculation method for computing reinforced concrete building constructions with external reinforcement has been created using basic principles of classical laminate theory for package of layers with significantly differing physically mechanical features, which gives possibility to calculate most efficient amount of cover plates for strengthened beams,
- using the layered structure of building elements, such calculating model is developed, which changes during loading process and let to take into account nonlinearity of concrete deformative features and development of cracks;
- the practical realization of method is carried out using numerical method by creating electronic spreadsheets, the results obtained theoretically are compared to experimental values for reinforced concrete beams with and without external reinforcement;
- numerically obtained curves in representative points both according to values and to shape, approximates the experimental results, it allows to state that proposed composite beam deflection calculation method can be successfully used to calculate different cross section elements and in cases of oriented components,
- the methodology for calculating strengthening of existing beams developed in this thesis is used to perform an additional reinforcement of twenty years exploited three span reinforced concrete bridge of Jurmala ring road crossing Varkaju channel, stretching from Lielupe river and Babītes lake in Rīga district, Latvia.

The methodology allows predicting redistribution of stresses in combined beam components during the development of cracks and deformations.

It is possible to determine the most rational amount of additional reinforcement for a particular size concrete beam with particular steel reinforcement bars. Strength margin can be determined in particular level of loading; failure of composite element can be predicted.

#### LIST OF PUBLICATIONS

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