RIGA TECHNICAL UNIVERSITY

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DECISION-MAKING UNDER PROBABILISTIC UNCERTAINTY AND FUZZINESS

Ph.D. Thesis Summary

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RIGA TECHNICAL UNIVERSITY
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PH.D. THESIS
IS NOMINATED IN RIGA TECHNICAL UNIVERSITY FOR TAKING A DOCTOR'S DEGREE IN ENGINEERING SCIENCE

Defence of the Ph.D. Thesis for obtaining a doctor's degree in engineering science will take place on November 13, 2006 at Riga Technical University, Department of Computer Science and Information Technology, Meža street 1/3, room 512.

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DECLARATION

I, Aleksandrs Vališevskis, declare that I have written this Thesis, which is submitted for reviewing in Riga Technical University for taking a doctor's degree in engineering science.

Aleksandrs Vališevskis ____________________ (signature)

Date: September 12, 2006

Ph.D. Thesis is written in Latvian, it contains introduction, 4 chapters, conclusion, a list of references, 1 appendix, 45 figures, 18 tables, 55 formulae. There are 156 pages in total. There are 54 references.

GENERAL DESCRIPTION OF THE THESIS

Topicality of problem

Traditionally decision analysis problems consider only one source of uncertainty. However, in the real world events usually are both fuzzy and nondeterministic. More precisely, events are non-deterministic, as they can be defined with the help of some subjective probability and they are fuzzy, as the event itself can be fuzzy. In the classical probability theory it is adopted that an event has either completely occurred or has not occurred. In the real world an event can occur only partially and the event itself can be fuzzy. One of the newest and fastest growing branches in fuzzy logic research is perception-based decision making that considers this sort of problem under several sources of uncertainty. Fuzzy granule is one of tools that are employed in perception-based decision making. The method that is developed in the Ph.D. Thesis allows one to use both fuzzy and nondeterministic values in the description of problem domain.

Effective decision making method is able to give a positive stimulus for companies that operate in national economy, as the method enables them to make better and more informative decisions that take into account diverse uncertainty of the real world.

Goals of research

Aim of the Ph.D. Thesis is to develop a decision making method that is intended for use in environment with two sources of uncertainty (randomness and fuzziness). We consider the case when the number of alternatives and criteria is finite. The method consists of several steps. More specifically, the following subtasks should be solved in order to develop the method:

1. It is necessary to study existing decision making methods that are intended for use in environment with one or several sources of uncertainty.
2. It is necessary to develop an adaptive network for fuzzy granule processing. It is necessary to develop a training algorithm for this adaptive network that can be used to determine importance of individual parameters of alternatives.
3. Shannon's entropy is used to determine informativeness of alternatives. However, interval probabilities are used in the method. Hence, Shannon's entropy should be generalized to the interval case and it should be proven that it is additive.
4. It is necessary to develop a method for producing complete and partial ranking of alternatives.
5. Individual tools that are developed in the Thesis should be joined in a decision making method that is intended for use in environments that are both nondeterministic and fuzzy.
6. It is necessary to develop software in order to carry out experiments.
7. The practical usability of the method developed should be proven with the help of a decision making task that is close to the real world conditions.

Object of the research

Object of the research is decision making method, which is intended for use in environment with several sources of uncertainty, i.e., in an environment, which is both nondeterministic and fuzzy. This method is intended for use when the number of criteria and alternatives is finite.

Research hypotheses

During the research the following hypotheses are made.

1. The considered decision making environment can be sufficiently precisely described with the help of fuzzy granules, i.e, using probability theory and fuzzy set theory tools.
2. During the evaluation of alternatives' informativeness it is assumed that the created systems are independent.

Methods of the research

Fuzzy set theory, possibility theory, probability theory, calculus, information theory, optimization theory is used in the Ph.D. Thesis. Moreover, fuzzy granules, Shannon entropy, Lagrange multiplier method, gradient descent method, fuzzy inference methods, decision models sensitivity analysis is used.

Scientific novelty

1. The main novelty of the Ph.D. Thesis is decision making method that is intended for use in environment with two sources of uncertainty, i.e., in an environment that is both fuzzy and nondeterministic. From the existing methods this methods differs because it enables one to include fuzzy and nondeterministic information into the analysis, as well as because it needs comparatively small amount of computational resources and it is practically usable in the case when the number of parameters is high.
2. Other novelties are related to the problems that were encountered and solved during research. More specifically, during the development of a method for evaluating informativeness of alternatives Shannon's entropy was generalized to the case of interval probabilities, as interval probabilities are used in the developed method. It is proven that the generalized entropy is additive.
3. Moreover, an adaptive network ANGIE and its training algorithm have been developed in order to process fuzzy granules. The training algorithm is based on gradient descent method. Adaptive network ANGIE is used to analyse fuzzy granules. Multi-parameter sensitivity analysis task is solved using adaptive network ANGIE training algorithm.

Practical use of the Thesis and approbation

A decision making method that is intended for use in environments with two uncertainty sources is developed in the Ph.D. Thesis. In contrast to other methods it is not necessary to construct criteria value matrix. Moreover, the method developed can be used in cases when the environment is so uncertain that such a matrix with precise values cannot be completed. From the other side, the method developed requires less computational power than other existing methods, which allow one to use several types of uncertainty. During research several practical decision making problems under uncertainty have been considered, which show how the method developed can be used in practice. Moreover, a programme has been developed in MS Visual C++ environment that enables one to make all the calculations that are necessary during the use of the method developed.

Results of the Ph.D. Thesis have been presented at the following scientific conferences:

1. International Conference on Operational Research "Simulation and Optimisation in Business and Industry", SOBI 2006, Tallinn, Estonia, May 17-20, 2006.
2. The Tenth Estonian Winter School in Computer Science, Palmse, Estonia, February 27 – March 4, 2005.
3. International Conference “Managing Uncertainty in Decision Support Models” (MUDSM 2004), Coimbra, Portugal, September 22-24, 2004.
4. International Conference on Fuzzy Sets and Soft Computing in Economics and Finance, FSSCEF 2004, St. Petersburg, Russia, June 17-20, 2004.
5. The Ninth Estonian Winter School in Computer Science, Palmse, Estonia, February 29 – March 5, 2004.
6. International Conference on Soft Computing and Computing with Words in System Analysis, Decision and Control, ICSCCW’2003, Antalya, Turkey, September 9-11, 2003.
7. The Second Estonian Summer School on Computer and Systems Science, Taagepera, Estonia, August 10 – 14, 2003.

8. International conference "Environment. Technology. Resources.", Rezekne, Latvia, June 26-28, 2003.
9. Seventh International Conference on Cognitive and Neural Systems, ICCNS'2003, Boston, USA, May 28 – 31, 2003.
10. International Conference "Modelling and Simulation of Business Systems", MoSiBuS'2003, Vilnius, Lithuania, May 13-14, 2003.
11. The Eighth Estonian Winter School in Computer Science, Palmse, Estonia, March 2–7, 2003.
12. International Conference on Computational Intelligence for Modelling, Control and Automation CIMCA'2003, Vienna, Austria, February 12-14, 2003.
13. International conference "Information Society and Modern Business", Ventspils, Latvia, January 31 - February 1, 2003.
14. Fifth International Conference on Application of Fuzzy Systems and Soft Computing ICAFS-2002, Milan, Italy, September 17-19, 2003.
15. International scientific conference Traditions and Innovations in Sustainable Development of Society, Rezekne, Latvia, February 28-March 2, 2002.

Main results of the Ph.D. Thesis

A decision making method is developed in the Ph.D. Thesis that is intended to be used in an environment with two sources of uncertainty, i.e., in an environment that is both fuzzy and nondeterministic. The method is based on fuzzy granules, which enable one to use fuzzy values and probabilities during description of alternatives. During research additional problems have emerged and have been solved in the Ph.D. Thesis. For example, when a method for evaluating informativeness of alternatives was developed, Shannon's entropy had been generalized to the case of interval probabilities and it was proven that it is additive, because in the developed method interval probabilities are used. Moreover, adaptive network ANGIE (*Adaptive Network for Granular Information and Evidence Processing*) has been developed in order to process fuzzy granules. Network ANGIE training algorithm is used to solve multi-parameter sensitivity analysis task. Practical examples of the developed method usage are considered in the Ph.D. Thesis, which show how the method developed can be used in practice. Below you can find a more detailed description of main results of the Ph.D. Thesis:

1. Decision making methods that are intended to be used in uncertain environments are analysed in the Ph.D. Thesis and it was concluded that the existing methods either do not take into account more than one type of uncertainty, or they are too complicated computationally, or their aggregation operators unjustifiably reduce the available information.
2. It is shown that fuzzy granules can be used in order to combine fuzziness and randomness. More specifically, fuzzy evidences can be considered as probability distribution of fuzzy values. Hence, the decision model enables one to include two different types of uncertainty into the description of the problem domain. In a case if one of these uncertainties is not related to the given parameter, it is possible to assign it a precise and/or deterministic value.
3. Adaptive network ANGIE is developed, which can be used to process fuzzy evidences. Calculations in this network are parallel, which enables one to increase calculation speed if the programme is run in a system that supports parallel calculations. Adaptive network ANGIE training algorithm can be used in decision model sensitivity analysis.
4. Shannon's entropy is generalized to the case of interval probabilities. Moreover, it is proven that the generalized entropy is additive.
5. A method for complete or partial ranging of alternatives is developed. Like in PROMETHEE method, it is based on negative and positive flows.

6. Decision making method has been developed that includes all the mentioned tools, which are necessary for the analysis of alternatives and for the decision making. The method developed is intended for use in environment that is both nondeterministic and fuzzy.
7. The method developed is tested on two practical examples that show how the method developed can be used in practice.

Publications

Results of the research have been published in 13 scientific papers. List of publications can be found in the list of references for the Ph.D. Thesis that can be found in the end of this summary.

Structure and contents of the thesis

Ph.D. Thesis consists of introduction, 4 chapters, conclusion, list of references and 1 appendix. Ph.D. Thesis occupies 153 pages, it includes 43 figures and 13 tables. There are 49 sources in the list of references.

Structure of the Ph.D. Thesis is the following:

INTRODUCTION – General description of the decision making domain.

CHAPTER 1. DECISION MAKING IN NONDETERMINISTIC AND FUZZY ENVIRONMENT – Description of tools that can be used to model randomness and fuzziness in decision making tasks.

CHAPTER 2. DECISION MAKING METHODS FOR UNCERTAIN ENVIRONMENT – Description of decision making methods and additional tools that are used therein.

CHAPTER 3. DEVELOPMENT OF GRANULAR-INFORMATION-BASED DECISION MAKING METHOD – Description of the decision making method developed by the author of the Ph.D. Thesis, as well as description of other Ph.D. Thesis results.

CHAPTER 4. SOLUTION OF PRACTICAL PROBLEMS – Several practical problems are analysed in this chapter that show how the method developed can be used in practice.

PH.D. THESIS RESULTS

PH.D. THESIS NOVELTY

APPENDIX – Source of the developed software can be found in the appendix.

SUMMARY OF THESIS CHAPTERS

First chapter

In the first chapter general problem statement of the decision making task is given. Moreover, tools that are used to model randomness and fuzziness in the decision making tasks are described. Aim of the Ph.D. Thesis is defined in this chapter.

Decision making tasks emerges when you can obtain the needed result in different ways. General problem statement of a decision making task can be defined as follows. Let us assume that a set of alternatives X and a set of outcomes Y are given. Each alternative $x_i \in X$ is related to a corresponding outcome $y_i \in Y$. Moreover, there is a selection quality assessment mechanism. Usually quality of the outcome is assessed. It is necessary to select an alternative, which has the best quality assessment. Quality notion has a wide meaning in this case and it is assessed in the framework determined by the available information and preferences of the decision maker [54].

In the first chapter of the Ph.D. Thesis classification of decision-making methods is given based on these features (see Figure 1):

1. Attribute value type. It can be precise, fuzzy, non-deterministic and both fuzzy and non-deterministic.
2. Number of criteria. Alternatives can be evaluated by one of more criteria.
3. Number of alternatives. Set of alternatives can be finite or infinite.
4. Type of weights. Weights can be precise of fuzzy.
5. Number of steps. The decision has to be made one or more times.

The decision making method developed in the Ph.D. Thesis in Figure 1 is marked with bold text.

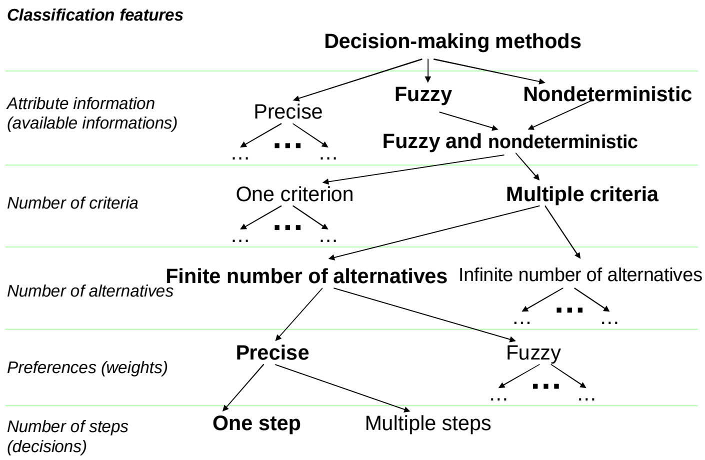

Figure 1. Classification of decision-making methods

Subject of the research is a decision making method that is intended for use in a environment with several sources of uncertainty, more specifically, in nondeterministic and fuzzy environments. A multi-criteria decision making method is developed, which should be used when the number of alternatives is finite.

Decision making environment has sources of both fuzzy and probabilistic uncertainty. In order to model the decision problem, one should use fuzzy and probabilistic values to describe alternatives. Union of these uncertainties is accomplished by using fuzzy granules in order to describe alternatives, as fuzzy granules can be considered as probability distribution of fuzzy values.

This decision making environment is typical for tasks when there is very little information available about the alternatives. For example, if future values are used in the description, which are not know precisely at the moment (e.g., evaluation of a project that has not been started yet, expert information), or if the actual information is not know precisely or only approximate expert information is available. For example, during the development of a selling strategy actual selling data are not available, as first of all they should be collected and it is obtained with a delay.

Fuzzy granules are generalization of Dempster-Shafer theory of evidence to the fuzzy case [45]. In the framework of Dempster-Shafer theory random sets are considered and you can model lack of knowledge. This is main point in which Dempster-Shafer theory differs from probability theory. Main feature of the fuzzy evidence theory is that is combines fuzzy logic and probability theory. Evidences in this theory can be considered as probability distribution of fuzzy values.

Fuzzy evidences contain information granules, each granule is given as a IF-THEN rule. Left side of the rules is given as a probability distribution and the rules are considered to be distribution of conditional possibility. Hence, fuzzy evidences can be considered to be probabilistic distribution of fuzzy values.

Moreover, the first chapter defines aims of the Ph.D. Thesis, which can be found in chapter "General description of the thesis".

Second chapter

The second chapter describes decision making methods and additional tools that are used therein. We tried to consider as many different types of decision making methods as possible.

Generalization with the results of comparative analysis of different methods can be found in Table 1, it shows name of the method, its features and drawbacks that were found in that method. When drawback were analysed the usability of a method in environment with fuzzy and non-deterministic uncertainty has been taken into account.

Table 1

Decision making methods that have been analysed in the Ph.D. Thesis and their drawbacks

Method	Features	Drawbacks
PROMETHEE, ELECTRE	Alternative ranking is built based on outranking relation	Input data should be precise
AHP	Hierarchy of criteria is built and criteria of each level are compared	A big number of binary comparisons
Decision analysis based on utility theory	Based on decision trees and utility theory	If the number of criteria is big, determination of utility functions and weights is laborious
Stochastic-dominance-based method for mixed evaluation aggregation	Precise, nondeterministic and fuzzy information can be used	All the values, including fuzzy values, are converted to probability density functions
PRUF – fuzzy-logic-based method for natural language description	Is based on a generalized relation, which can be used to describe different types of uncertainty	Computationally is a complicated method, when number of input data increases it becomes practically unusable

Moreover, additional tools are described in the second chapter, which are used in decision making methods: morphological analysis and sensitivity analysis.

Morphological analysis is used to create set of alternatives. It is a tool that can be used to generate new alternatives. Idea of the morphological analysis is the following: it is necessary to create a list of attributes and to define values of these attributes. In order to generate a new alternative it is necessary to choose a value from each column. The obtained set of parameters will describe one alternative.

Sensitivity analysis of decision models is also called robustness analysis, as it can be used to determine how stable is decision model and what is the influence of changes in model attribute values [4]. During sensitivity analysis decision model parameters are changed, for example, probability values, criteria values, criteria weight values and it is analysed how these changes influence ranking of alternatives. This analysis can be used to determine whether the decision that is to be made is stable, i.e., minor changes do not change the ranking. It is especially important when a decision under uncertainty is made.

Two types of sensitivity analysis can be used in practice: one-way and two-way. In one-way sensitivity analysis only one parameter value is changed at a time, but in two-way sensitivity analysis two values are changed at a time. Results of these analyses can be represented using the so-called Tornado diagrams and two-dimensional drawings.

Third Chapter

The third chapter considers the decision making method developed by the Ph.D. Thesis author, as well as it describes other results obtained during development of the mentioned method.

In the beginning adaptive network ANGIE (*Adaptive Network for Granular Information and Evidence Processing*) is described, which is used for fuzzy granules processing [36]. One of the advantages of the network is that calculations are parallel, thus it is possible to accelerate computation. It is obvious that increase in the speed will happen only if the network will be implemented on a hardware that supports parallel computation.

Since the network developed is adaptive, it is possible to teach it or, in other words, to automate adjustment of its parameters [37]. Thus, the learning algorithm for adaptive network ANGIE is developed. Similar to neural network learning algorithms, adaptive network ANGIE learning algorithm is based on gradient descent method.

Due to the specifics of the calculations performed within the network it is possible to adjust only probability values. Since the algorithm adjusts only probability values and does not take into account consistency, it is necessary to normalize probability values. Three possibilities are available: 1) normalize values after each iteration; 2) normalize after each *n*-th iteration and 3) normalize after the system has been trained. Analysis results shows that in this case most appropriate is the third possibility [33].

After training system ANGIE it is possible to determine which parameter and to what extent influences that a criterion will be equal to the desired value. As is shown below this task is related to decision model sensitivity analysis task.

Adaptive network that corresponds to the following system is shown on Figure 2.

1. IF (X_1 is B_{11}) with p_{11} THEN (Y is G_{11})
   IF (X_1 is B_{12}) with p_{12} THEN (Y is G_{12})
2. IF (X_2 is B_{21}) with p_{21} THEN (Y is G_{21})
   IF (X_2 is B_{22}) with p_{22} THEN (Y is G_{22})

Explanation of the meaning of the network processing elements follows. Oval elements are meant for crisp information processing and diamond elements are meant for fuzzy information processing.

Layer 1.

The output of element G_{ij} is a fuzzy value which corresponds to the right-hand side of the *j*-th granule in the *i*-th evidence.

The output of element p_{ij} is the value of the probability associated with the left-hand side of the *j*-th granule in the *i*-th evidence.

The output of element Q is the desired value of the criterion.

Layer 2.

The element denoted *sup* is used for determining the supremum of the intersection of membership functions fed to the element. Fuzzy values received through dendrites are aggregated by the function of intersection. The transfer function of this element is the supremum function. Thus, the output of this element is the supremum of the intersection of the membership functions received through dendrites.

Layer 3.

The output of element Π is the product of its input values.

Layer 4.

The output of the element Σ is the sum of its input values.

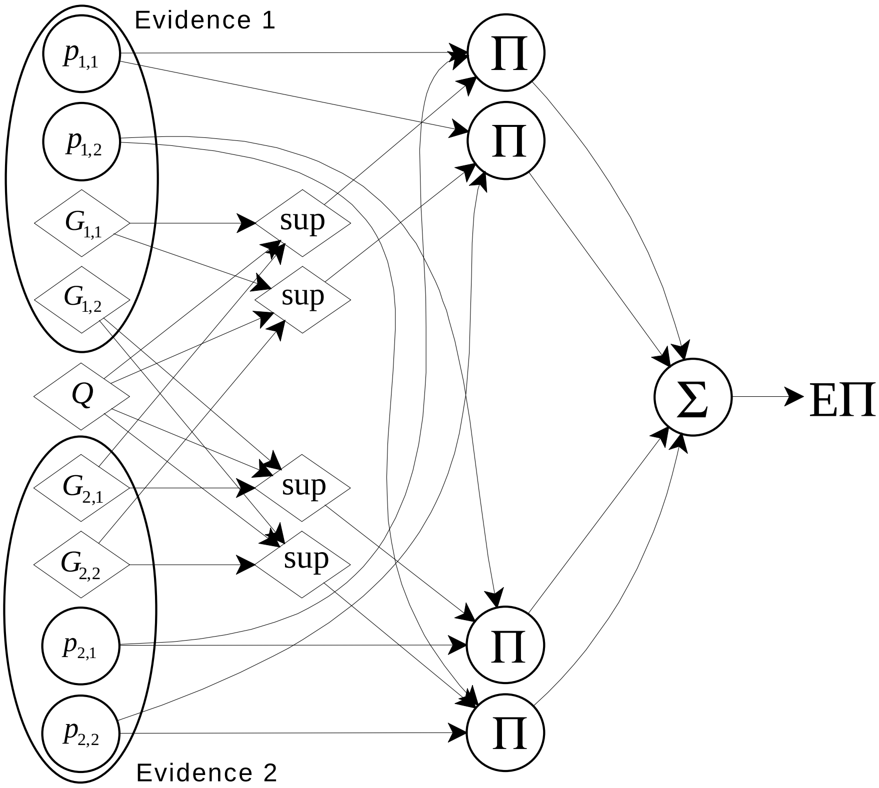

Figure 2. Adaptive network ANGIE

The gradient descent method is used to tune the parameters of ANGIE. That means that in order to tune the parameters we have to determine the partial derivatives of the error function with respect to the parameters we want to tune.

In the proposed method interval entropy is used, i.e. it is calculated based on interval probabilities. Thus, Shannon's entropy should be generalized to the interval case. In the Ph.D. Thesis generalization problem is described as an optimization problem with inequality constraints. This problem is solved analytically using method of Lagrange multipliers [31].

Let us assume that a system can be in *n* states, and the probability that the system is in the *i*-th state is interval and is equal to $[p_i^{min}, p_i^{max}]$. In case when probabilities are single-valued rather than interval-valued, it is required that probabilities sum to 1, i.e.

$$\sum_i p_i = 1.$$

If probabilities are interval-valued, then this can be rewritten as:

$$\sum_i p_i^{\min} \le 1 \le \sum_i p_i^{\max}.$$

The entropy is interval as well: $H = [H^{\min}, H^{\max}]$. General problem statement for the calculation of the interval-valued entropy can be stated in the following way (1).

Let $[p_1^{min}, p_1^{max}]$, $[p_2^{min}, p_2^{max}]$, … , $[p_n^{min}, p_n^{max}]$ be interval-valued probabilities, then lower and upper boundary of entropy can be calculated as follows:

$$H^{\min} : -\sum_{i=1}^{n} p_i \ln p_i \to \min \quad \text{and} \quad H^{\max} : -\sum_{i=1}^{n} p_i \ln p_i \to \max, \quad \text{subject to} \tag{1}$$

$$p_i^{\min} \le p_i \le p_i^{\max}, \qquad i = \overline{1,n}; \qquad \sum_{i=1}^{n} p_i = 1$$

We use method of Lagrange multipliers in order to solve program (1). It is necessary to convert all inequality constraints $\geq$ into $\leq$ by multiplying by -1. Moreover, it is necessary to convert a minimization problem into maximization problem. First let us consider solution of the maximization problem (2).

$$
\boxed{\begin{aligned}
&H^{\max}: -\sum_{i=1}^{n} p_i \ln p_i \to \max \\
&\text{subject to} \\
&\qquad -p_i \leq -p_i^{\min}, \quad p_i \leq p_i^{\max}, \qquad i=\overline{1,n}. \\
&\qquad \sum_{i=1}^{n} p_i = 1
\end{aligned}} \tag{2}
$$

The Lagrangian for program (2) is the following:

$$
\begin{aligned}
&L(p_1,\dots,p_n,\lambda,\mu_{11},\mu_{12},\mu_{21},\mu_{22},\dots,\mu_{n1},\mu_{n2},) = \\
&\qquad = -\sum_{i=1}^{n} p_i \ln p_i + \lambda\left(\sum_{i=1}^{n} p_i - 1\right) + \sum_{i=1}^{n}\left(\mu_{i1}\left(p_i - p_i^{\min}\right) + \mu_{i2}\left(p_i^{\max} - p_i\right)\right)
\end{aligned} \tag{3}
$$

Partial derivatives of the Lagrangian (3) are:

$$
\frac{\partial L}{\partial p_i} = -\ln p_i - 1 + \lambda + \mu_{i1} - \mu_{i2}, \qquad i=\overline{1,n}.
$$

Thus, it is necessary to solve the following problem:

$$
-\ln p_i - 1 + \lambda + \mu_{i1} - \mu_{i2} = 0, \qquad i=\overline{1,n}, \tag{4}
$$

$$
\sum_{i=1}^{n} p_i = 1, \tag{5}
$$

$$
\mu_{i1}\left(p_i - p_i^{\min}\right) = 0, \qquad \mu_{i2}\left(p_i^{\max} - p_i\right) = 0, \qquad i=\overline{1,n}, \tag{6}
$$

$$
p_i^{\min} \leq p_i \leq p_i^{\max}, \ \mu_{ij} \geq 0, \qquad i=\overline{1,n}, \ j=1,2. \tag{7}
$$

Now it is necessary to consider 2^{2n} cases depending on values of μ_{ij} in complementary slackness conditions (8), namely, depending whether a particular μ_{ij} is or is not equal to zero.

1) If $\mu_{ij} = 0, \ i=\overline{1,n}, \ j=1,2$ and each interval probability contains value $\frac{1}{n}$, then the maximum entropy is reached at this point. Otherwise proceed to the next step.

2) Check each configuration in which a number of μ_{ij}'s are not equal to zero for some *i* and *j*. It is useful to note that if μ_{i1} or μ_{i2} is not equal to zero, then $\left(p_i - p_i^{\min}\right)$ or $\left(p_i^{\max} - p_i\right)$ must be equal to zero according to (6). Thus, if μ_{i1} or μ_{i2} is not equal to zero, then, accordingly, $p_i = p_i^{\min}$ or $p_i = p_i^{\max}$.

Hence, first for all positive μ_{i1} or μ_{i2} determine values of probabilities. Values of all other probabilities must be equal, which follows from (4) and from the fact that $\mu_{j1} = 0$ and $\mu_{j2} = 0$ for all p_j that cannot be determined. Values of p_j can be determined using (5) and the fact that all unknown probabilities are equal. At this point values of all probabilities for a particular configuration have been determined.

Now it is necessary to check whether probabilities sum to *1* and whether all μ_{ij}'s are nonnegative according to (7). In order to determine the latter it is necessary to calculate a system of linear equations derived from (4).

If all conditions are satisfied, calculate value of entropy at the corresponding point. After all the configurations have been processed choose the biggest value, which is the sought upper boundary of entropy $H^{\max}$.

Solution of the minimization problem is very similar to that of (2).

Further in the thirds chapter the developed method is described [25]. Distinctive feature of the approach is that alternatives are evaluated based not on their performance on criteria, as in most methodologies, but on probabilities that performance of an alternative on a criterion will be high. The probabilities are interval, which makes ranking of alternatives a non-trivial task and allows for alternatives to be non-comparable. Thus, in general only a partial ranking of alternatives is available. Which is in full consent with intuition – one cannot expect a machine to give a precise answer to an extremely imprecise question – and it replicates the situation in such decision aid methodologies as PROMETHEE or ELECTRE. Usually it is not possible to obtain a complete ranking without loosing some information, as the decision problem can be too hard for any methodology to be able to make a smart trade-off between conflicting alternatives. Thus, it can be advantageous to build a partial ranking and to delegate trading-off to a decision-maker.

As with any decision aid methodology that is used for decision-making under uncertainty, in our setting the notion of optimality is vague. Thus, the main task of the methodology is not to extract an optimal alternative, in whatever way it is defined, but rather to use such tools as ranking and sensitivity analysis to help decision-maker to make a better and more informative decision.

This methodology differs from similar decision aids in several aspects. The main difference is that the proposed decision aid can be used when the available information is very uncertain. It may be so uncertain that it will not be able to construct a performance table or to describe the problem domain using any crisp methodology or framework, including interval probabilities.

As was mentioned before, the so-called fuzzy granules are used in the proposed method. It enables one to used both fuzzy and nondeterministic information in the description of the alternatives. This can be considered as a trade-off between non-fuzzy decision making methods and such methods as PRUF, which use fuzzy information so extensively that the problem becomes too complex computationally.

As you can see in Figure 3 the proposed method can be divided into the following steps:

1. Determine set of alternatives and criteria.
2. Describe criteria with the help of f-granules.
3. Determine probabilities that criteria will take desired values.
4. Determine informativeness of alternatives.
5. Perform preference modelling.
6. Perform partial or complete ranking.
7. Perform sensitivity analysis.
8. Analyse the results obtained.
9. Show results.

Let us consider the steps involved in greater details.

Step 1: Determine a set of alternatives and criteria

First it is necessary to determine what alternatives are there and which criteria will be used to evaluate these alternatives. The nature of the proposed decision aid methodology requires the set of alternatives to be finite and each alternative to be discrete and definite.

Like in other approaches, different alternatives and criteria can be determined by morphological analysis, during brainstorming sessions or by careful analysis of decision problem [53].

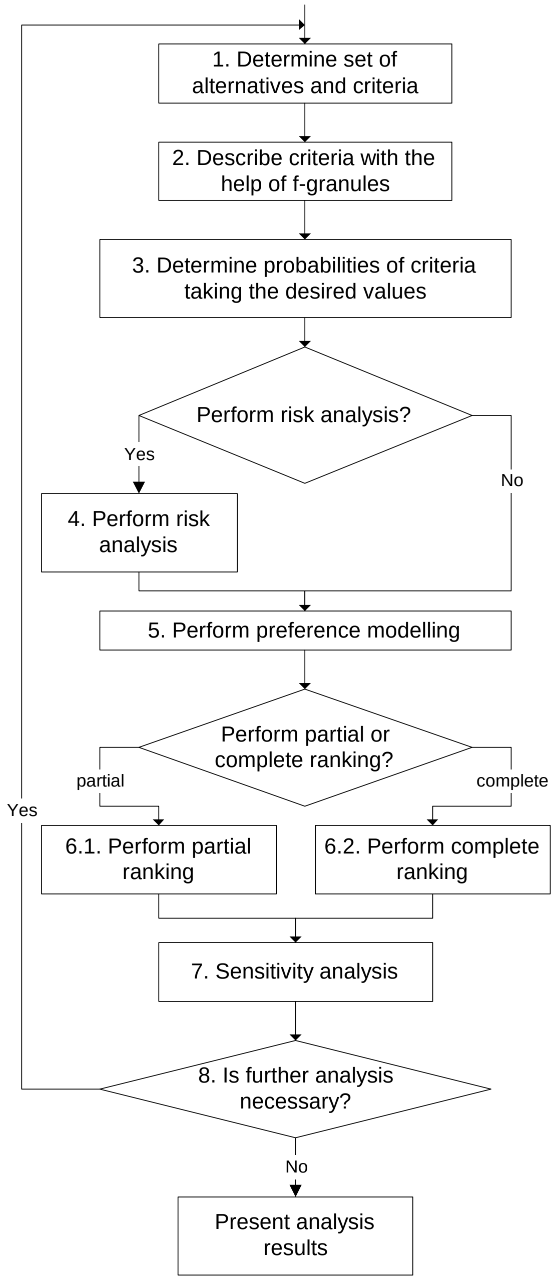

Figure 3. Workflow of the developed decision making method

Step 2: Describe criteria with the help of f-granules

After having determined a set of alternatives and criteria it is necessary to construct evidences that consist of f-granules, as described earlier in the paper. Each evidence contains granules that describe possible outcomes of some non-deterministic event and consequence of these outcomes on the value of a criterion. Usually description of an alternative contains a set of evidences, which describe performance of the alternative on criteria, defined in the first step. However, as will be seen later, alternatives are compared based on the probabilities that their performance on criteria will be high.

Information about one criterion can be distributed among several evidences. However, each evidence should describe only one criterion. When a set of evidences is defined and agreed upon, if there is a group of decision makers, we can extract information from these evidences by determining what is the probability that a criterion will be equal to some fuzzy value.

As was mentioned before, conditional fuzzy granules can be viewed as probability distribution of fuzzy values, or to be more exact, they introduce probabilistic constraints on possibility distribution of criteria.

Fuzzy granules can be represented in the form of IF-THEN rules, where the premise part corresponds to different possible non-deterministic outcomes. Outcomes that correspond to the same event or phenomenon are grouped into evidence. The consequence part contains fuzzy value of a criterion, which is a possibilistic constraint on its value.

To clarify this picture let us consider an example. Assume that we assess risk of a construction project based on the type of soil, which we are uncertain about. In this setting risk is the criterion and the type of soil is a non-deterministic phenomenon. After consulting experts and examining samples of soil we may have to come to the following evidence that contains three granules:

IF (soil is rocky) with probability 0.8 THEN risk is VERY LOW
IF (soil is sandy) with probability 0.15 THEN risk is ABOVE AVERAGE
IF (soil is swampy) with probability 0.05 THEN risk is VERY HIGH

Each evidence is a probability distribution of fuzzy values. For example, we can interpret the aforementioned evidence as the following probability distribution:

P(risk is VERY LOW) = 0.8
P(risk is ABOVE AVERAGE) = 0.15
P(risk is VERY HIGH) = 0.05

Given a set of such evidences, which describe criterion “risk” and maybe also other criteria, we can determine what is the probability that the risk will be LOW for a particular construction project. Value of Prob(risk=LOW) is an interval value.

This is the main tool for describing alternatives and for assessing their performance, which is actually the probability that a particular alternative will perform well.

Step 3: Determine probabilities that criteria will take desired values

Given a set of evidences that describe some criteria, we can determine what is the probability that criterion will take a particular fuzzy value; e.g., considering the example presented above, we can determine what is the probability that risk will be LOW. The obtained probability is interval. It is in consent with our intuition, as the description of alternatives is uncertain – with randomness and fuzziness present – so we cannot expect the system to give a precise answer.

Step 4: Entropy-based evaluation on alternatives' informativeness

This chapter describes how one can determine how much information is available about each alternative. This method is based on entropy, which is used to calculate systems' uncertainty. Entropy value shows how much information is missing. Thus, it can be used as an evaluation of alternatives' uncertainty [27].

In order to calculate entropy the problem has to be presented in the information theoretic form – a set of systems is constructed, which can be in different states. Each system corresponds to a criterion and its states are values that the corresponding criterion can be equal to. Probabilities of systems' states are interval. Thus, it is necessary to used interval entropy calculation method developed in the Ph.D. Thesis.

As a result an additional criterion will be obtained, which will represent lack of knowledge about alternatives.

The proposed entropy calculation method can be divided into four steps, and the first two steps are the same as for the proposed decision making method. Thus, if the informativeness is determined in the decision method framework, then first of all the first two steps should be performed.

1) Determination of set of criteria. In this step it is necessary to determine which criteria will be used to assess alternatives.
2) Description of alternatives with the help of fuzzy granules. In this step all the alternatives are described with the help of fuzzy granules.
3) Creation of systems. In this informativeness analysis step the problem is represented in information theoretic terms.
4) Determination of entropy value. Since in the developed method the entropy is used the measure of informativeness, with the calculation of entropy we obtain assessment of alternative's informativeness.

Let us consider the last two steps in greater detail. Evidences described in the second step can be used in order to determine probability that a criterion will be equal to a particular value. This probability is interval value, which determines upper and lower boundary of the probability.

In such a way we can calculate probabilities that a criterion will be equal to different fuzzy values. Moreover, this operation must be repeated for other criteria, in order to determine probabilities of their criteria values.

Now let us see how this all relates to the informativeness assessment method, which is shown in Figure 4 and how the problem is represented in information theoretic terms.

Different criteria correspond to different systems. States of these systems correspond to the possible values of these parameters. Thus, probability that the system S_i is in state q_k is equal to the probability that the criterion that corresponds to the system S_i is equal to the value that corresponds to the state q_k. In other words expression "system is in a state" is equivalent to the expression "criterion is equal to a value"

All the criteria should be described in such a way. As a result we will obtain a set of systems and each system corresponds to a criterion. This can be regarded as the end of the definition stage and the beginning of the calculation phase. In other words, after the systems have been defined it is necessary to calculate entropy of these systems [35].

Thus the informativeness evaluation problem has been reduced to the information theoretic problem. Moreover, the informativeness evaluation process does not end at this point. Now it is necessary to obtain an overall evaluation of entropy for the alternative. In this case it is possible to use the fact that the generalized entropy is additive. Entropy additivity means that if we join several independent systems then the entropy of the joint system is equal to the sum of the entropy of individual systems. More specifically, if two systems are given X, Y, and the entropy of these systems is $H(X)$, $H(Y)$, then additivity means that entropy of the joint system $H(X, Y)$ is equal to $H(X, Y) = H(X) + H(Y)$. In the Ph.D. Thesis it is shown that the generalized entropy is additive. Thus, the overall entropy evaluation of an alternative can be obtained by summing entropy values of individual systems.

The obtained overall entropy evaluation is informativeness evaluation for the given alternative. In order to obtain evaluations for other alternatives it is necessary to repeat this procedure.

Step 5: Perform preference modelling

We propose to use weighting scheme in order to model preferences. There are a number of weighting schemes readily available for use. For example, it is possible to adapt a value-tree-based approach used in AHP. Moreover, we can use such methods for weight evaluation as SMART, SWING or their generalization to interval case. The basic idea of the latter approach is to fix one so-called reference attribute, to assign it some value and then to determine relative importance of other attributes by assigning them evaluations relative to that of the reference attribute. Weights are then determined by normalizing the sum of these evaluations to one.

In its turn, AHP consists of constructing a hierarchy of criteria, sub-criteria. Weights are assigned based on pair-wise comparisons of criteria that have a common parent.

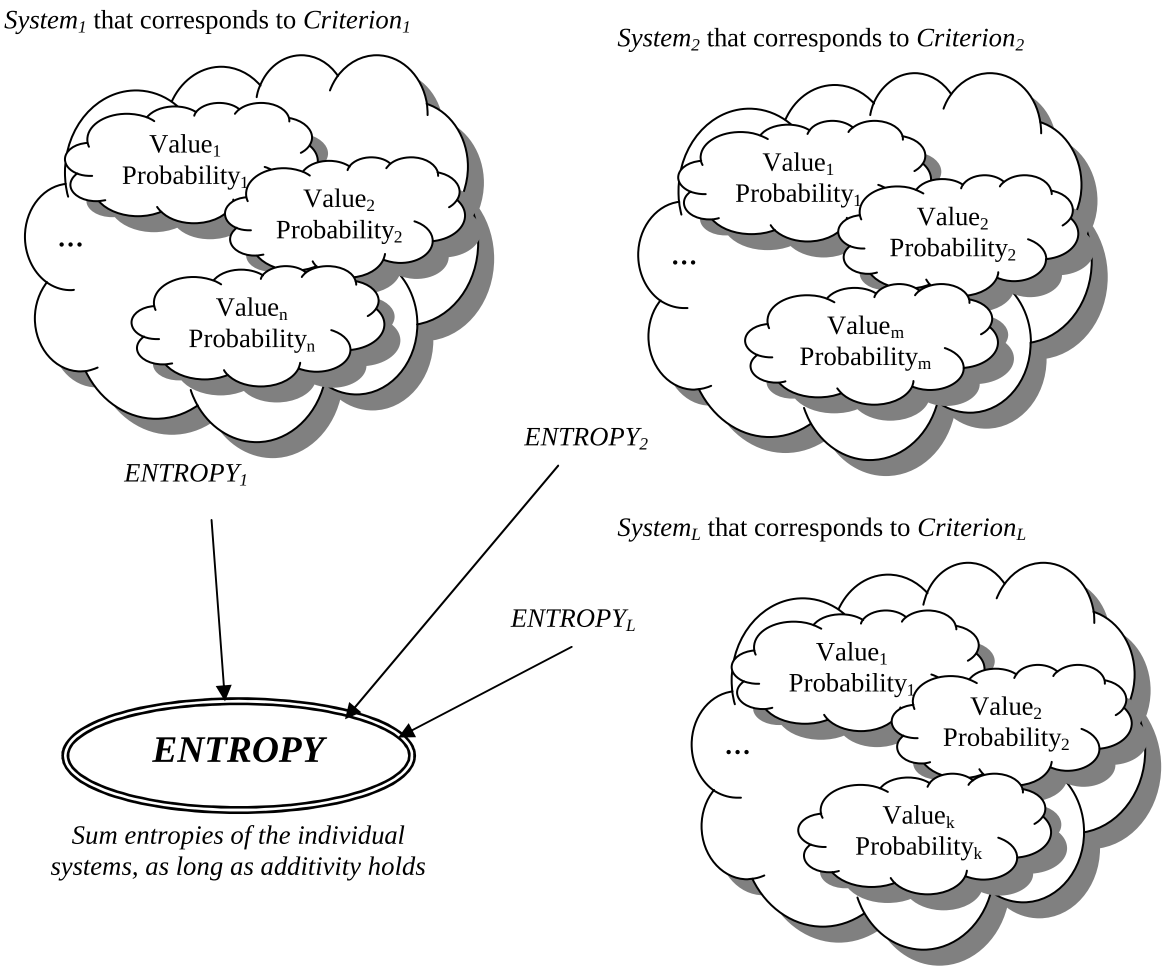

Figure 4. Informativeness assessment method

Step 6: Perform ranking

Below the partial ranking procedure is described, it is based on the evaluation of incoming and outgoing flows [31]. First let us introduce several definitions.

Let us assume that we have a set of alternatives A = $\{a_1, a_2, \ldots, a_n\}$, where n is the number of alternatives. A set of criteria that is used to evaluate alternatives is C = $\{c_1, c_2, \ldots, c_m\}$, where m is the number of criteria. Let us assume that using a weighting scheme the following weights have been assigned to criteria: $w_1, w_2, \ldots, w_m$. Each alternative is evaluated on *m* criteria and the evaluations are interval-valued probabilities. For alternative A_i we have the following evaluations: $E_1^i, E_2^i, \ldots, E_m^i$. Evaluations are interval, so: $E_j^i = \left({}^{\min}e_j^i,\ {}^{\max}e_j^i\right)$.

We define the outgoing and incoming flows as follows:

$$\phi^-(A_i) = \sum_{k=1}^{m} w_k \cdot {}^{\min}e_k^i$$

$$\phi^+(A_i) = \sum_{k=1}^{m} w_k \cdot {}^{\max}e_k^i$$

Partial ranking is performed by the following pre-order, where a and b are two alternatives from set A, while P is outranking relation, I is indifference relation and R is incomparability relation:

$$\begin{cases} aPb, & \text{ja } \phi^-(a) > \phi^+(b), \\ aIb, & \text{ja } \phi^-(a) = \phi^-(b) \text{ un } \phi^+(a) = \phi^+(b), \\ aRb, & \text{pretējā gadījumā,} \end{cases} \quad (8)$$

If we present some folding of interval evaluations, then a complete ranking would be possible. But, as noted elsewhere in the literature, complete ranking is possible only if some information is hidden or ignored. A natural way to fold interval probabilities is the following:

$$e_i^j = \frac{{}^{\min}e_i^j + {}^{\max}e_i^j}{2}$$

Now let us define performance p_i of an alternative a_i as follows:

$$p_i = \sum_{k=1}^{m} w_k e_k^i$$

Then the complete ranking is defined according to the following preorder, where a_i and a_j are two alternatives from set A:

$$\begin{cases} a_i P a_j, & \text{ja } p_i > p_j \\ a_i I a_j, & \text{ja } p_i = p_j \end{cases}$$

Once again we would like to point out that the complete ranking might be deceptive, as it hides away features of alternatives that may make them incomparable.

Step 7: Sensitivity analysis of the created decision model and use of the adaptive network ANGIE for evaluation of criteria importance

Sensitivity analysis is an important and powerful tool of a decision maker, as it helps to determine how sensitive a decision model is to changes in its parameters.

Like in other approaches, in the proposed decision aid methodology it is possible to change values of weights and to follow alternations in ranking of alternatives.

Should it be necessary, definition of alternatives or desired criteria values can be changed as well. These changes require that probabilities be re-calculated (step 3).

There is another approach that can aid in performing sensitivity analysis. It is based on determining which features of alternatives are desirable [28]. It can be considered as a sort of reverse engineering: first we define which values of criteria are desirable and then we determine a ‘dream alternative’, which is close to satisfying these desires. Results of such an analysis can be used for pinpointing critical features of alternatives, which we should pay our attention to. This kind of analysis is based on the adaptive network ANGIE (Adaptive Network for Granular Information and Evidence processing).

Step 8: Analysing results

At this point it should be decided whether further analysis is required. Decision analysis is necessarily an iterative task, so one should not neglect data adjustment and revision. Several reasons for revising data are: risk analysis shows that there is too little information available about an alternative; there are critical features that have not been properly examined; further sensitivity analysis is required; new data is available and so on.

Fourth Chapter

In the fourth chapter several practical problems are considered, which show how the developed decision-making method can be used in practice. Summary of one of these problems is presented below.

Let us assume that we have to choose a pulp mill construction project from three available alternatives. The first alternative is to build it in a town called Liepusala, the second is to build it in Daugavbuda and the third is to build it in Mazupe. The steps involved in the solution of this problem are as follows:

- it is necessary to determine set of criteria;
- each alternative must be described using f-granules;
- it is necessary to determine desired values of criteria;
- it is necessary to assess informativeness of each alternative;
- preferences among criteria should be modelled;
- it is necessary to construct partial and complete ranking of alternatives;
- results obtained should be analyzed.

First of all it is necessary to create a set of alternatives. We use the following criteria groups: Logistics, Cost of construction, Commercial viability, Influence on environment, Recruitment possibilities. Criteria are listed below:

- Logistics
    - How convenient is it for suppliers to deliver raw material
    - How convenient is it to ship production
    - Development of local infrastructure
- Cost of construction
- Commercial viability
    - ROI forecast
    - Number of new potential jobs
    - Fixed costs
    - Variable costs
- Influence on environment
    - Influence on forests
    - Influence on rivers
    - Influence on humans
    - Influence on animals
    - Influence on fish
- Recruitment possibilities
    - Availability of qualified personnel
    - General economic status of the region
    - Mobility costs related to personnel transfer

The next step is to describe the criteria defined using fuzzy granules. In order to do so we have to define possible fuzzy values of criteria. Abstract qualitative values are used to describe such criteria as "influence on environment". On the other hand, fuzzy values for such criteria as "ROI forecast" are based on the numerical values. Let us give examples of fuzzy values for two criteria: (1) ROI forecast and (2) Influence on humans, see Figure 5 and 6 respectively.

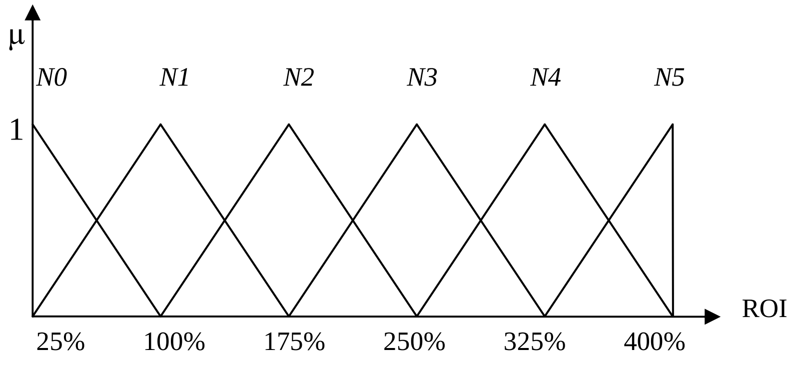

Figure 5. Fuzzy values for the criterion "ROI forecast"

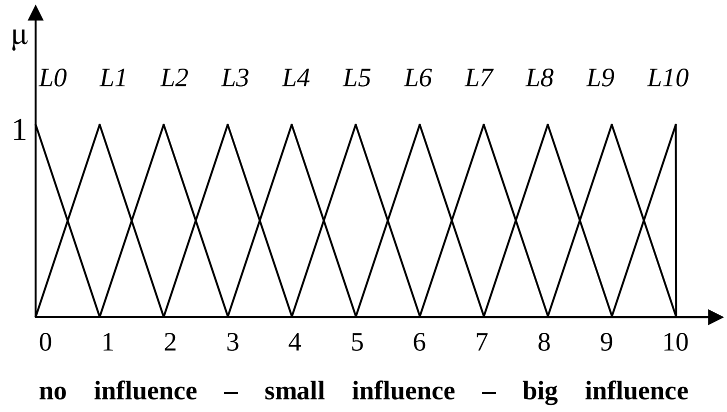

Figure 6. Fuzzy values for the criterion "Influence on humans"

Fuzzy evidence (a group of fuzzy granules) should be constructed for each criterion. Let us describe fuzzy evidences for two criteria: (1) ROI forecast and (2) Influence on humans. Please note that we define fuzzy evidences as probability distribution of fuzzy values.

First let us construct the evidences for the alternative "Liepusala":

P(ROI_forecast = N2) = 0,3
P(ROI_forecast = N4) = 0,6
P(ROI_forecast = N5) = 0,1

P(Influence_human = L2) = 0,1
P(Influence_human = L3) = 0,2
P(Influence_human = L6) = 0,4
P(Influence_human = L7) = 0,3

For the alternative "Daugavbuda" these criteria are defined as follows:

P(ROI_forecast = N2) = 0,1
P(ROI_forecast = N4) = 0,5
P(ROI_forecast = N5) = 0,4

P(Influence_human = L4) = 0,1
P(Influence_human = L5) = 0,2
P(Influence_human = L7) = 0,4
P(Influence_human = L9) = 0,3

For the alternative "Mazupe" these criteria are defined as follows:

P(ROI_forecast = N2) = 0,3
P(ROI_forecast = N4) = 0,6
P(ROI_forecast = N5) = 0,1

P(Influence_human = L5) = 0,1
P(Influence_human = L7) = 0,2
P(Influence_human = L9) = 0,4
P(Influence_human = L10) = 0,3

After defining all the criteria for all the alternatives using the method described above, it is necessary to define the *desired value* of each alternative. For example, for the criterion "ROI forecast" it is equal to the following fuzzy subset DN:

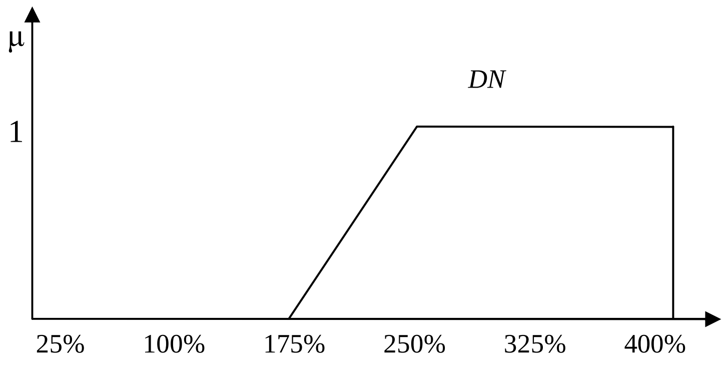

Figure 7. Desired value of the criterion "ROI forecast"

Desired value of the criterion "Influence on humans" is defined as follows:

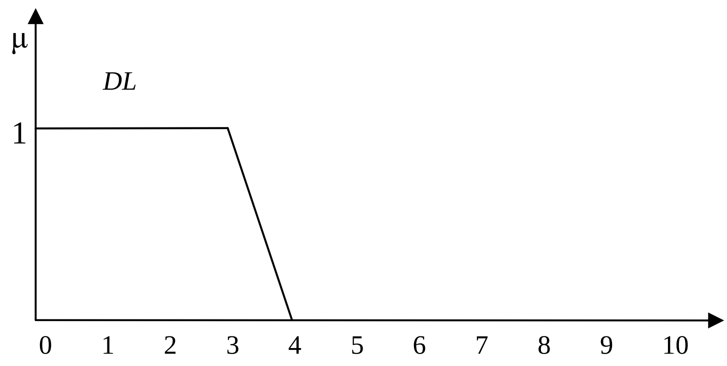

Figure 8. Desired value of the criterion "Influence on humans"

We are solving a decision-making problem under uncertainty. This means that only partial, incomplete and uncertain information is available. It would be advantageous to evaluate how much information is available about each alternative. We can use the entropy-based method described in [Vališevskis&Borisov, 2006] in order to assess how much information is available about each alternative. Due to limitations on the length of the paper we present only the calculated informativeness values:

Informativeness for the alternative "Liepusala": $\boldsymbol{H_L = [12{,}2\ ;\ 12{,}57]}$
Informativeness for the alternative "Daugavbuda": $\boldsymbol{H_D = [12{,}07\ ;\ 12{,}61]}$
Informativeness for the alternative "Mazupe": $\boldsymbol{H_M = [12{,}27\ ;\ 12{,}56]}$

The next step is preference modelling. We can use any suitable weighting scheme in order to model preferences among criteria. For example, we can assign some criteria a relative importance equal to 100 points or any other arbitrary value. Based on this value, relative importance values of other criteria are assigned. The procedure ends with normalizing assigned values so that they sum up to one. We can use more sophisticated methods, such as SMART, SWING, etc.

In order to construct partial ranking of alternatives we have to determine the incoming and outgoing flows. Based on the data available about the alternatives we have obtained the following flow values:

$$\phi_E^-\left(A_{Liepusala}\right) = 0.177\,, \qquad \phi_E^+\left(A_{Liepusala}\right) = 0.266\,,$$

$$\phi_E^-\left(A_{Daugavb\bar{\imath}da}\right) = 0.151\,, \qquad \phi_E^+\left(A_{Daugavb\bar{\imath}da}\right) = 0.233\,,$$

$$\phi_E^-\left(A_{Mazupe}\right) = 0.207\,, \qquad \phi_E^+\left(A_{Mazupe}\right) = 0.332\,.$$

As one can note, in this case there are no dominating alternatives, thus all the alternatives are incomparable. Thus, it is necessary to calculate performance values of alternatives and to create a complete ranking:

$$p_{Liepusala}^E = 0.221\,, \qquad p_{Daugavb\bar{\imath}da}^E = 0.192\,, \qquad p_{Mazupe}^E = 0.269$$

Thus, the complete ranking of alternatives is the following:

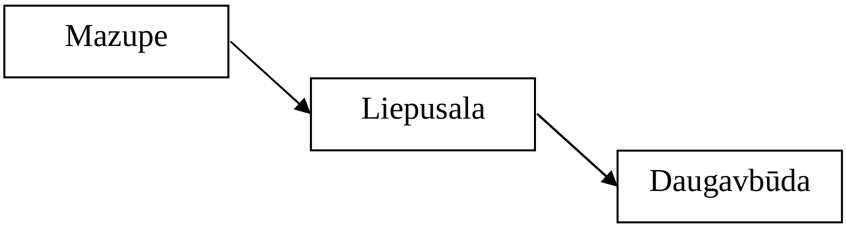

Figure 9. Complete ranking of alternatives

From the complete ranking shown in Figure 9 it follows that the most preferred alternative is to build a pulp mill in Mazupe. However, during the construction of the complete ranking part of the information is lost, since we have folded interval probabilities. During the solution of this problem we have analysed all the three alternatives and in the end we have obtained the ranking that represents effectiveness of alternatives, based on the input data.

In the **conclusion** of the Ph.D. Thesis main results are considered.

In the **Appendix** description of software can be found that was developed by the Ph.D. Thesis author in order to perform experiments; it enables one to automate calculations that are necessary to use the proposed method. Software is developed in MS Visual C++ environment.

MAIN RESULTS OF THE THESIS

A decision making method is developed in the Ph.D. Thesis that is intended to be used in an environment with two sources of uncertainty, i.e., in an environment that is both fuzzy and nondeterministic. The method is based on fuzzy granules, which enable one to use fuzzy values and probabilities during description of alternatives. During research additional problems have emerged and have been solved in the Ph.D. Thesis. For example, when a method for evaluating informativeness of alternatives was developed, Shannon's entropy had been generalized to the case of interval probabilities and it was proven that it is additive, because in the developed method interval probabilities are used. Moreover, adaptive network ANGIE (*Adaptive Network for Granular Information and Evidence Processing*) has been developed in order to process fuzzy granules. Network ANGIE training algorithm is used to solve multi-parameter sensitivity analysis task. Practical examples of the developed method usage are considered in the Ph.D. Thesis, which show how the method developed can be used in practice. Below you can find a more detailed description of main results of the Ph.D. Thesis:

1. Decision making methods that are intended to be used in uncertain environments are analysed in the Ph.D. Thesis and it was concluded that the existing methods either do not take into account more than one type of uncertainty, or they are too complicated computationally, or their aggregation operators unjustifiably reduce the available information.
2. It is shown that fuzzy granules can be used in order to combine fuzziness and randomness. More specifically, fuzzy evidences can be considered as probability distribution of fuzzy values. Hence, the decision model enables one to include two different types of uncertainty into the description of the problem domain. In a case if one of these uncertainties is not related to the given parameter, it is possible to assign it a precise and/or deterministic value.
3. Adaptive network ANGIE is developed, which can be used to process fuzzy evidences. Calculations in this network are parallel, which enables one to increase calculation speed if the programme is run in a system that supports parallel calculations. Adaptive network ANGIE training algorithm can be used in decision model sensitivity analysis.
4. Shannon's entropy is generalized to the case of interval probabilities. Moreover, it is proven that the generalized entropy is additive.
5. A method for complete or partial ranging of alternatives is developed. Like in PROMETHEE method, it is based on negative and positive flows.
6. Decision making method has been developed that includes all the mentioned tools, which are necessary for the analysis of alternatives and for the decision making. The method developed is intended for use in environment that is both nondeterministic and fuzzy.
7. The method developed is tested on two practical examples that show how the method developed can be used in practice.

LIST OF REFERENCES

1. Ben Amor S., Jabeur K., Martel J.-M. Multiple criteria aggregation procedure for mixed evaluations// Proceedings of MUDSM-2004, Portugal, Coimbra, September 22-25, 2004.
2. Brans J.P., Vincke Ph., Mareschal B. How to select and how to rank projects: The PROMETHEE method// European Journal of Operations Research 24, 1986, 228-238 p.
3. Chaitin G. From Philosophy to Program Size. Key Ideas and Methods// Lecture Notes on Algorithmic Information Theory. Estonian Winter School in Computer Science, Palmse, Estonia, March 2-7, 2003, available for download at http://www.umcs.maine.edu/~chaitin/eesti.html
4. Clemen R.T., Reilly T. Making Hard Decisions. Duxbury Press., 2004, 752 p.
5. Clifford D. Managing the Product Life Cycle// European Business Journal, July 1969.
6. Comninos I. Product Life Cycle Management, Aristotle University of Thessaloniki, 2002.
7. Cornuejols G., Trick M. Course Notes on Quantitative Methods for the Management Sciences. Carnegie Mellon University, 1998, 43-53 p.
8. Edwards W. How to use multiattribute utility measurement for social decisionmaking// IEEE Transactions on Systems, Man and Cybernetics, 7, 1977, 326-340 p.
9. Einstein A. Letter to Max Born (1926)// In: The Little Oxford Dictionary of Quotations. Ratcliffe S. (Ed.), Oxford University Press, 1994.
10. Hawking S. Does God Play Dice? Public lectures, 2000, http://www.hawking.org.uk/
11. Ishibuchi H., Nakashima T., Nii M. Classification and Modeling with Linguistic Information Granules, Springer-Verlag Berlin Heidelberg, 2005.
12. Keeney R.L., Raiffa H. Decisions with Multiple Objectives. John Wiley and Sons, New York, 1976.
13. Kosko B. Neural Networks and Fuzzy Systems: A Dynamical Systems Approach to Machine Intelligence, Prentice Hall, 1992.
14. Kyburg H. Interval-Valued Probabilities. SIPTA, http://www.sipta.org/
15. Mustajoki J., Hamalainen R.P., Salo A. Decision support by interval SMART/SWING – methods to incorporate uncertainty into multiattribute analysis, Manuscript. December 17, 2001.
16. Pickett J.P. The American Heritage® Dictionary of the English Language. Fourth edition. Boston, Houghton Mifflin Company, 2000.
17. Power D.J. Decision Support Systems: Concepts and Resources for Managers. Greenwood Publishing, 2002.
18. Roy B. The Outranking Approach and the Foundations of ELECTRE Methods. Theory and Decision 31, 1991, 49-73 p.
19. Rumelhart D.E, Hinton G.E. et al. Learning Internal Representation by Error Propagation. Parallel Distributed Systems, MIT Press, Massachusetts, 1986, 318.-362. p.
20. Saaty T.L. The Analytic Hierarchy Process. McGraw-Hill, New York, 1980.
21. Salo A., Hamalainen R.P. Preference programming through approximate ratio comparisons, European Journal of Operational Research 82, 458-475 p.
22. Shafer G. Mathematical Theory of Evidence. Princeton Univ Press, 1976.

23. Shannon C., Weaver W. The mathematical theory of communication, Urbana: University of Illinois Press, 1949.
24. Turban E., Aronson J.E., Liang T.-P. Decision Support Systems and Intelligent Systems. Prentice Hall, 2004.
25. Vališevskis A. Granular-Information-Based Decision Aid Methodology// Proceedings of the International Conference “Managing Uncertainty in Decision Support Models” (MUDSM 2004), Coimbra, Portugal, September 22-24, 2004.
26. Vališevskis A. Granular Information-Based Decisions// Proceedings of the international conference Information Society and Modern Business, Ventspils, Latvia, January 31 - February 1, 2003, 327-333 p.
27. Vališevskis A. Granular Information-Based Risk Analysis in Uncertain Situations// Proceedings of the international conference "Environment. Technology. Resources", Rezekne, Latvia, June 26-28, 2003, 385-391 p.
28. Vališevskis A. Using Granular-Evidence-based Adaptive Networks for Sensitivity Analysis// In: Merkuryev J, Dzemyda G et al. (eds), Scientific Proceedings of Riga Technical University, vol. 10, 2002, 150-155 p.
29. Vališevskis A. Adaptive Learning Algorithm for Hybrid Fuzzy System// Proceedings of the International scientific conference Traditions and Innovations in Sustainable Development of Society, Rezekne, Latvia, February 28-March 2, 281-288 p.
30. Vališevskis A. Comparative Analysis of Different Approaches towards Multilayer Perceptron Training// Scientific Proceedings of Riga Technical University, vol. 5, 157-167 p.
31. Vališevskis A., Borisov A. F-Granule-Based Decision-Making// Proceedings of the International Conference on Operational Research "Simulation and Optimisation in Business and Industry", SOBI 2006, Tallinn, Estonia, May 17-20, 2006, 164-169 p.
32. Vališevskis A., Borisov A. Risk Analysis in Granular-Information-Based Decision Aid Models// Proceedings of the International Conference on Fuzzy Sets and Soft Computing in Economics and Finance, FSSCEF 2004, St. Petersburg, Russia, June 17-20, 2004, 466-473 p.
33. Vališevskis A., Borisov A. Normalization Issues in Granular Evidence-Based Adaptive Network ANGIE// Proceedings of the International Conference on Computational Intelligence for Modelling, Control and Automation CIMCA'2003, Vienna, Austria, February 12-14, 2003, 117-124 p.
34. Vališevskis A., Borisov A. Using Interval-Valued Entropy for Risk Assessment// Proceedings of “Modelling and Simulation of Business Systems”, Kaunas University of Technology Press – Technologija, Lithuania, 2003, 74-78 p.
35. Vališevskis A., Borisov A. Fuzzy-Granule-Based Information Theoretic Approach towards Risk Analysis// Proceedings of the Second International Conference on Soft Computing, Computing with Words and Perceptions in System Analysis, Decision and Control ICSCCW-2003, Antalya, Turkey, September 9-11, 2003, 275-281 p.
36. Vališevskis A., Borisov A. Design Considerations of the Granular-Evidence-Based Adaptive Network ANGIE// Scientific Proceedings of Riga Technical University, vol. 14., 28-35 p.
37. Vališevskis A., Borisov A. ANGIE: Adaptive Network for Granular Information and Evidence Processing// Proceedings of the Fifth International Conference on Application of Fuzzy Systems and Soft Computing ICAFS-2002, Milan, Italy, September 17-19, 2002, 166-173 p.
38. Von Winterfeldt D., Edwards W. Decision analysis and behavioral research. Cambridge University Press, 1986.

39. Whalen T. Interval Probabilities Induced by Decision Problems// Advances in the Dempster-Shafer Theory of Evidence. Editors: Fedrizzi, Kacprzyk, Yager. Wiley, N.Y., 1994, pp. 353-374.
40. Yager R.R. Approximate Reasoning and Possibilistic Models in Classification// International Journal of Computer and Information Sciences, Vol. 10, No. 2, (1981), 141-175 p.
41. Zadeh L.A. Fuzzy Logic = Computing with Words, IEEE transactions on fuzzy systems, vol. 4, No. 2, May 1996.
42. Zadeh L.A. Fuzzy Logic// Computer Magazine, No.4, 1988, 83-93 p.
43. Zadeh L.A. Is probability theory sufficient for dealing with uncertainty in AI: A negative view// Uncertainty in AI, L.N. Kanal and J.F. Lemmer, eds., Elsevier Science Publishers, New York, N.Y., 1986, 103-116 p.
44. Zadeh, L.A., A simple view of the Dempster-Shafer theory of evidence and its implication for the rule of combination. AI Magazine, 7(2), pp. 85-90, 1986.
45. Zadeh L.A. Fuzzy Sets and Information Granularity// Advances in Fuzzy Set Theory and Applications, Editors: Gupta M.M., Ragade R.K et al., North-Holland Publishing Company, 1979, 3-18 p.
46. Zadeh L.A. PRUF-a meaning representation language for natural languages// Int. J. Man-Machine Studies 10, 1978, 395-460 p.
47. Zadeh L.A. Fuzzy Sets and Systems// Proceedings of Symposium on System Theory, Polytechnic Institute of Brooklyn, 1965, 29-37 p.
48. Zavadskas E., Peldschus F., Kaklauskas A. Multiple Criteria Evaluation of Projects in Construction. Vilnius, "Technika", 1994.

49. Андрейчиков А. В., Андрейчикова О. Н. Анализ, синтез, планирование решений в экономике. «Финансы и статистика», 2004, 464 с.
50. Вентцель Е.С. Теория вероятностей. Седьмое издание, Москва, «Высшая школа», 2001, 575 с.
51. Геращенко А.Л. Принципиальная модель эволюции природы, 2002, http://www.principics.narod.ru/
52. Колмогоров А.Н. Три подхода к определению понятия "Количество информации"// Проблемы передачи информации, т. I , вып. 1, 1965, 3-11 с.
53. Половинкин А.И. Основы инженерного творчества , Москва, «Машиностроение», 1988, 368 с.
54. Черноруцкий И.Г. Методы принятия решений, Санкт-Петербург, «БХВ-Петербург», 2005, 416 с.