Transport and Telecommunication, 2007, Volume 8, No 1, 20–23
Transport and Telecommunication Institute, Lomonosov 1, Riga, LV-1019, Latvia

OPTIMUM ALGORITHM OF SEMI-ANALYTICAL INERTIAL SYSTEM IN GEOGRAPHICAL COORDINATE SYSTEM

Tatyana Trifonova-Bogdanova, Oleg Zarianski, Peter Trifonov-Bogdanov

Aviation Institute, Riga Technical University
Lomonosov Str.1, Riga, LV-1019, Latvia
Ph.: (+371)67269574, (+371)26005007. E-mail: ptb@inbox.lv

The algorithm of an inertial system in geographical coordinate system is developed, where correction implements on parameters of motion in the absolute space. The developed algorithm contains smaller number of operations concerning classical version.

Keywords: *inertial system, coordinate system, vector of the absolute velocity of movement, vector of the carrying velocity, vector of relative velocity*

Geographical system of coordinates NEH is connected to the Earth surface. Axis N and E are located in the plane of local horizon. Axis N is directed to the North Pole of the Earth. Axis E is directed to the East and perpendicular to the N axis. Axis H is situated in a vertical direction to the place. The system of coordinates is realized by gyro-stabilized platform, on which the sensors of primary information of navigation system are fitted.

In order to get the algorithms of the semi-analytic type INS with the correction of the parameters of absolute movement functioning in the geographical system of coordinates it is necessary to project the vector equation of relative velocity changes on the axis's of the geographical system of coordinates:

$$\frac{d\vec{U}}{dt} = \vec{\alpha}_{acc} - \left(\vec{\Omega}_{SC} \times \vec{V}_a\right) - \frac{d\vec{V}_e}{dt}, \tag{1}$$

$$\vec{V}_a = \vec{U} + \vec{V}_e\,;\; \vec{V}_e = \vec{V}_3 = \vec{\Omega}_3 \times \vec{R}_3,$$

$$\vec{\Omega}_{SC} = \frac{\vec{V}_a}{R_3};$$

where: $\vec{\alpha}_{acc}$ – vector of the output signal of the accelerometer,

$\vec{V}_a$ – vector of the absolute velocity of movement,

$\vec{V}_e$ – vector of the carrying velocity,

$\vec{U}$ – vector of relative velocity,

$\vec{\Omega}_3$ – vector of the angular velocity of rotation of the Earth,

$\vec{V}_3$ – vector of the linear velocity of rotation of the Earth,

$\vec{\Omega}_{SC}$ – vector of the angular velocity of the geographical system of coordinates' rotation.

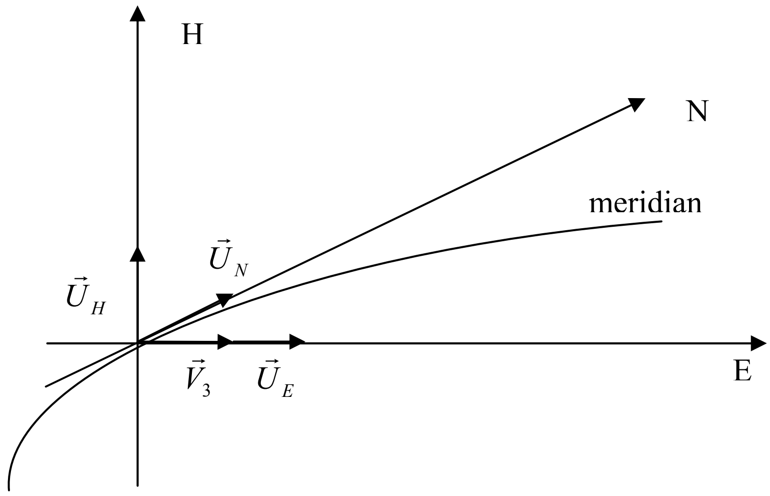

Figure 1

The vector of the linear velocity of the Earth's movement V_3, directed by the E axis. The vector of the relative velocity of the aircraft movement has projections on all axes of the geographical system of coordinates. According to that projections of the vector of the absolute movement velocity $\vec{V}_a$ on the axes of the geographical system of coordinates will be determined as follows:

$$V_{aE} = V_3 + U_E\,; \quad V_{aN} = U_N\,; \quad V_H = U_H\,. \tag{2}$$

The geographical system of coordinates is the moving reckoning system in the world's absolute space. Geographical system of coordinates rotating with the angular velocity Ω_{SC}, that depends on absolute movement velocity of the aircraft. North component of the absolute velocity V_{aN} will produce the East component of the angular velocity of movement of the system of coordinates: $\Omega_{SCE} = -\frac{V_{aN}}{R_3}$.

The East component of absolute velocity V_{aE} will produce angular velocity of movement of the system of coordinates $\Omega_{SCj3} = \frac{V_{aE}}{R_3 \cos\varphi}$, which have components by both N axis (Ω_{SCN}) and H axis (Ω_{SCH}).

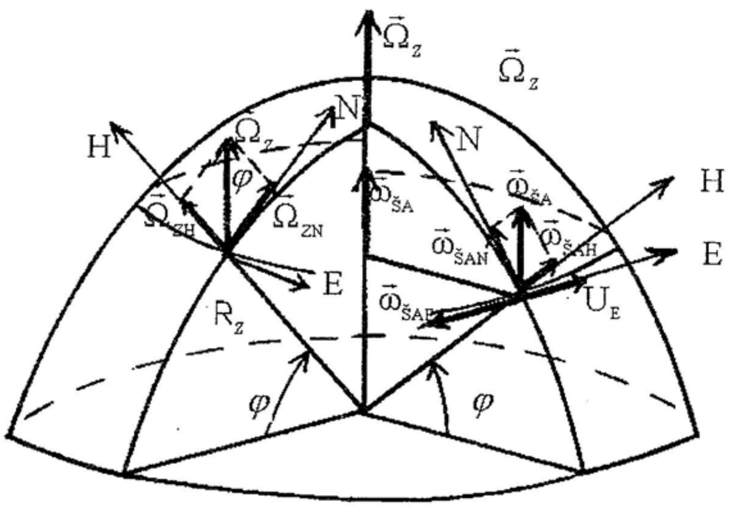

Figure 2

The projections of vector of angular velocity of system of coordinates $\vec{\Omega}_{SC}$ on their axes will be determined as follows:

$$\Omega_{SCN} = \Omega_{SCj3} \cdot \cos\varphi = \frac{V_{aE}}{R_3};$$

$$\Omega_{SCH} = \Omega_{SCj3} \cdot \sin\varphi = \frac{V_{aE}}{R_3} tg\varphi; \tag{3}$$

$$\Omega_{SCE} = -\frac{V_{aN}}{R_3};$$

where φ is latitude angle.

In the equation (1) the member $\frac{d\vec{V}_e}{dt}$ determined as the value changes of the vector $\vec{V}_3$. At the movement of the aircraft by the meridian (when latitude angle φ changes) will change vector's $\vec{V}_3(\Delta\varphi)$ value. Let's find out the value of the vector $\vec{V}_3$ changes:

$$\begin{aligned}\Delta V_3 &= V_{32} - V_{31} = \Omega_3 R_3 \cos\varphi_2 - \Omega_3 R_3 \cos\varphi_1 = \Omega_3 R_3 \cos\varphi_2 - \Omega_3 R_3 \cos(\varphi_2 - \Delta\varphi) = \\ &= \Omega_3 R_3 \cos\varphi_2 - \Omega_3 R_3 \cos\varphi_2 \cos\Delta\varphi - \Omega_3 R_3 \sin\varphi_2 \sin\Delta\varphi = \Omega_3 R_3 \cos\varphi_2 - \Omega_3 R_3 \cos\varphi_2 - \\ &- \Omega_3 R_3 \sin\varphi_2 \cdot \Delta\varphi = -\Omega_3 R_3 \sin\varphi_2 \cdot \frac{U_N \Delta t}{R_3}.\end{aligned} \tag{4}$$

From the equation (4) we will get the value of the differential $\frac{d\vec{V}_e}{dt}$:

$$\left|\frac{d\vec{V}_e}{dt}\right| = \left|\lim_{\Delta t \to 0} \frac{\Delta \vec{V}_3}{\Delta t}\right| = -\Omega_3 U_N \sin\varphi_2. \tag{5}$$

The differential $\frac{d\vec{V}_e}{dt}$ has a projection $\Delta\vec{V}_3(\Delta\varphi)$ only by axis E of geographical system of coordinates:

$$\frac{dV_e}{dt} = \frac{dV_{eE}}{dt} = -\Omega_3 U_N \sin\varphi_i, \tag{6}$$

where φ_i is a current value of the latitude angle.

When we find separate components in equation (1), we can calculate projection of the vectors multiplication $\left(\vec{\Omega}_{SC} \times \vec{V}_a\right)$ on axis's of geographical system of coordinates. Projections of the vectors multiplication $\left(\vec{\Omega}_{SC} \times \vec{V}_a\right)$ on axis's N,E,H will be equal:

$$\begin{aligned}\left(\vec{\Omega}_{SC} \times \vec{V}_a\right)_{1N} &= V_{aE}\Omega_{SCH} + V_{aH}\Omega_{SCE}, \\ \left(\vec{\Omega}_{SC} \times \vec{V}_a\right)_{1E} &= V_{aN}\Omega_{SCH} + V_{aH}\Omega_{SCN}, \\ \left(\vec{\Omega}_{SC} \times \vec{V}_a\right)_{1H} &= V_{aE}\Omega_{SCN}.\end{aligned} \tag{7}$$

Now if we will collect all components of equation (1) by the appropriate axes, will get the full algorithm of Inertial Navigation System operating in geographical system of coordinates with the correction by the absolute movement parameters:

$$\frac{dU_N}{dt} = \alpha_{accN} - V_{aE}\Omega_{SCH} - V_{aH}\Omega_{SCE},$$
$$\frac{dU_E}{dt} = \alpha_{accE} + V_{aN}\Omega_{SCH} - V_{aH}\Omega_{SCN} + \Omega_3 U_N \sin\varphi,$$
$$\frac{dU_H}{dt} = \alpha_{accH} - q - V_{aE}\Omega_{SCN},$$
$$V_{aE} = V_3 + U_E;\ V_{aN} = U_N;\ V_H = U_H;\ V_3 = \Omega_3 R_3 \sin\varphi, \quad (8)$$
$$\Omega_{SCN} = \frac{V_{aE}}{R_3};\ \ \Omega_{SCE} = -\frac{V_{aN}}{R_3};\ \ \Omega_{SCH} = \frac{V_{aE}}{R_3} tg\varphi;$$
$$S_N = \int_0^t U_N dt;\ \ S_E = \int_0^t U_E dt;\ \ H = \int_0^t U_H dt,$$
$$\varphi = \varphi_0 + \frac{S_N}{R_3};\ \ U_N = \int_0^t \frac{dU_N}{dt}dt;\ \ U_E = \int_0^t \frac{dU_E}{dt}dt;\ \ U_H = \int_0^t \frac{dU_H}{dt}dt.$$

Without a vertical channel the algorithm of the semi-analytic type Inertial Navigation System, this is operating in geographical system of coordinates with the correction by the absolute movement parameters:

$$\frac{dU_N}{dt} = \alpha_{accN} - V_{aE}\Omega_{SCH};\ \ \frac{dU_E}{dt} = \alpha_{accE} + V_{aN}\Omega_{SCH} + \Omega_3 U_N \sin\varphi;$$
$$U_N = \int_0^t \frac{dU_N}{dt}dt;\ U_E = \int_0^t \frac{dU_E}{dt}dt;$$
$$V_{aE} = V_3 + U_E;\ V_{aN} = U_N;\ V_3 = \Omega_3 R_3 \sin\varphi; \quad (9)$$
$$\Omega_{SCN} = \frac{V_{aE}}{R_3};\ \ \Omega_{SCE} = -\frac{V_{aN}}{R_3};\ \ \Omega_{SCH} = \frac{V_{aE}}{R_3} tg\varphi;$$
$$S_N = \int_0^t U_N dt;\ \ S_E = \int_0^t U_E dt;\ \varphi = \varphi_0 + \frac{S_N}{R_3}.$$

Developed algorithm contains less number of operations (23) in comparison with classical variant of INS (39). This will cause the decreasing of the number of the structural technical elements and therefore increasing of the reliability of the INS operation.

References

1. Bromberg, P. V. *The theory of inertial systems of navigation.* M.: Nauka, 1979. (In Russian)
2. Zakharin, M. I., Zakharin, F. M. *Kinematics of inertial systems of navigation*. M.: Mashinosroyeniye, 1968. (In Russian)
3. Ishlinskiy, A. Y. *Orientation gyroscopes and inertial navigation.* M.: Nauka, 1976. (In Rusiian)
4. Pelypor, D. S., Mikhalev, I. A., Bauman, V. A. *Gyroscopic systems*. M.: Visshaya Shkola. 1988. (In Russian)
5. Trifonov-Bogdanov, P. I. *Inertial navigating systems of partly analytical type.* Riga: RAU, 1998. (In Russian)
6. Trifonov-Bogdanov, P. I. *Inertial navigating system of a wide class of air courts. The application for the invention 93036974. (037054).* RJ. 1993. (In Russian)
7. Trifonov-Bogdanov, P. I. *Inertial navigating system with simplified correction. Application P-98-70.* LV. 1998. (In Latvian)