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MODEL OF DECISION MAKING IN ADAPTIVE MULTICRITERIAL OPTIMIZATION PROCEDURE

LĒMUMU PIEŅEMŠANAS MODELIS ADAPTĪVĀ DAUDZKRITERIĀLĀS OPTIMIZĀCIJAS PROCEDŪRĀ

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Multicriterial optimization, adaptive approach, decision maker, the best solution, criteria priority system

1. Introduction

Multicriterial optimization is a typical mathematical approach for solving different designing and planning tasks. Such tasks do not have objective formal solution (formal objective solution is Pareto set, what is too large for problem solver needs). To solve the task with several conflicting criteria of quality, we must use additional information from experts or decision makers (DM). But this information is subjective, with the all followed aftereffects. Different approaches for usage of additional DM information are worked out [1-4]. Widely are used a posteriori information usage methods and iterative or interactive methods [5]. Works [6-7] introduce adaptive interactive methods. Adaptive methods not only realize search of more preferable solutions in the Pareto set [8], but also analyze behavior of DM and try to correct unwished subjectivity utterance.

There is a big problem with use of interactive and adaptive interactive methods. It is hard and mostly impossible to make comparison of methods, because real DM could solve multicriterial optimization task only once. If he tries to solve the same task with another method, he is not the previous DM, because his knowledge is changed (increased). To compare the methods, we need DM model, which simulate DM behavior and its starting knowledge about optimization task always are the same. Such formal model could help to analyze different subjective features of real DM.

2. Multicriterial interactive and adaptive optimization approaches

Multicriterial tasks $U_r(r=1,\dots,m)$ are formed by set of contradictory criteria $q_1(X),\dots,q_k(X)$ which values need to optimize (desirable to achieve minimal or maximal values of criteria), changing project or plan variables $X=(X_1,X_2,..,X_n)$. Of course, additionally we have technological, physical and cost equality and inequality constraints $D: h_j(X)\geq 0 \ \ (j=1,...,m)$,

$D: g_i(X)=0 \ \ (i=1,...,n)$. They define the feasible set D of possible solutions. The best solution from DM viewpoint is a Pareto set P solution X^{**}. It means that the best solution search it is worth to accomplish only within Pareto set solutions [1-4].

Multicriterial optimization interactive approach methods are realised as iterative goal seeking process. DM analyse one or some $(\mathrm{v}=1,\dots,E, E\geq 1)$ near Pareto solutions and forms his wishes $I_1,I_2,I_3,\dots$. This information is transformed in the parameters values of Pareto solutions search method and method obtain new estimations $X_1^{*V}, X_2^{*V},\dots$ of the best solution X**. Experience of real tasks solving shows, that number of iterations for one task solution process must not overran, approximately, 20 till 30. We must take into account, that in the solution process DM get tired and his information $I_1,I_2,I_3,\dots$ about criteria priorities becomes unreasoned and inadequate. The process of multicriterial optimization using the interactive approach could be presented as follows:

$$\{X_0^{*V}\in \mathrm{P}, \mathrm{Q}_0^{V}(X_0^{V})\}\xrightarrow{I_1}\{X_1^{*V}\in \mathrm{P}, \mathrm{Q}_1^{V}(X_1^{V})\}\xrightarrow{I_2}\dots\xrightarrow{I_s}\{X_s^{*V}\in \mathrm{P}, \mathrm{Q}_s^{V}(X_s^{V})\}, \quad (1)$$

with conditions:

$$X^{**}\in\{X_s^{V}\}\ (\mathrm{v}=1,\dots,E,E\geq 1) \text{ and } X_0^{V}\prec X_1^{V}\prec\cdots\prec X_s^{V}=X^{**}. \quad (2)$$

Adaptive approach add to interactive approach schema two main adaptation processes:

1) DM adapts to multicriterial task criteria interdependences and potentialities. At start DM is not aware of behaviour of criteria functions or behaviour of criteria values in the multidimensional space.
2) searching method must adapt to DM's system of criteria priorities. For that reason we must use formal model of DM's criteria priorities system.

The third adaptation process in multicriterial optimization is adaptation of searching algorithm to criteria function features, to accelerate the searching process of new Pareto solution.

In adaptive approach the current DM's information $I_2,I_3,\dots$ about criteria priorities is combined with previous (historical) DM's considerations H_1, H_2**, … .** This information comes from formal model of DM's criteria priority system in search method.

Adaptive approach is improvement of (1) (interactive approach) with conditions form (2):

$$\{X_0^{*V}\in \mathrm{P}, \mathrm{Q}_0^{V}(X_0^{V})\}\xrightarrow{I_1}\{X_1^{*V}\in \mathrm{P}, \mathrm{Q}_1^{V}(X_1^{V})\}\xrightarrow{I_2,H_1}\dots\xrightarrow{I_s,H_{s-1}}\{X_s^{*V}\in \mathrm{P}, \mathrm{Q}_s^{V}(X_s^{V})\}.$$

(3)

It enables DM's inconsequence and errors filtration, in performing next Pareto solution (-s) search.

3. Multicriterial adaptive optimization procedure

Its need to perform four main tasks in multicriterial adaptive optimization:
1) extraction of Pareto set solutions $\{X_j^* \in P, j = 1, \ldots\}$;
2) obtaining estimations of DM's conception about solutions priorities $I_1, I_2, I_3, \ldots$;
3) as much as possible accurate DM's information transformation in search method parameter values;
4) control of decision making process, to guarantee the quality of acquired solution and quickness of process.

Figure 1 presents the steps of multicriterial adaptive optimization procedure. There are two search processes in procedure. One for search feasible Pareto solutions (steps 3, 4, 5, 6), another (7, 8, 9, 10, 11, 12, 13, 6, 7), search for Pareto solution with prescribed features (transition from one Pareto solution to another). In the adaptive multicriterial optimization methods the model of DM solutions priority system must be used [9]. With the help of it, algorithm could correct DM's current wishes to reduce DM's inconsistency and uncertainty in solutions estimation.

Different classes of methods to find the Pareto solution are used:

1) Methods of global criterion [1, 2] $W(Q(X), \varpi)$ where $Q(X) = (q_1(X), \ldots, q_k(X))$ and $\varpi = (\lambda_1, \ldots, \lambda_k)$ are weights or significance of criteria. This class includes method of weighted sum, goal attainment method, epsilon-constraint method.

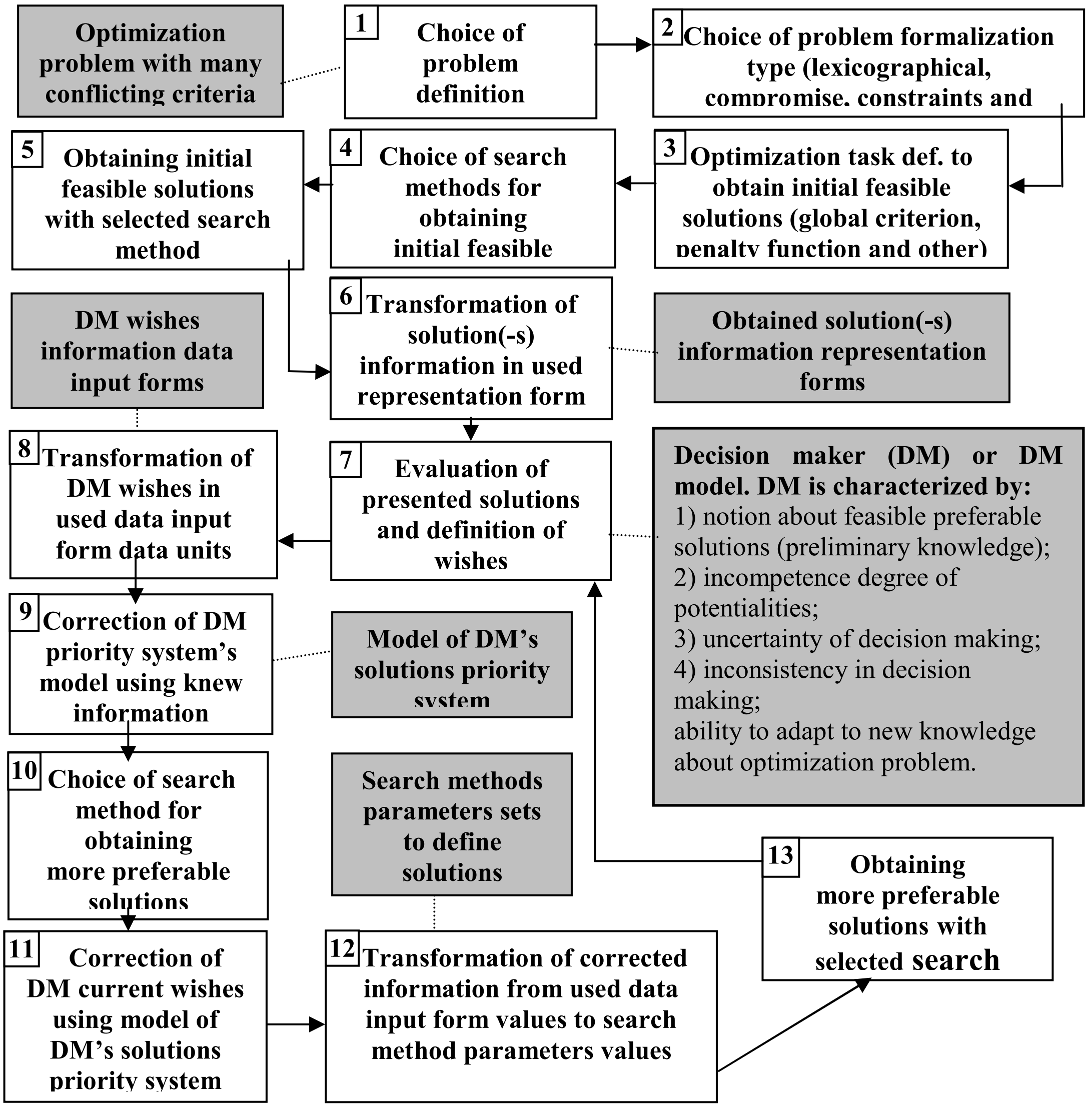

Figure 1. Elements of multicriterial adaptive optimization procedure.

2) lexicographic methods [4], when is possible to define strong criteria priority range
   $q_1(X) \prec q_2(X) \prec q_3(X) \prec .. \prec q_n(X) \prec q_{k+1}(X)$ (4)
3) vector-relaxation methods [10];
4) stochastic search methods [11];
5) stochastic programming methods [1];
6) multiobjective genetic algorithms and multiobjective evolutionary algorithms [12-14].

In practice global criterion and vector-relaxation methods are mainly used [15,7]. In global criterion methods all criteria are mapped in single global criterion of quality. Different types of global criterion function are used. For instance the following ones:

1) The sum of weighted criteria $W(Q(X),\varpi) = \sum_{i=1}^{i=k} \lambda_i q_i(X) \to optimum$;

2) Minimax principle $W(Q(X),\varpi) = \max \min_{i=1,\dots,k} \lambda_i q_i(X)$.

With the help of weights value combination it is possible to address every Pareto sets solution.

In vector-relaxation methods [10] the search direction $V = (v_1, \ldots, v_n)$ from solution X_j (point in the variables space) is chosen from the considerations, that $\forall i \in \{1,...,k\}[q_i(X_j + g \cdot V) \geq q_i(X_j)]$ and $\exists i \in \{1,...,k\}[q_i(X_j) + g \cdot V) > q_i(X_j)]$, where g is a little step size in variables space and for all criteria the greater value means, that it is more preferable.

Decision makers are people with different experience and imagination. In order to get better possibilities of presented Pareto solutions evaluation, it is necessary to deal with different forms of information representation for different DM. It means, that we must use in concrete situation (task, DM) information presentation form, whish is the best for appropriate DM.

In a solution process of multicriterial optimization, DM operates with his previous experience knowledge Ψ_E and knowledge obtained during optimization process Ψ_1, Ψ_2, ...

Presentation forms $F_z^{PR}(z = 1, \ldots)$ of obtained current results $\{X_j^V \in \mathrm{P}, Q_j^V(X_j^V)\}$ assists DM in making next decision. From the data organization mode in the presentation often is dependent the quality of decision making. Multidimensionality of criteria and variables spaces and search history (search trajectories in variables and in criteria spaces) are objects which presentation request use not only alphabetical – numbering data, but desirable is use of graphic schemas. In multicriterial optimization is popular usage of different multidimensional depiction types. For example, “spider web” frames (Figure 2). In this picture we could easily see, that criteria q_2 and q_5 are contrary. q_2 criterion value growth causes decrease of criterion q_5. Such depiction help perceive the trends of criteria or variables values. It assists in comparison of different vector type objects.

In the multicriterial optimization take place two information transformation processes:

1) DM wishes are transformed in the data units of the used data input form;
2) data unit’s values of data input form are transformed in the search algorithm parameters values.

Existence of second transformation process request, that DM wishes data input form must be concordant with search algorithm parameters. There are many good data input forms from DM viewpoint, but the use of its makes difficult problematical transformations of forms data to search algorithm parameter values.

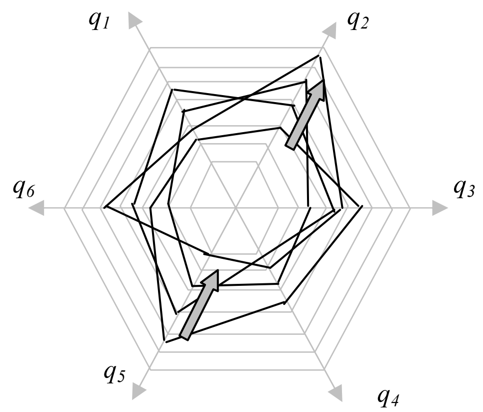

Figure 2. “Spider web” information presentation picture.

In the multicriterial optimization systems mainly are used such general forms for DM wishes data input [15, 7]:

1) use of criteria weights $\lambda_i (i = 1,\dots,k)$. This form is mathematically correct only for linear criteria functions, but despite of this, in real life it is the most popular form.
2) use of desired criteria values $q_1^d,\dots,q_k^d$ estimations;
3) selection of criteria which values must be improved and for which criteria values can worsen;
4) use of dynamic constraint system for desired criteria values.

Working with different multicriterial optimization algorithms and DM wishes data input forms we often come to contradictions. The main rule in data input form choice is: form must be convenient for DM, it must be easy to formulate DM wishes in the terms of form. But next, we must transform information from form to algorithm parameters and in many cases it not easy or is impossible. Often we could lose appropriate amount of information (Figure 3):

$$I_j^{DM} = I_j^F + \Delta_j^F = I_j^{Me} + \Delta_j^F + \Delta_j^{Me} \quad (5)$$

where Δ_j^F and Δ_j^{Me} are the information loses in transformations.

Sometimes the loss of information is not critical for solution process, however in other situations it can lead up to incorrect result.

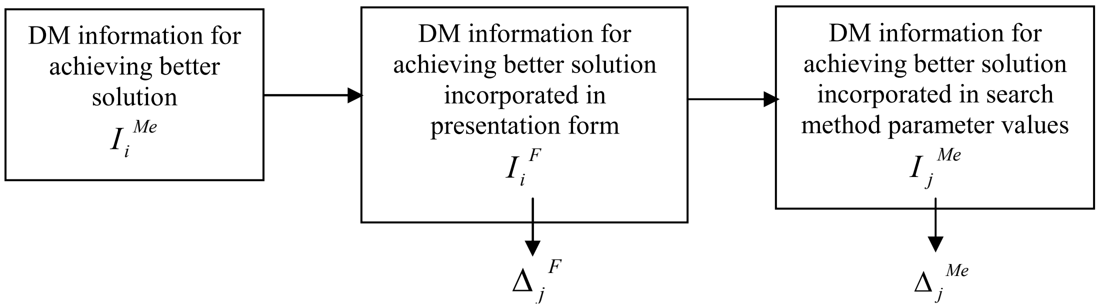

Figure 3. DM's information transformation process.

If we have an essential DM information loses Δ_j^F and Δ_j^{Me}, it means that we persecute DM activities. DM does not know, what part of his furnished information is not included in solutions search rules. DM seems that he receives inadequate information.

4. Decision making model in adaptive multicriterial optimization

In the iterative process of multicriterial optimization different DM used dissimilar tactics for the most preferable solution search. But each DM is characterized not only with his tactic. There is some DM behaviour features and knowledge level, which determine the quality of searching process:

α_1- DM's preliminary knowledge or grounding about potentialities of solutions;
α_2- DM's uncertainty when decision should be made;
α_3- DM's inconsistency in decision making process;

α_4- ability to adapt to information obtained in the solution process with the help of current results presentation form $F_z^{PR}(z=1,\ldots)$.

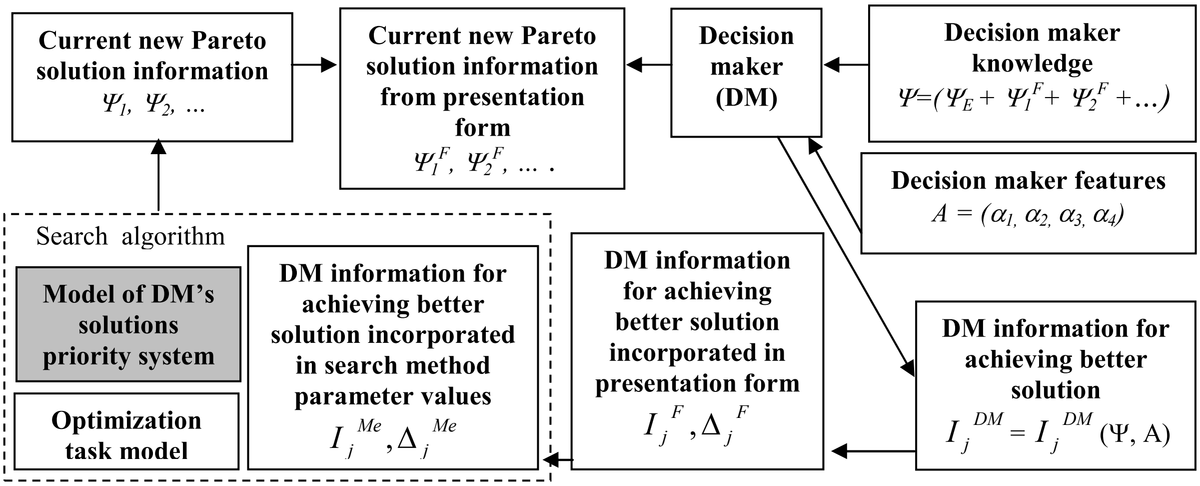

Figure 4. Multicriterial optimization decision making process.

Decision maker adjusts his system of criteria priorities according to newly obtained knowledge Ψ_j^F $(j = 1, \ldots)$, analyses it and makes a decision about further search (Figure 4). But his decisions frequently are not consistently. Sometimes it's needed to adjust these decisions. For that, it could be used DM model. The model "collects" DM decisions and tries to define the search trend. Knew DM wishes is compared with the trend and, if needs, a little adjustment take place. Search method must inform DM about this, otherwise DM could not interpret the results achieved in this search step. Calculation of trend could be used for the best solution forecasting. Use of such additional information could accelerate the process and allow to correct mistakes of information furnished by DM.

5. Testing of interactive and adaptive multicriterial optimization algorithms

Many interactive multicriterial optimization algorithms are worked out [16, 17]. They used different DM wishes data input forms, the best solution estimations representation forms and Pareto solutions search methods. The topic of the day is testing and comparison of multicriterial optimization. There are very few works devoted to this problem [4, 18, 19].

Table 1. Interactive solution process of multicriterial task.

x_1	x_2	$q_1(X)$	$q_2(X)$	$q_3(X)$	$q_4(X)$	$W(X, \Lambda)$	λ_1	λ_2	λ_3	λ_4	
6.40	**3.20**	**60.32**	**53.92**	**18.72**	**6.92**	**21.68**	**0.10**	**0.10**	**0.40**	**0.40**	**Goal**
3.90	1.30	10.37	43.97	54.77	37.57	25.33	0.60	0.10	0.20	0.10	
4.30	1.85	18.98	46.18	39.78	27.38	27.22	0.50	0.05	0.25	0.20	
5.62	2.57	39.55	45.64	26.21	12.17	26.88	0.30	0.05	0.30	0.40	
6.70	3.50	71.09	59.89	16.29	6.29	16.21	0.05	0.05	0.40	0.50	
6.50	3.00	56.25	48.25	22.25	6.25	20.65	0.15	0.05	0.30	0.50	
6.44	3.25	61.96	54.92	18.20	6.80	18.86	0.12	0.03	0.35	0.50	
6.25	3.10	56.50	52.50	19.50	7.60	21.35	0.15	0.05	0.35	0.45	

6.33	3.23	60.57	55.24	17.94	7.33	19.97	0.12	0.05	0.37	0.45	
6.61	3.00	57.25	47.49	22.81	5.71	21.72	0.12	0.10	0.33	0.45	
6.34	3.16	58.78	53.34	19.02	7.18	21.56	0.12	0.08	0.38	0.42	
6.36	3.32	63.10	57.34	16.86	7.38	19.74	0.10	0.06	0.40	0.44	

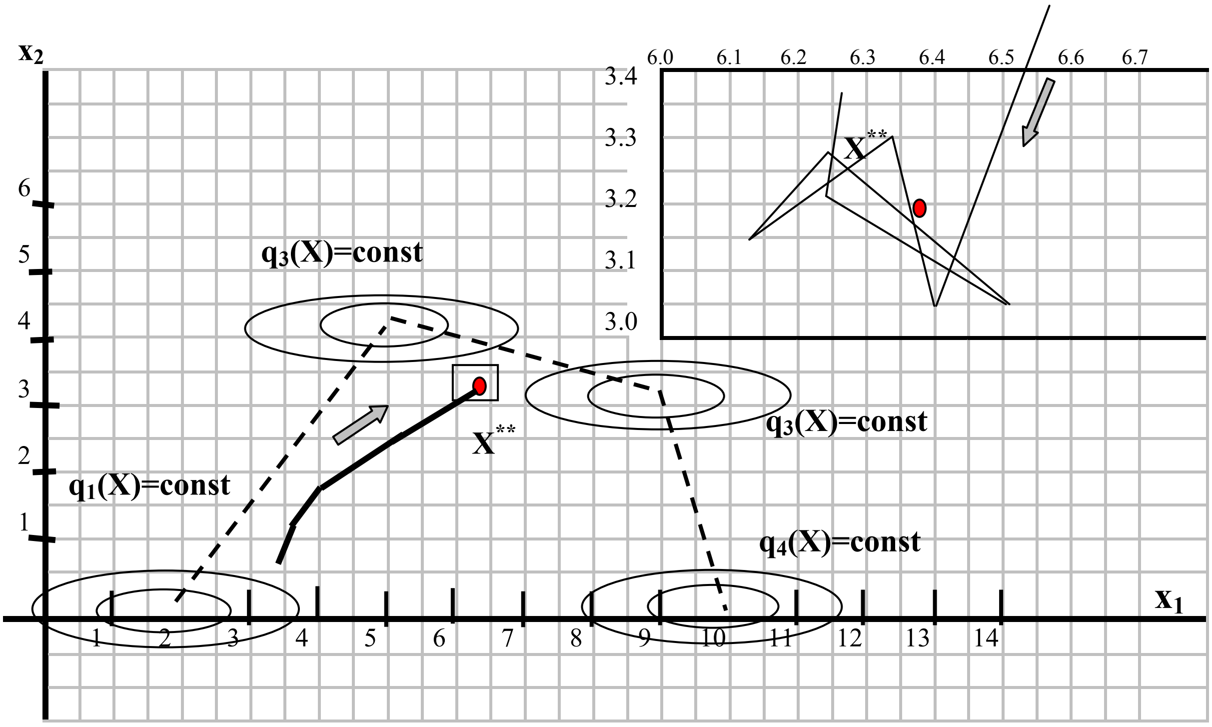

Figure 5. Geometrical representation of interactive solution process of multicriterial task.

On the bases of performed analyses of adaptive multicriterial optimization procedure and discussed decision making model, it is possibly to define testing procedure for interactive multriceterial optimization algorithms:

1) for the test task is detected Pareto solution, which is announced as goal of the task – the most preferable solution X**;
2) the DM wishes data input form is defined;
3) the Pareto solutions searching method is selected;
4) the DM formal model is formed. Model compares current iteration criteria and goal Q(X**) values and generates wishes for solution improvement. This model must simulate such DM behaviour features as: uncertainty, inconsistency and adaptation possibility. Changing the parameters values, which determine the utterance size of these features, it is possibly to test the working manner of algorithms with different types of DMs.

Figure 5 shows the graphical interpretation of multicriterial task with four square function criteria solution process. As a Pareto solution search method is used weighted sum method. Formal DM model generate criteria weights comparing goal values with criteria values in current iteration. Table 1 presents the searching results and parameters values.

5. Conclusions

Adaptive multicriterial optimization raises not only scalar and vector optimization mathematical methods problems, but makes someone to think about DM features, that have influence on decision making process. Understanding of this logic could help to investigate new multicriterial adaptive methods and improve existing. One of the "weak points" of multicriterial adaptive methods are forms for DM wishes data input and achieved estimations representation. Introduction of graphical schemes for information representation could essentially improve an information exchange between methods implementations and DM.

Worked out decision making model can help to solve very "painful" multicriterial adaptive optimization problem – testing and comparing of methods. The main problem is existence of DM, whose knowledge and characteristics must be identical every time we solve one and the same multicriterial optimization task.

On the ground of worked out decision making model arrangements are in train to implement the multicriterial adaptive optimization methods test polygon. The main element of it is formal DM model, which simulate many properties typical to real decision makers: preliminary knowledge, incompetence of potentialities, uncertainty and inconsistency in decision making, ability to adapt to new knowledge about optimization problem. It is not easy to evaluate values for these properties in real DM situation, but it is not a very essential problem for the methods testing.

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Zujevs A., Eiduks J. Lēmumu pieņemšanas modelis adaptīvās daudzkriteriālās optimizācijas procedūrā
Daudzkritēriālā optimizācija risina optimizācijas uzdevumus ar diviem vai vairākiem pretrunīgiem kritērijiem. Meklēšanas metodēm jānosaka risinājumi, kuri pēc lēmumu pieņēmēja personas (LPP) subjektīvajām domām ir vislabākie. Risinot adaptīvās daudzkritēriālās optimizācijas uzdevumus, notiek intensīva sadarbība ar LPP, kura optimizācijas laikā iegūst jaunu pieredzi un skatījumu uz optimizācijas uzdevumu. Adaptīvā pieeja daudzkriteriālās optimizācijas uzdevumu risināšanai apvieno iteratīvu risinājuma precizēšanas procesu ar divu veidu adaptācijām: LPP adaptējas optimizācijas uzdevumam un optimizācijas metode adaptējas LPP kritēriju prioritāšu sistēmai. Otrā tipa adaptācijas realizēšanai meklēšanas metodē tiek iekļauts LPP kritēriju prioritāšu sistēmas modelis. Modelis ļauj izvērtēt LPP sniegto informāciju un veikt precizējumus. Tas paātrina risināšanas procesu un nodrošina tā konverģenci. Darbā analizēta adaptīvās daudzkriteriālās optimizācijas procedūra un tās elementi. Izveidotais adaptīvās daudzkritēriālās optimizācijas procedūras modelis raksturo optimizācijas procesa posmus un to elementu savstarpējās saites, atkarību un nozīmību, kā arī raksturo dažādas problēmas un to iespējamos atrisinājumus. Modelis parāda ar kādām iespējamām problēmām var sastapties lēmumu pieņemšanas persona un optimizācijas metodes, izmantojot adaptīvo optimizācijas pieeju. Darbā izveidots lēmumu pieņemšanas procedūras modelis un noteikti faktori, kuri raksturo LPP darbību. Tas ļauj realizēt formālu LPP darbību simulējošu procedūru, kura var tikt izmantota daudzkriteriālās optimizācijas metožu testēšanai, tā atrisinot vienu no "sasāpējušākām" problēmām daudzkriteriālajā optimizācijā.

Zujevs A., Eiduks J. Model of decision making in multicriterial adaptive optimization procedure
Multicriterial optimization deals with optimization problem that include two or more contradictory criteria. The best compromise solution from decision maker (DM) viewpoint must be selected. Adaptive multicriterial approach implies two-way adaptation: DM adapts to optimization task, while optimization method adapts to DM criteria priorities system. During solving multicriterial optimization problem, intensive cooperation with DM and system, which implement algorithm, takes place. Created adaptive multicriterial optimization procedure model characterizes stages of optimization process and interrelations of its elements, their interdependence and importance, sheds light on different problems and possible methods that can be used to solve these problems. Decision process model for adaptive multicriterial optimization is developed in the work. Model can help to solve very "painful" multicriterial adaptive optimization problem – testing and comparing of methods. The main problem is existence of DM, who knowledge and characteristics are identical every time we solve one and the same multicriterial optimization task. On the ground of worked out decision making model, arrangements are in train to implement the multicriterial adaptive optimization methods test polygon. The main element of its is formal DM model, which simulate many properties typical to real decision makers: preliminary knowledge,

incompetence of potentialities, uncertainty and inconsistency in decision making, ability to adapt to new knowledge about optimization problem.

Зуев А., Эйдук Я. Модель принятия решений в процедуре адаптивной многокритериальной оптимизации

Многокритериальная оптимизация решает задачи, где существуют два и более противоречивых критериев качества и где необходимо найти наилучшие компромиссные решения по мнению лица принимающего решения. Адаптивный многокритериальный подход подразумевает двухсторонюю адаптацию, т.е., лицо принимающее решения (ЛПР) адаптируется к задаче оптимизации, в то время как метод оптимизации адаптируется к системе приоритетов критериев ЛПР. Созданная модель процедуры адаптивной многокритериальной оптимизации характеризует этапы процесса оптимизации, связи элементов модели, взаимозависимости и важность отдельных элементов модели, а так же характеризует различные проблемы и возможные варианты их решения. Модель описывает проблемы с которыми сталкивается ЛПР и методы поиска решения в процесе адаптивной оптимизации. В работе создана модель процесса принятия решений в адаптивной многокритериальной оптимизации. Модель является фундаментом для тестирования методов адаптивной многокритериальной оптимизации с использованием формальной модели ЛПР.