



FINITE ELEMENT MODELLING FOR STATIC BEHAVIOUR ANALYSIS OF SUSPENSION BRIDGES WITH VARYING RIGIDITY OF MAIN CABLES

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Abstract. The paper presents a summary of numerical analysis on static behaviour of suspension bridges with varying rigidity of cables. The primary purpose of this study was to compare suspension systems with flexible and rigid cables and to determine the influence of varying rigidity of cables on the response of bridge members under the action of uniformly distributed symmetrical and unsymmetrical static loading. The finite element analysis of a three-dimensional bridge model was performed. In the first model, the cable is modelled as TRUSS3D element, in the second model as BEAM3D element. In both models, the hangers and backstays are TRUSS3D elements and stiffening girder as BEAM3D element. It is shown that a suitable increase of main cable's bending stiffness can effectively reduce the displacements, internal forces and stresses of suspension systems. Recommendations for appropriate stiffness are given.

Keywords: suspension bridges, flexible cables, rigid cables, FEM, displacements, moments and stresses.

1. Introduction

Suspension bridges are very attractive structures possessing a number of technical, economical and aesthetic advantages. The main problem of them is their excessive deformability, especially under action of traffic loads, changes in environmental temperature and the wind effects. Deformability of suspension systems depends on the kinematical character of displacements of a flexible suspension cable. Various structural measures are taken to improve the stiffness and stability of suspension structures (Juozapaitis *et al.* 2006, 2007; Krishna 2001; Palkowski 2006).

As is shown in our previous investigations, one of the ways to increase the rigidity of a bridge suspension system is to transfer a part of stiffening girder's rigidity to a suspension cable (Grigorjeva *et al.* 2004; 2006). The cables can be made of standard steel profiles or have a composite section. The steel hollow sections filled with concrete can be an attractive solution (Kuranovas, Kvedaras 2007; Soundararajan *et al.* 2008). By changing the cable to stiffening girder bending stiffness ratio it is possible to reduce considerably the vertical displacements of the loaded suspension system and to retain its original geometric form. The stabilization of the deformability of the suspension bridges by giving a certain bending stiffness to the sus-

pension cables are also known in practice. The examples of suspension bridge in Pittsburgh (Качурин *et al.* 1971) and famous Tower Bridge in London (Bennett 1997) can be mentioned.

The analysis of suspension bridges with flexible cables is based on the deflection theory and solution of the differential Eqs. In all cases, the existing well-known theories of suspension system's analysis are tedious and involve several operations and approximations with complex numbers. In an analysis of suspension bridges, main cables are generally assumed to have no flexural stiffness and to be subject to axial tension only (Gimsing 1997).

Although several research papers focus on the bending problem of suspension or stay cables (Furst *et al.* 2001; Prato, Ceballos 2003), to the best of the author's knowledge little information on theoretical analysis of suspension bridges with rigid cables is available. Some results of influence of the main cable stiffness on the deformability of suspension bridge decks are presented in references (Качурин *et al.* 1971; Ladret *et al.* 2002). A simplified engineering analytical method and design of suspension bridges with varying rigidity of cables is proposed by the authors (Grigorjeva *et al.* 2006). An analysis shows that there is a close relationship between the rigidity of the main cable, its layout, the character of applied load, and

Table 1. Geometric characteristics of cable and stiffening girder

$\xi = EI_c/EI_g$	Cable section, $b \times h \times t$, mm	Cable tension rigidity, EA_c , kN	Cable flexural rigidity, EI_c , kNm ²	Stiffening girder section $b \times h \times t$, mm	Stiffening girder rigidity EI_g , kN/m ²	Amount of steel, kg
0						
0.1	100×400×25	4 725 000	74 484	400×700×25	718 922	117 750
0.2	100×480×25	5 565 000	123 967	400×660×25	622 213	120 890
0.4	120×560×25	6 615 000	206 296	400×610×25	513 219	124 815
0.6	120×620×25	7 245 000	273 627	400×580×25	453 912	127 170
0.8	120×650×25	7 560 000	312 112	400×540×25	381 647	126 385
1.0	120×670×25	7 770 000	339 650	400×510×25	332 374	125 600

the displacements of the stiffening girder. It is possible to reduce deformability of suspension systems substantially by variation of the flexural stiffness of the main cables.

From a mechanics standpoint, suspension systems present a complex hyperstatic behaviour. It was clear that there was a need to use more refined method of analysis based on finite element modelling (FEM) as the only way to confirm fully the structural concept and to check the overall behaviour of the suspension system with rigid cables. Two models were developed: the first one with flexural cable and the other with cables of varying rigidity. Two and three-hinged rigid cables were considered. In this paper, the results of such analysis are presented.

2. FE model

Analysis of suspension bridges was performed with a relatively simple FE model using commercial finite element programs CosmosM and Midas/Civil. The three-dimensional models, consisting of beam and bar elements, were constructed to examine the overall behaviour of the bridge system (Fig. 1). The models were developed using the following assumptions:

- Bar elements TRUSS3D were used to represent the flexible cable (Model 1), the hangers and backstays;
- Beam elements BEAM3D were used for the rigid cables (Model 2) and stiffening beam;
- The girder deck and the cables are hinged at the tower footings and tower saddles, respectively;
- The deck ties (if any) are excluded from analysis;
- The main rigid cable, stiffening girder, backstays are steel box profiles with the Young's modulus $E = 2.1 \times 10^5$ N/mm² and Poisson's ratio $\nu = 0.3$; suspenders are steel circular rods with the tension rigidity $EI = 412\,125$ kN; pylons are wide-flange shapes with compression rigidity $EA_{pyl} = 1\,274\,280$ kN;
- The cable possesses geometrical non-linearity.

**Fig. 1.** Finite element space model of the bridge

The model studies had several objectives:

- Investigate the deflections, internal forces and stresses caused by uniformly distributed static loads;
- Study the influence of cable to girder stiffness ratio and live to dead load ratio on behaviour of suspension system;
- Compare the behaviour of suspension system with flexible and two- and three-hinged rigid cables.

Tables 1 and 2 summarize the geometric and loading characteristics of bridge models used in analysis. The basic structural system is shown in Fig. 2.

Table 2. Loading characteristics

$\gamma = v/g$	Live load v , kN/m ²	Permanent load g , kN/m ²
1.0	5.0	5.00
1.5	5.0	3.33
2.0	5.0	2.50
2.5	5.0	2.00
3.0	5.0	1.66

3. Structural behaviour

The suspension bridge models with flexible and rigid cables were analyzed through the comparison of vertical displacements, bending moments and normal stresses under the action of symmetrical and unsymmetrical static distributed loadings. These parameters were done, when the flexural stiffness is imposed on the cables from the very beginning of bridge erection, i.e. before it is loaded by the permanent and variable loads.

3.1. Symmetrical loading

In Figs 3 and 4 the main parameters are shown for the two studied system cases.

The vertical displacements, moments and stresses in the stiffening girder are reduced due to participation of the rigid cable, as part of the suspension system. The magnitude of reduction will depend on the bending stiffness of the main cables. The main reduction of displacements and moments at mid-span by about 30 and 70%, respectively, is observed when the stiffness ratio ξ is up to 0.5–0.6. It seems that the bending stiffness of the stiffening girder should be about two times higher than the flexural stiffness of the rigid cables. On the other hand, the moments

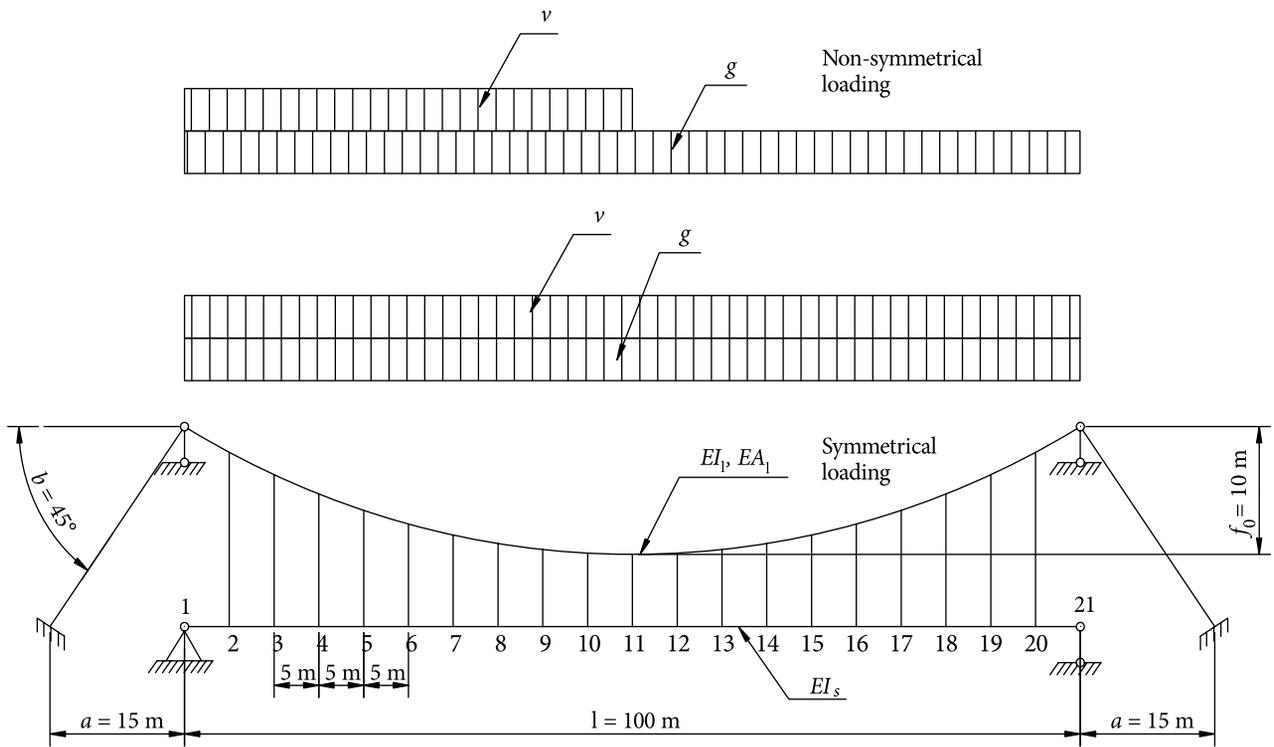


Fig. 2. Structural system of a bridge model and loading configuration

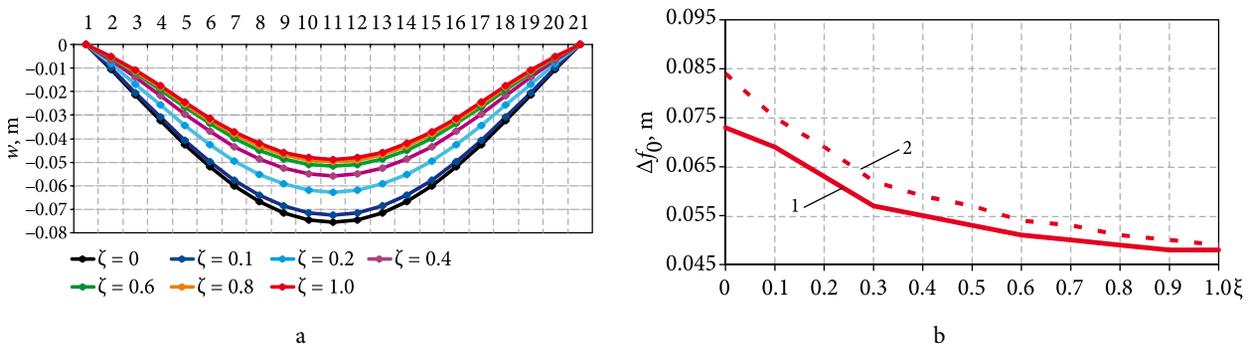


Fig. 3. Vertical displacement curves (a) and mid-span deflection (b) of the stiffening girder under symmetrical loading ($\gamma = 1$): 1 – two-hinged cable; 2 – three-hinged cable

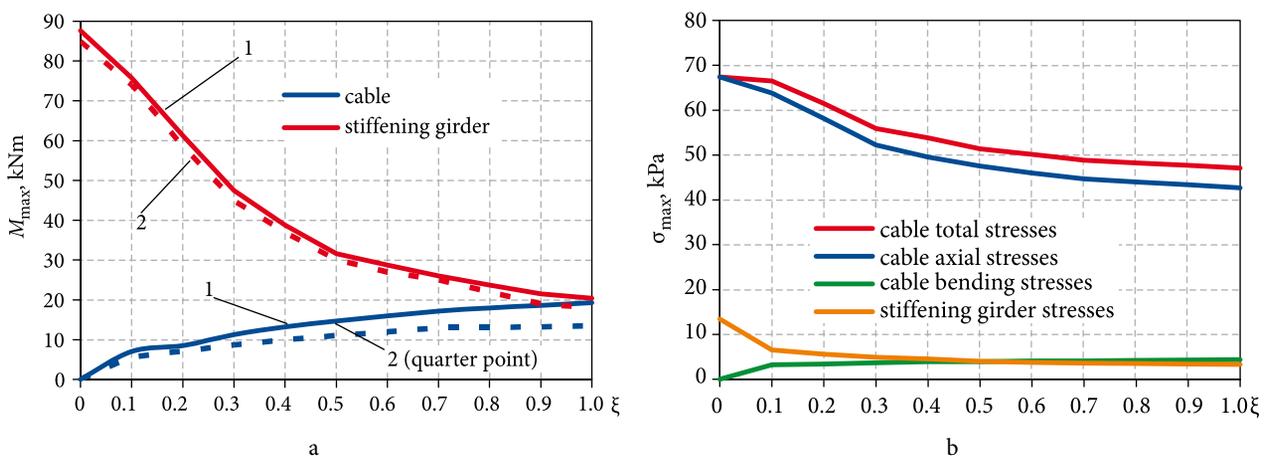


Fig. 4. Max moments (a) and stresses (b) in the midspan of the stiffening girder and cable under symmetrical loading ($\gamma = 1$): 1 – two-hinged cable; 2 – three-hinged cable

and bending stresses are increasing in the cables, when the stiffness ratio ξ is increased. However, these moments and stresses are relatively small. The main action effect in the rigid cable remains its axial force.

Comparison of the suspension system with two and three-hinged cables shows that displacements follow the same pattern. Max displacements and bending moments in the stiffening girder for the system with three-hinged cable are up to 10% lower (Figs 3, 4). The max moment in the three-hinged cable is in the quarter point of the span

and is up to 30% (depending of the values of ζ) lower, than that in the mid-span of the two-hinged cable (Fig. 4).

Fig. 5 shows the finite element results for one of the models of suspension bridge.

3.2. Unsymmetrical loading

Fig. 6 shows the displacements of stiffening girder under the action of unsymmetrical live loading as a function of the ratio ξ and γ . Under the action of unsymmetrical loading, deflection curves become S-shaped. Such a stiffening girder is found to be essential lack of suspension bridges.

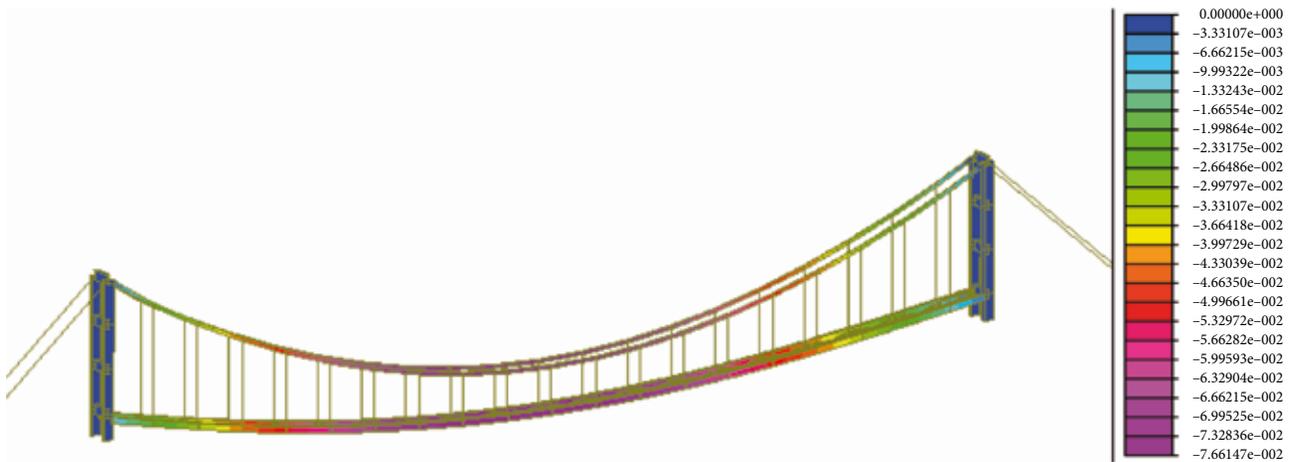


Fig. 5. Deformed FEM of the bridge under symmetric loading ($\xi = 0.6, \gamma = 1$)

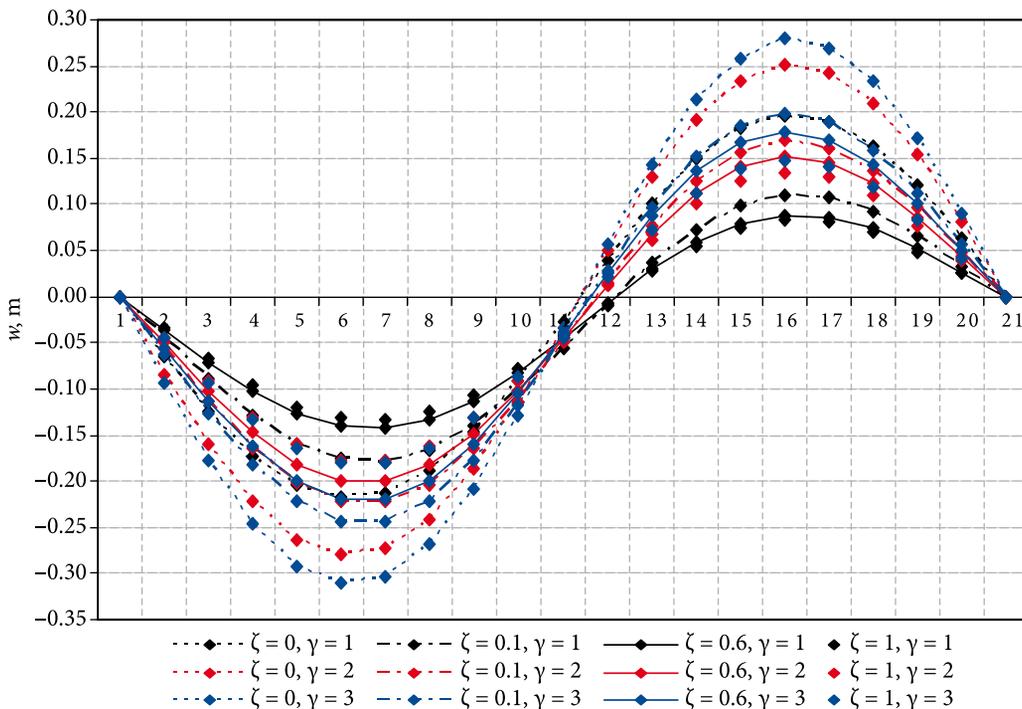


Fig. 6. Vertical displacement curves under unsymmetrical loading of two-hinged system

The largest displacements occur in suspension bridges with flexible cable ($\xi = 0$). If the ratio ξ increases, the vertical displacements of the girder decrease leading to considerable reduction of suspension system's deformability. Referring to Fig. 7, the useful limit value of stiffness ratio ξ is somewhat of 0.5–0.6. When the ratio $\zeta = 0.6$, the reduction of the vertical displacements of the left quarter point of the span is on average of 30% and that of right quarter point of 60%. The higher ratio ξ has only a little influence on vertical displacements.

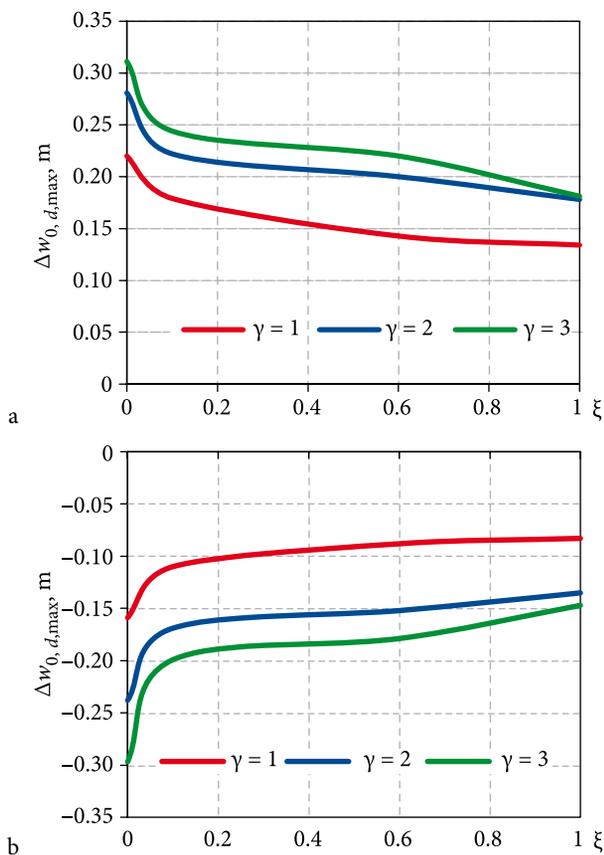


Fig. 7. Vertical deflection in the left (a) and right (b) quarter points of the span in the stiffening girder under unsymmetrical loading

The rigidity of cables also influences the bending moments and stresses in the stiffening girder and cables as illustrated in Figs 8 and 9.

As can be expected, the bending moments in the stiffening girder decrease and in the rigid cable increase with the rigidity ratio, ζ . Referring to Fig. 9, it should be noted that the max stresses change rapidly when ζ increases from 0 to 0.1 but the higher values of ζ appear to have little effect on the stresses.

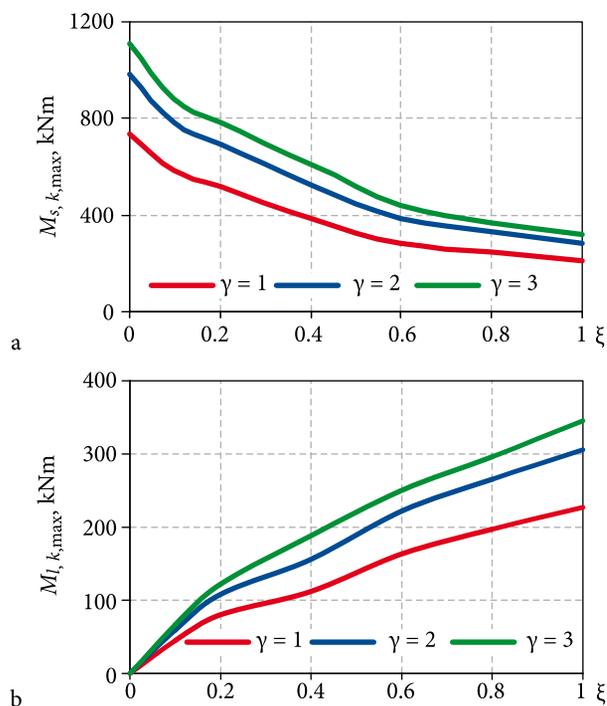


Fig. 8. Max moments at the quarter point in the stiffening girder (a) and cable (b) under unsymmetrical loading

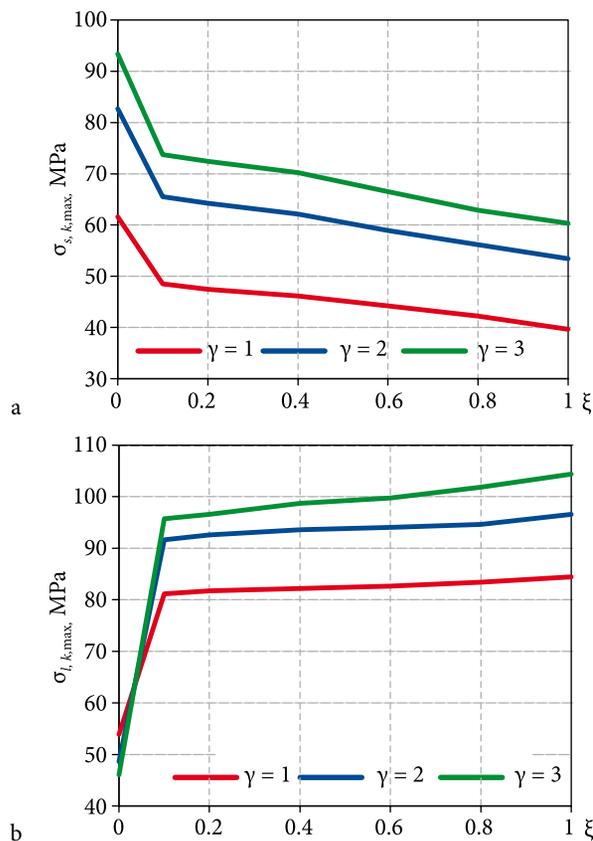


Fig. 9. Max stresses at the quarter point in the stiffening girder (a) and cable (b) under unsymmetrical loading

Fig. 10 shows the influence of hinges in the rigid cable on the bending moments of the members as a function of the ratio γ ($\zeta = 0.6$). It is seen that internal hinge slightly reduces the vertical displacements (up to 20%) and max bending moments (up to 10%) in the stiffening girder and cable.

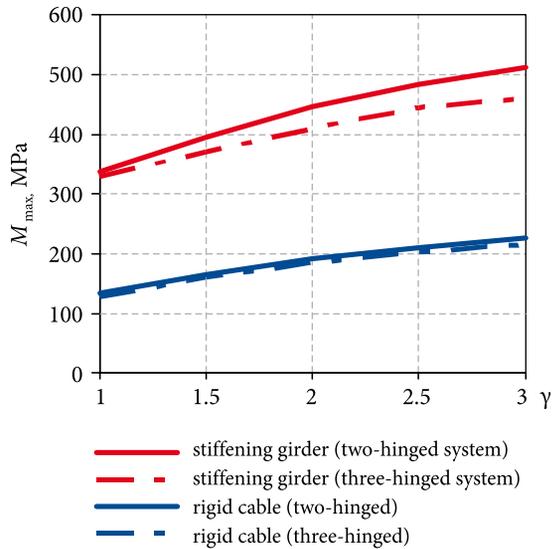


Fig. 10. Comparison between the max bending moments of the stiffening girder and rigid cable in two and three-hinged system vs ratio γ

Fig. 11 shows the deformed FEM of the bridge under unsymmetrical loading.

4. Influence of cable shape

The original outline of the cables of a final bending stiffness is very important factor in the behaviour of suspension bridges. It influences the stability of the main cables as well as the structural performance of a whole bridge.

The influence of the initial shape of the axis of the three-hinge cables was analyzed on the bridge’s model with varying initial sag from $f_1 = 0.25 f_0$ up to $f_5 = 0$ in a quarter of the bridge span (Fig. 12).

All the cables illustrated in Fig. 12 would deform in a similar way under the action of the load, but the actual displacements of the stiffening girder would be different.

Fig. 13 shows the displacements of a stiffening girder with various initial shapes of the main cable under the action of symmetrical and unsymmetrical uniformly distributed loading. There is a close relationship between the rigidity of main cable, the original shape of the rigid cable, and the displacements of the stiffening girder.

For a uniformly loaded bridge deck, the shape of the rigid cable is ideally parabolic. Min displacements are developed in the stiffening girder.

Under action of unsymmetrical loading the efficiency of the initial shape of the cables becomes obvious. The displacements of stiffening girder form S-shape curve. By changing the initial shape or curve of a cable, it is possible to reduce or almost eliminate the upward displacement curve. For example, if initial sag of a suspension cable in a quarter span is $f_4 = 0.0625 f_0$, then the shape of deflections in the loaded part of stiffening girder is similar to a parabola. At the part of stiffening girder free of loading very small deflections are observed.

It should be pointed out that the shape of the cable axis may be governed by the distribution and magnitude of the superimposed loads. On the other hand, curved cable can be replaced by a combination of straight members.

5. Conclusions

The main behaviour problem of suspension bridges is their excessive deformability, especially under the action of traffic loads, changes in environmental temperature and the effects of wind. The use of cables with finite bending stiffness is proposed by the authors of the present paper with the aim to increase the rigidity of suspension bridges.

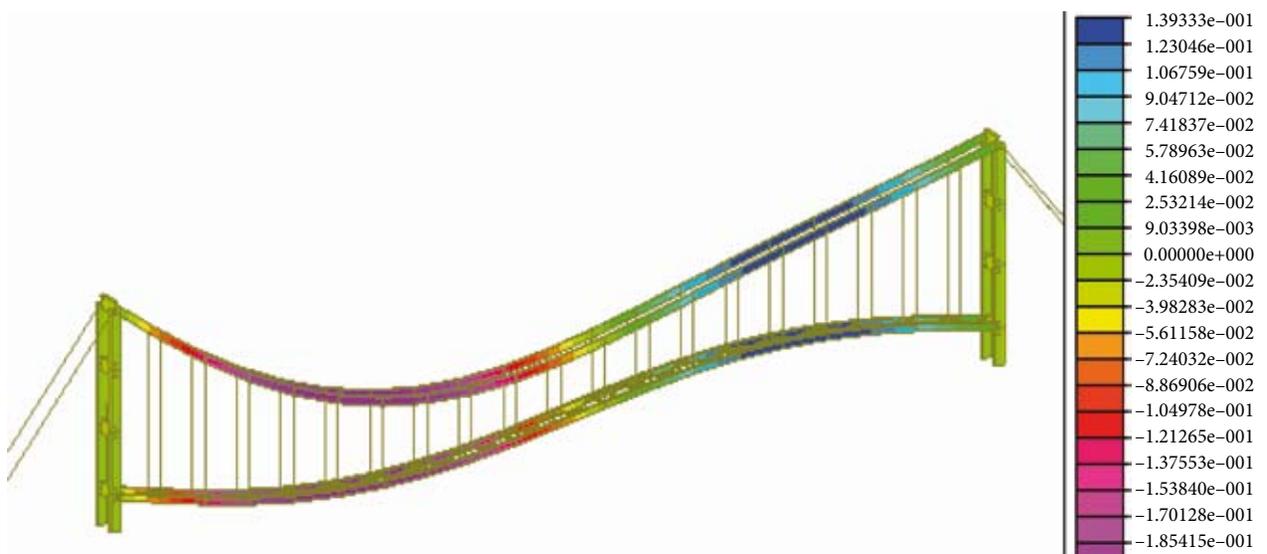


Fig. 11. Deformed FEM of the bridge under unsymmetrical loading ($\xi = 0.6, \gamma = 1$)

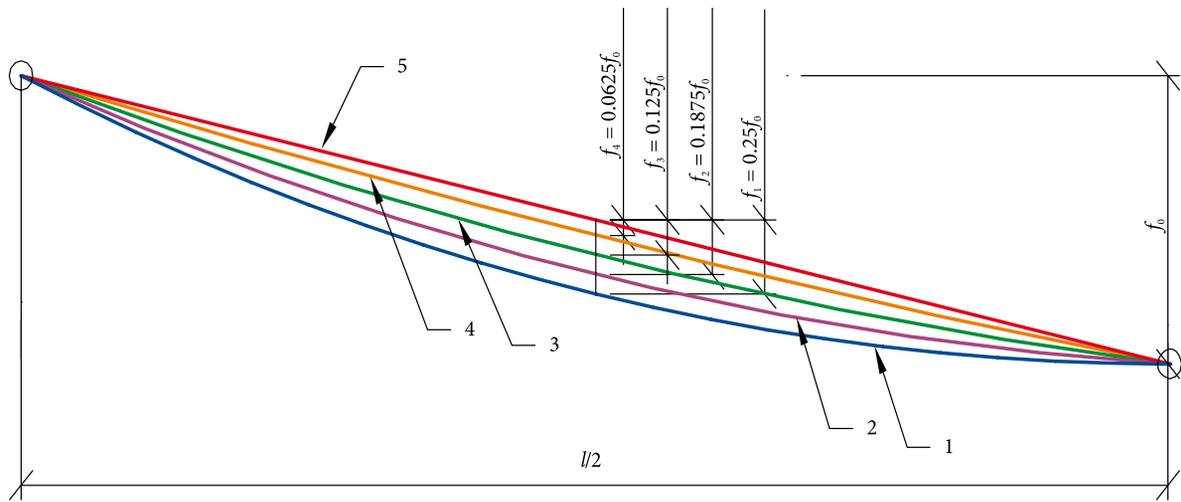


Fig. 12. Geometrical shapes of a rigid cable in a half of bridge span: 1 - $f_1 = 0.25 f_0$; 2 - $f_2 = 0.1875 f_0$; 3 - $f_3 = 0.125 f_0$; 4 - $f_4 = 0.0625 f_0$; 5 - $f_5 = 0$

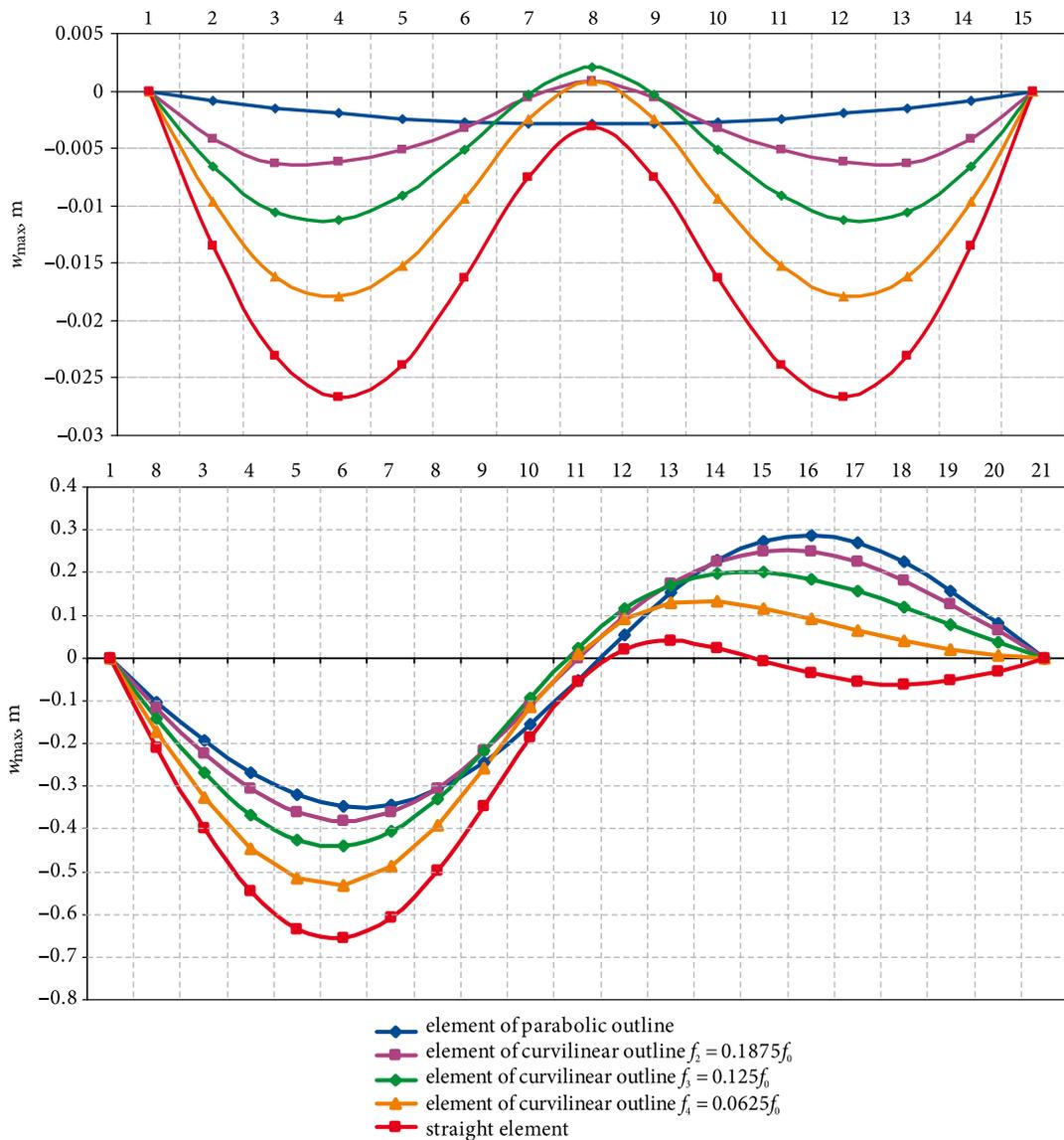


Fig. 13. Displacements of stiffening girder of suspension bridge with various initial shapes of the main cable under action of symmetrical (top) and unsymmetrical (bottom) loading

Finite element analysis using commercial finite element programs CosmosM and Midas/Civil of the suspension bridge models was performed. Two finite element space models were developed to simulate suspension system with flexible and rigid two and three-hinged main cables. The suspension bridge models were analyzed through the comparison of vertical displacements, bending moments and normal stresses under the action of symmetrical and unsymmetrical static uniformly distributed loads.

By the results of the simulation analysis performed it has been shown that vertical displacements, moments and stresses in the stiffening girder are reduced due to participation of the rigid cable as a part of the suspension system. The magnitude of reduction varies with the bending stiffness and shape of the cable, loading characteristics and presence of internal hinge in the cable. Loading effects depend to a large extent on the cable to girder stiffness ratio, ζ . The rational cable to girder stiffness ratio is somewhat 0.5–0.6. The vertical deflections can be reduced up to 30% and the bending moments up to 70%.

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