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THE RESEARCH OF STABILITY OF SYNCHRONIZATION  
PROCESS WITH MATHEMATICAL MODEL'S  
APPLICATION OF TWO GENERATORS

DIVU ĢENERATORU SINHRONIZĀCIJAS PROCESA  
STABILITĀTES IZPĒTE IZMANTOJOT MATEMĀTISKO  
MODELĒŠANU

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## Introduction

Modern electro technical progress now is determinable wherewith developing and initiating safe and economical alternating current electrical equipment is simply maintenance.

When we develop and initiate to new electrical equipments, we have too mach exploitation and other kind of problems, who are needed for theoretic and practice solutions.

The synchronization process theory is progress now and it makes possible to develop practice assignments.

Nonetheless the concept about synchronization process in technical literature is describing incomplete [1, 2].

That is why it is becoming more and more important to estimate synchronization conditions of generators. For check of stability of any synchronization method it is necessary to determine first of all allowable angular speed of rotor sliding at the connected machine, and at exact or rough synchronization with restriction of an angle of switch-on and a limiting angle of switch-on. For opinion about size of the currents proceeding in system during synchronization, it is necessary to find an angle on which the generator deviates at switch-on, and also to analyze influence of superfluous moment changes, rotation frequency and excitation on synchronization process.

Application with mathematical models allows envelop all basic calculation complexes with synchronous generator's current and electromagnetic moment.

For correct description regime parameters fluidity and electrical energy quality indices, mathematic model have to take need for not only generators electromechanical process regularity, but also energetic engine (prime engine) mechanical process.

The most interesting is generators synchronization process with limiting power system. At that publication is offering power system model that contains of two synchronous generators with primary engines.

This question wasn't so completely consider in the publication "Model of synchronous generators switching on the ship network" [5].

Systems load modulation grapple with statically active – inductive load (Fig.1.).

Synchronization process modulating condition components from two steps:

- First step - SG1 generator stationary condition calculation to active – inductive load;
- Second step - SG2 generator connection withal synchronization conditions noncompliance.

For an estimation of synchronization stability the mathematical model is developed. The model includes the synchronous generator model with use of the full differential Park-Gorev's equations and the statically active – inductive load's equations [3, 4].

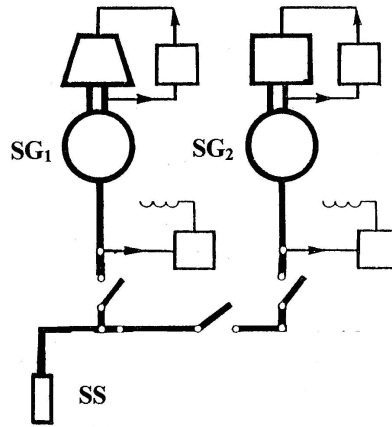


Figure1. Model's schema

The synchronous generator's mathematical model in axes d, g, 0 rotational with rotor angular frequency is:

$$\left. \begin{aligned}
 u_d &= \frac{d\Psi_d}{d\tau} - \Psi_q \omega + i_d R; \\
 u_q &= \frac{d\Psi_q}{d\tau} + \Psi_d \omega + i_q R; \\
 u_f &= \frac{d\Psi_f}{d\tau} + i_f R_f; \\
 0 &= \frac{d\Psi_D}{d\tau} + i_D R_D; \\
 0 &= \frac{d\Psi_Q}{d\tau} + i_Q R_Q; \\
 T_M \frac{d\omega}{d\tau} &= M_T - (\Psi_q i_d - \Psi_d i_q).
 \end{aligned} \right\} \quad (1)$$

Where

$$\Psi_d = X_d i_d + X_{ad} i_f + X_{ad} i_D;$$

$$\Psi_q = X_q i_q + X_{aq} i_Q;$$

$$\Psi_f = X_{ad} i_d + X_f i_f + X_{ad} i_D;$$

$$\Psi_D = X_{ad} i_d + X_{ad} i_f + X_D i_D;$$

$$\Psi_Q = X_{aq} i_q + X_Q i_Q;$$

Substituting (1) we shall receive the modeling algorithm of the synchronous generator in alternating currents. In the matrix form we shall receive:

$$\begin{aligned} \frac{d}{d\tau} \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_D \\ i_Q \end{bmatrix} &= \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 \\ 0 & 0 & 0 & Q_4 & 0 \\ 0 & 0 & 0 & 0 & Q_5 \end{bmatrix} \times \begin{bmatrix} u_d \\ u_q \\ u_d \\ u_d \\ u_q \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \times \begin{bmatrix} i_d \\ i_q \\ i_f \\ i_D \\ i_Q \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} = \\ &= \begin{bmatrix} Q_1 & 0 & 0 & 0 & 0 \\ 0 & Q_2 & 0 & 0 & 0 \\ 0 & 0 & Q_3 & 0 & 0 \\ 0 & 0 & 0 & Q_4 & 0 \\ 0 & 0 & 0 & 0 & Q_5 \end{bmatrix} \times \begin{bmatrix} u_d \\ u_q \\ u_d \\ u_d \\ u_q \end{bmatrix} + \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \\ H_5 \end{bmatrix} \end{aligned} \quad (2)$$

$$\frac{d\omega}{d\tau} = \frac{1}{T_M} [M_{PD} - (\Psi_q i_d - \Psi_d i_q)];$$

$$Q_1 = \frac{x_D x_f - x_{AD}^2}{\Delta d}; Q_2 = \frac{x_Q}{\Delta q}; Q_3 = -\frac{x_{SD} x_{ad}}{\Delta d}; Q_4 = -\frac{x_{sf} x_{ad}}{\Delta d}; Q_5 = -\frac{x_{aq}}{\Delta q};$$

$$\Delta d = x_d x_f x_D - x_{ad}^2 (x_d + x_D + x_B - 2x_{ad}); \Delta q = x_q x_Q - x_{aq}^2;$$

$$a_{11} = -r_a Q_1, a_{12} = x_q \omega Q_1, a_{13} = -r_f Q_3, a_{14} = -r_D Q_4, a_{15} = x_{aq} \omega Q_1;$$

$$a_{21} = -\omega x_d Q_2, a_{22} = -r_a Q_2, a_{23} = -x_{ad} \omega Q_2, a_{24} = -x_{ad} \omega Q_2, a_{25} = -r_Q Q_5;$$

$$a_{31} = -r_a Q_3, a_{32} = x_q \omega Q_3, a_{33} = -\frac{x_d x_D - x_{ad}^2}{\Delta d} r_f, a_{34} = \frac{x_s x_{ad}}{\Delta d} x_D, a_{35} = x_{aq} \omega Q_3;$$

$$a_{41} = -r_a Q_4, a_{42} = x_q \omega Q_4, a_{43} = \frac{x_s x_{ad}}{\Delta d} r_f, a_{44} = -\frac{x_d x_B - x_{ad}^2}{\Delta d} r_D, a_{45} = x_{aq} \omega Q_4;$$

$$a_{51} = -x_d \omega Q_5, a_{52} = -r_a Q_5, a_{53} = -x_{ad} \omega Q_5, a_{54} = -x_{ad} \omega Q_5, a_{55} = -\frac{x_q}{\Delta q} r_Q;$$

$$B_1 = Q_3 U_B, B_2 = 0, B_3 = -\frac{x_d x_D - x_{ad}^2}{\Delta d} U_B, B_4 = \frac{x_s x_{ad}}{\Delta d} U_B, B_5 = 0;$$

Focal point voltage by all connected elements could be calculated at active – inductive load.

$$[u_1] = -[L_{StL}] \frac{d}{dt} [I_{StL}] - [Z_{StL}] \times [I_{StL}] \quad (3)$$

Where:  $[u_1] = -\begin{bmatrix} u_{d1} \\ u_{g1} \end{bmatrix}; [L_{StL}] = \begin{bmatrix} X_{StL} & 0 \\ 0 & X_{StL} \end{bmatrix}; [Z_{StL}] = \begin{bmatrix} R_{StL} & -X_{StL} \\ X_{StL} & R_{StL} \end{bmatrix}.$

If we put to use full elements equation system, it combines to systems (1) at the synchronous generator stator circuit equation system. The equation at the first Kirchhoff law for focal point 1 can be written down:

$$[I_{SG1}] + [I_{SG2}] = [I_{StL}] \quad (4)$$

At mathematical model we use a full synchronous generators equation, that's why currents are uninterrupted differential function, and equation (3) will be differentiated:

$$\frac{d[I_{SG1}]}{dt} + \frac{d[I_{SG2}]}{dt} = \frac{d[I_{StL}]}{dt} \quad (5)$$

To put to use first two equations for stator system current fluxion (2), we will write it's at general form:

$$\frac{d[I_{SGi}]}{dt} = [Q_{SGi}] \times [U_1] + [H_{SGi}] \quad (6)$$

At equation (3) instead  $\frac{d[I_{StL}]}{dt}$  we use items from (5) and put it at equation (6) we shall write:

$$[U_1] = -[L_{StL}] \times [m_{SG1} Q_{SG1} + m_{SG2} Q_{SG2}] \times [U_1] - [L_{StL}] \times [m_{SG1} H_{SG1} + m_{SG2} H_{SG2}] - [Z_{StL}] \times [m_{SG1} I_{SG1} + m_{SG2} I_{SG2}] \quad (7)$$

If we transfigure it we shall receive:

$$([1] + [L_{StL}] \times [m_{SG1} Q_{SG1} + m_{SG2} Q_{SG2}]) \times [U_1] = -[L_{StL}] \times [m_{SG1} H_{SG1} + m_{SG2} H_{SG2}] - [Z_{StL}] \times [m_{SG1} I_{SG1} + m_{SG2} I_{SG2}] \quad (8)$$

If we develop (8) equation respectively to  $[U_1]$ , we will have systems voltage value for synchronous generators model.

$$[U_1] = ([1] + [L_{StL}] \times [m_{SG1} Q_{SG1} + m_{SG2} Q_{SG2}])^{-1} \times (-[L_{StL}] \times [m_{SG1} H_{SG1} + m_{SG2} H_{SG2}] -$$

$$- [Z_{sL}] \times [m_{SG1} I_{SG1} + m_{SG2} I_{SG2}] \quad (9)$$

At the modulated system synchronous generators parameters are at own relative quantity. That is why it necessary to take into account (10) condition.

$$\sum m_{SGi} = 1 \quad \text{That is:} \quad m_{SG1} + m_{SG2} = 1 \quad (10)$$

For work conditions it was accepted the synchronization rate following algorithm:

1. Mixed statically load switching on generator number one. At this stage  $m_{SG1} = 1.0$ ;
2. At one generator with load switching on second generator. At this stage  $m_{SG1} = 0.9$ , but  $m_{SG2} = 0.1$ .

As well for synchronization switching on possibility switching goes on us don't count exact synchronization condition. You can see dependence at figures from 2 till 5.

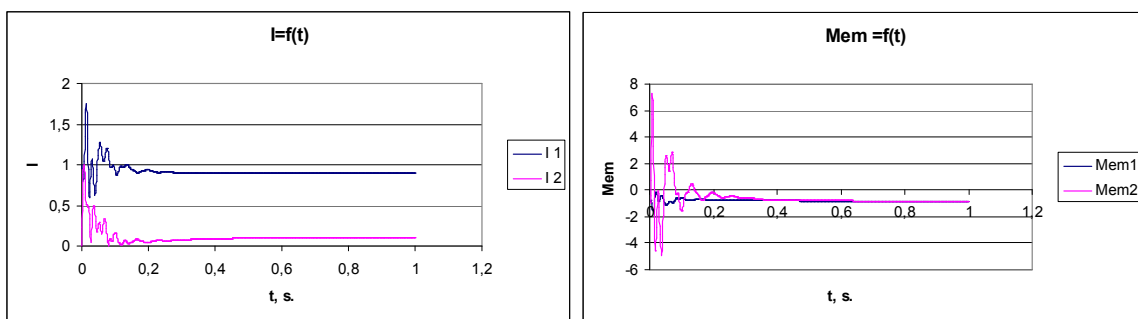


Figure 2. Results of two generators synchronization currents in time:  
I<sub>1</sub>- first generator,  
I<sub>2</sub>- second generator.

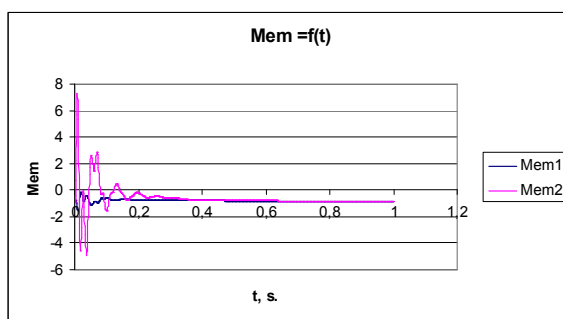


Figure 3. Results of two generators synchronization moments in time:  
Mem<sub>1</sub>- first generator,  
Mem<sub>2</sub>- second generator.

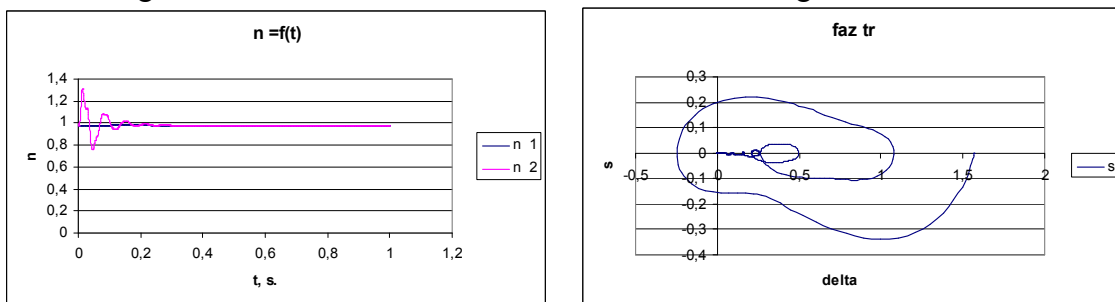


Figure 4. Results of two generators synchronization frequency in time:  
n<sub>1</sub> – first generator,  
n<sub>2</sub> – second generator.

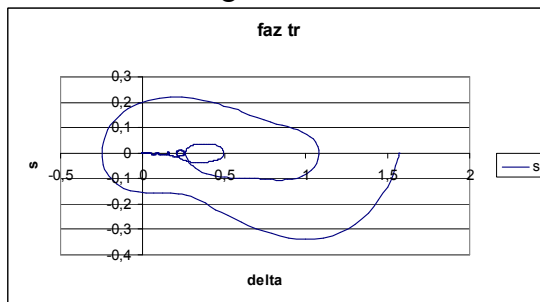


Figure 5. Results of two generators synchronization total phase's trajectory.

## Conclusions

1. It is developing methodology, what can analyze possibility synchronous generator switching on parallel work with limited power system at deflection from ideals synchronization conditions.
2. It is shown, what the main character at this process is frequency cutting regulator setup, because frequency and phase deflection to generate voltage have a substantially effect on synchronization process.
3. Calculations results makes possible develop electro energetically automatically synchronization limited power system.

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*Bērziņa K., Ketnere E., Ketners K., Latve I. Divu ģeneratoru sinhronizācijas procesa stabilitātes izpēte, izmantojot matemātisko modelēšanu Pašreizējais elektrotehniskais progress ir virzīts uz ekonomisku, drošu un viegli apkalpojamo maiņstrāvas elektroiekārtu izstrādi un ieviešanu. Jauno iekārtu ieviešanas un ekspluatācijas laikā parādās dažādi sarežģīti uzdevumi, kas jārisina kā praktiski, tā arī teorētiski.*

*Darbā tiek aplūkoti līdzvērtīgas jaudas sinhrono ģeneratoru sinhronizācijas procesu matemātiskās modelēšanas jautājumi. Pētot pārejas procesus, ir izstrādāts ierobežotas jaudas sistēmas matemātiskais modelis, ieskaitot sinhrono ģeneratoru, kurš baro aktīvi induktīvo slodzi un papildu ģeneratoru, kas tiek paralēli pieslēgts sistēmai pie novirzes no ideālajiem sinhronizācijas nosacījumiem. Uz izstrādātā modeļa pamata ir veikta sinhronizācijas procesa analīze.*

***Berziņa K., Ketner E., Ketner K., Latve I. The research of stability of synchronization process with mathematical model's application of two generators***

*Modern electro technical progress now is determinable wherewith developing and initiating safe and economical alternating current electrical equipment is simply maintenance.*

*When we develop and initiate to new electrical equipments, we have too mach exploitation and other kind of problems, who are needed for theoretic and practice solutions. In the present work are considered questions of mathematical modelling of comparative power synchronous generators transient processes .For transient processes research is developing limited power system mathematical model including: synchronous generator with active – inductive load and collateral generator which is switching on parallel work to limited power system with deviation from ideal synchronization conditions. The model includes the synchronous generator model with use of the full differential Park-Gorev's equations. On the basis of the developed model are lead comparison and the analysis of influence of synchronous generator's cutting frequency regulator to synchronization process.*

***Берзина К., Кетнер К., Кетнер Э., Латве И. Исследование стабильности процесса синхронизации двух генераторов, используя математическое моделирование***

*Прогресс современной электротехники обусловлен созданием и широким внедрением экономичного и надежного электрооборудования переменного тока, отличающегося простым обслуживанием.*

*При внедрении и эксплуатации новых образцов электрооборудования приходится сталкиваться с многими трудными задачами, требующими теоретического и практического решения.*

*В настоящей работе рассматриваются вопросы математического моделирования переходных процессов синхронизации синхронных генераторов соизмеримой мощности. Для исследования переходных процессов разработана математическая модель системы ограниченной мощности, включающая в себя синхронный генератор, питающий статическую активно-индуктивную нагрузку и дополнительный генератор, подключаемый на параллельную работу к системе при отклонении условий синхронизации от идеальных. В основу модели положены уравнения Парка-Горева синхронного генератора. На основании разработанной модели проведен анализ процесса синхронизации с учетом влияния на процесс регуляторов частоты вращения синхронных генераторов.*