USING GRANULAR-EVIDENCES-BASED ADAPTIVE NETWORKS FOR SENSITIVITY ANALYSIS*

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1. Introduction

Sensitivity analysis helps to determine what does and what does not make a difference in the decision making process. Traditional sensitivity analysis involves graphical methods that are suitable for one-way or two-way analysis [Clemen, 1996]. This means that we can determine the relationship of just two parameters or probabilities, which influence our decision. This paper considers a problem that is similar to sensitivity analysis and in which we ought to determine the relationship of a bigger number of parameters.

Informally we can define this problem as follows. We are given a number of parameters and fuzzy values, which these parameters can assume. We know what is the value of the fuzzy criterion for all of the aforementioned parameters values. The problem is to define the *importance coefficients* of these values, which denote how *important* the parameter value is in order for the criterion to assume the *desired value*. The desired value of the criterion can indicate one of the possible alternatives. So, this problem can be viewed as an inverse problem to sensitivity analysis: we are trying to denote, which parameters values correspond to some alternative.

The adaptive network described in [Valishevsky et al., 2002] is being used to solve this problem. This network can be used in fuzzy-evidences-based decision support systems. It's possible to derive a training algorithm for this network, which allows to solve the aforementioned problem.

In the next chapter granular information and the adaptive network are considered. The possibility of using this network for sensitivity analysis is considered later in the paper. Case study concerning multi-parameter sensitivity analysis is presented as well.

2. Adaptive Network for Granular Information and Evidence Processing (ANGIE)

The considered network processes fuzzy evidences. Evidences consist of a collection of π-granules of form (1).

$$g_i \overset{\Delta}{=} \text{If } X = u_i \text{ then } Y \text{ is } G_i , \tag{1}$$

where G_i is a fuzzy subset of V, which is dependent on u_i. Variable X assumes its value with a specified probability. Thus, evidence is a probability distribution P_X of conditional π-granules. Moreover, each granule can be regarded as a conditional possibility $\Pi_{(Y|X=u_i)} = G_i$.

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Hence, evidence can be regarded as a conditional possibility distribution $\Pi_{(Y|X)}$. Thus, evidence can be considered as the following construction:

$$E = \{P_X, \Pi_{(Y|X)}\}.$$

Given a collection of evidences $E = \{E_1, ..., E_K\}$, one would like to ask questions about the information contained in these evidences. The main question is the following: "what is the probability of (*Y* is *Q*)?" where *Q* is some fuzzy value. The probability of such an event is an interval value, where the expected possibility $\text{E}\Pi(Q)$ is regarded as the upper bound, and the expected certainty $\text{EC}(Q)$ is regarded as the lower bound of the sought probability. In [Zadeh, 1979] the following formulae are proposed:

$$\text{E}\Pi(Q) = \sum_{i,j} p_{ij} \sup\left(Q \cap G_i \cap H_j\right), \tag{2}$$

$$\text{EC}(Q) = 1 - \text{E}\Pi(Q'), \tag{3}$$

where Q' is complement of the set *Q*. Formula (2) is for a special case, when there're two evidences in the system, but it wouldn't present any difficulty to induce a more general form of the formula.

Refer to [Zadeh, 1979] for further information on information granularity.

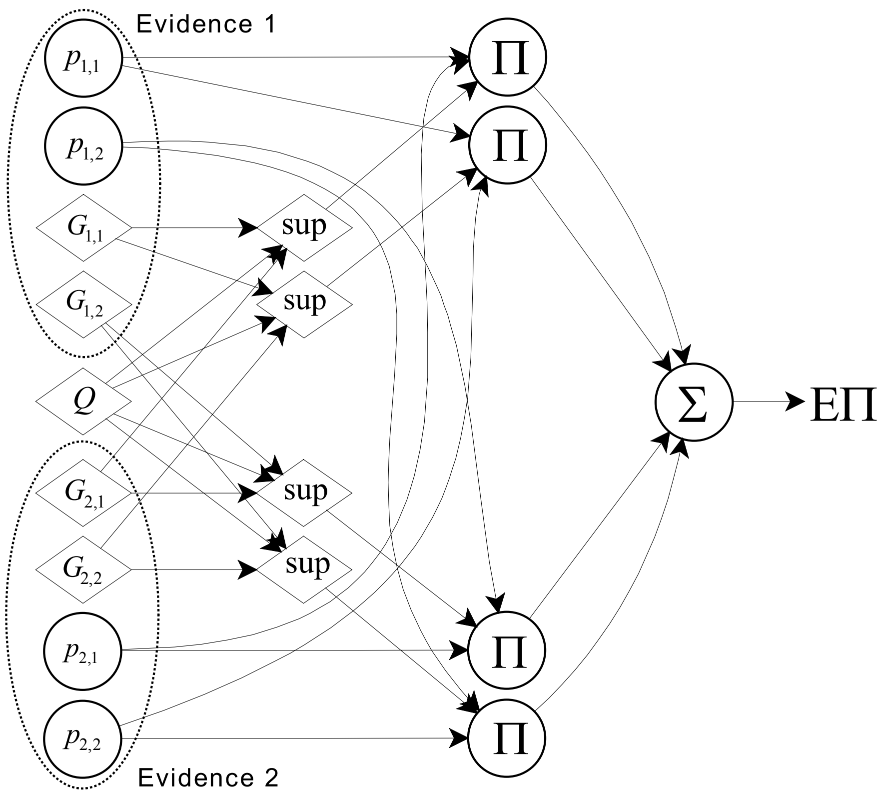

Figure 1. Adaptive network for the calculation of the upper bound of the probability

Let's consider a system that consists of two evidences:

I. Evidence 1:
IF (X_1 is B_{11}) with p_{11} THEN (Y is G_{11})
IF (X_1 is B_{12}) with p_{12} THEN (Y is G_{12})

II. Evidence 2:
IF (X_2 is B_{21}) with p_{21} THEN (Y is G_{21})
IF (X_2 is B_{22}) with p_{22} THEN (Y is G_{22})The adaptive network shown in Figure 1 can be used in order to determine the upper bound of the probability (2).

3. Using ANGIE for Sensitivity Analysis

A training algorithm for adaptive network ANGIE is based on the gradient descent method and is described in [Valishevsky et al., 2002]. The algorithm allows to tune the values of the probabilities contained in granules. This can be used in order to solve a problem that is similar to sensitivity analysis.

First of all we have to define evidences that describe possible values of the parameters and the criterion. In order to train the network we have to define a *desired criterion value*, which corresponds to one of the alternatives. After the training process is completed, the probability values can be considered as *importance coefficients*, which define how *important* the given parameter value is in order for the criterion to reach the desired value.

4. Case Study: Choosing an Optimal Gift

Let's consider an application example of the procedure described in the previous chapter. Assume that we have to determine the influence of different gift parameters on the evaluation of the gift. This problem is similar to sensitivity analysis. However, in the traditional sensitivity analysis only one or two parameters are taken into consideration, as the result is represented graphically. In this case study we will try to determine the relationship among about 20 different values of parameters. It's evident that we won't be able to represent the solution graphically, as the solution is a dot in the multidimensional space (dimension is equal to the number of parameters).

The gift will be evaluated using the criterion *"Y = Evaluation of the gift"*. Some of the possible values of this criterion are as follows:

1. *Y_1 = You would better not present this*
2. *Y_2 = Useless trinket*
3. *Y_3 = Trinket, but an application can be found*
4. *Y_4 = Useful gift*
5. *Y_5 = The receiver will be happy*
6. *Y_6 = I myself would like to receive such a gift*
7. *Y_7 = You would better not embarrass the receiver*

The membership functions for these criterion values are shown in Figure 2.

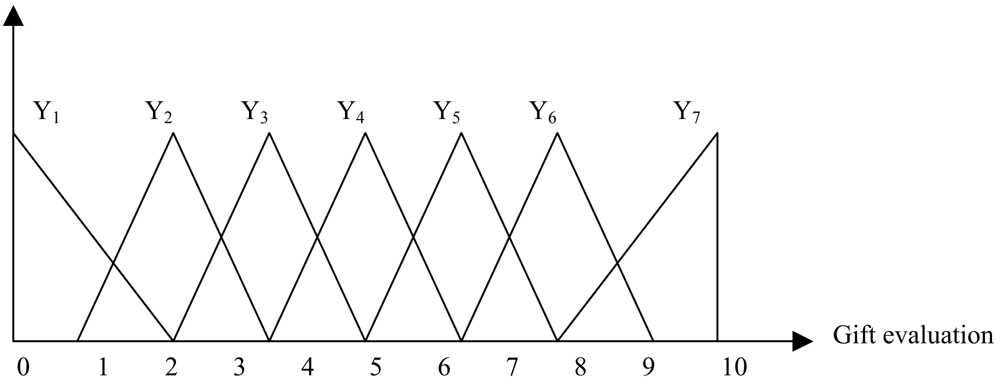

Figure 2. Membership functions of the criterion fuzzy values

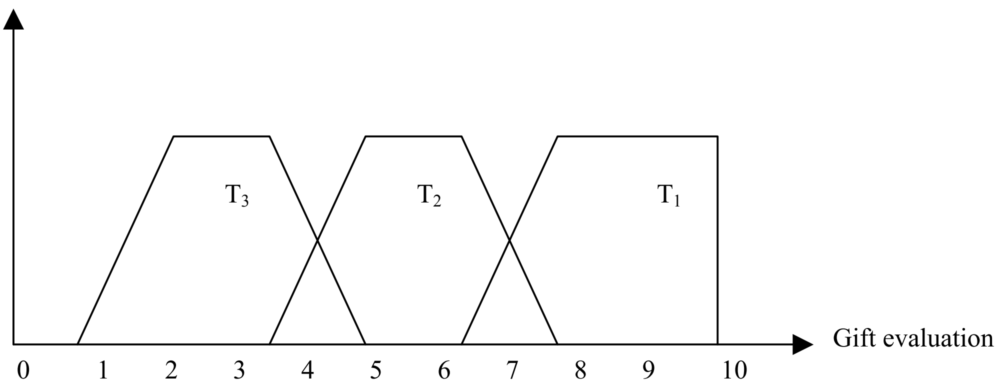

Figure 3. Additional membership functions

As was mentioned before, now we have to define the parameters and their values. We'll consider six different parameters, which are listed above. Possible values of the corresponding parameter and their identifiers are listed in brackets.

1. Price (*very high* = p_{11}, *high* = p_{12}, *acceptable* = p_{13}, *low* = p_{14}, *suspiciously low* = p_{15}).
2. Colour (*bright* = p_{21}, *moderate* = p_{22}, *pale* = p_{23}).
3. Expenses that appear during the exploitation (*correspond to gift's price* = p_{31}, *not high* = p_{32}, *no expenses* = p_{33})
4. For how long the gift will be interesting to the receiver (*duration is not limited* = p_{41}, *until the next birthday* = p_{42}, *a month* = p_{43}, *a week* = p_{44}, *half an hour* = p_{45})
5. Weight (*big* = p_{51}, *middle* = p_{52}, *light* = p_{53}, *weightless* = p_{54})
6. For how long you can use the gift without additional expenses (*expenses appear right away* = p_{61}, *not long enough* = p_{62}, *long enough*= p_{63}, *no additional expenses*= p_{64})

Now we have to define granular evidences that describe possible values of the aforementioned parameters. The system consists of 6 evidences, as there're 6 parameters. We'll give an example for the evidence that describe values of the parameter "Price":

Evidence for the parameter "X_1 = Price"
If X_1 = *VERY HIGH* with probability p_{11} , Then Y = Y_7
If X_1 = *HIGH* with probability p_{12} , Then Y = T_1
If X_1 = *ACCEPTABLE* with probability p_{13} , Then Y = T_2

If X_1 = *LOW* with probability p_{14}, Then Y = T_3
If X_1 = *SUSPICIOUSLY LOW* with probability p_{15}, Then Y = Y_1
T_1, T_2 and T_3 are additional fuzzy values shown in Figure 3.

Other evidences are defined in a similar manner.

For the first experiment, the desired value of the criterion was chosen to be "*Y_6 = I myself would like to receive such a gift*".

When the training process of the system was finished, the following probability or importance values have been obtained:

Table 1. Probability values after training

p_{11} =0.133561 p_{12} =0.149396 p_{13} =0.513799 p_{14} =0.101622 p_{15} =0.101622	p_{21} =0.178997 p_{22} =0.424877 p_{23} =0.396126	p_{31} =0.190155 p_{32} =0.570355 p_{33} =0.23949
p_{41} =0.166968 p_{42} =0.522816 p_{43} =0.103405 p_{44} =0.103405 p_{45} =0.103405	p_{51} =0.132685 p_{52} =0.529669 p_{53} =0.168823 p_{54} =0.168823	p_{61} =0.123005 p_{62} =0.272749 p_{63} =0.429368 p_{64} =0.174877

Further sensitivity analysis may involve finding the distance between the points in the space of probabilistic parameters (one of such points is defined in Table 1). To demonstrate this, additional experiments have been carried out and parameters values have been found for the criterion values of "*Y_4 = Useful gift*" and "*Y_7 = You would better not embarrass the receiver*".

Euclidean metrics has been used in order to determine the distance. The results are shown in Table 2.

Table 2. Distance between points representing different solutions

To / From	Y_4	Y_6	Y_7
Y_4	0	0.018	2.52
Y_6	0.018	0	1.918
Y_7	2.52	1.918	0

As can be seen from Table 2, the closer the desired criterion values are (see Figure 2), the lesser the distance between the points representing them in the space of probabilistic parameters is. But the solution may not be as intuitive, especially when we're considering a multicriteria decision model, which is the next topic that can be considered in the context of the multi-parameter sensitivity analysis using adaptive networks.

5. Conclusion

The possibility of using adaptive networks for sensitivity analysis of decision models is considered in this paper. One of the advantages of this approach is that it's possible to determine the relationship among a bigger number of parameters with the help of adaptive networks. Traditional sensitivity analysis allows to determine the influence of just one or two parameters on the decision, as the solution is represented graphically. Methodology that has been considered in this paper is not the only way to solve the multi-parameter sensitivity analysis task. But it can be useful in determining what does and what does not matter for some alternative. One of the main drawbacks is that the adaptive network that has been described in this paper requires a big amount of computational power. It should be noted that originally the network was designed for granular information based decision support systems. It was not designed as a tool for sensitivity analysis, although it's possible to derive a training algorithm, which allows to solve sensitivity analysis tasks.

One of the issues that have not been discussed in this paper concerns the normalization of probabilistic parameters in the adaptive network. Probability values are modified using gradient descent method, which does not care about their consistency, so probabilities should be normalized from time to time, to make sure that their sum is equal to one. In this paper probabilities have been normalized only after the training process has finished, as it allows to speed up the convergence time.

A case study, which demonstrates in what way the described network can be used in sensitivity analysis, has been considered as well.

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Vališevskis A. Uz granulārām liecībām bāzēto adaptīvo tīklu izmantošana jūtīguma analīzei

Dotajā darbā tiek apskatīta adaptīvo tīklu izmantošanas iespēja jūtīguma analīzes uzdevuma risināšanai. Tiek aprakstīts adaptīvais tīkls, kas apstrādā izplūdušās granulas. Aprakstītā tīkla apmācīšanai paredzēto algoritmu var izmantot lēmumu pieņemšanas modeļu jūtīguma analīzei. Turklāt, darbā tiek apskatīts jūtīguma analīzes piemērs, kas parāda kādā veidā adaptīvo tīklu var izmantot jūtīguma analīzes uzdevumu risināšanai.

Valishevsky A. Using Granular-Evidences-based Adaptive Networks for Sensitivity Analysis

This paper considers the possibility of using adaptive networks for sensitivity analysis. Adaptive network that processes fuzzy granules is described. The adaptive network training algorithm can be used for sensitivity analysis of decision making models. Furthermore, a case study concerning sensitivity analysis is described, which shows in what way the adaptive network can be used for sensitivity analysis.

Валишевский А. Использование адаптивных сетей, основанных на нечетких свидетельствах, в задачах анализа чувствительности

В работе рассматривается возможность использования адаптивных сетей при решении задачи анализа чувствительности. Описывается адаптивная сеть, которая обрабатывает нечеткие гранулы. Алгоритм, позволяющий проводить обучение описанной сети, можно использовать для анализа чувствительности моделей принятия решений. Кроме того, в работе приводится описание примера

анализа чувствительности, при помощи которого демонстрируется, каким образом адаптивную сеть можно применять в задачах анализа чувствительности.