THE USE OF MULTIVARIATE REGRESSION MODEL TO FORECAST THE FREIGHT AIR TRANSPORTATIONS IN THE MEMBER COUNTRIES OF THE EUROPE UNION

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The paper deals with the multivariate regression models for analysis and forecasting of freight and mail air international transportations by the member countries of the European Union. The air international transportations are divided into two parts, thus two foreseeable variables exist: internal and external transportations in relation to the EU. After that both variables are forecasted together owing to their strong correlation with each other, i.e. *the total forecast* of two considered dependent variables is obtained. The corresponding regression models contain main economical factors affected internal and external transportation for each country. Two different approaches are considered: the classical linear multiply regression model and the multivariate regression model. Various tests for hypothesis of explanatory variables insignificance and model correctness are carried out. The total forecast of internal and external transportations has been obtained for 2007 year. The confidence limit for this forecast is calculated as well. The purpose of the research is experimental proof of the significance of total forecast.

Keywords: *multivariate regression, freight air transportations, total forecast, confidence limit*

1. Introduction

The aim of research is elaboration of regression models for the freight and mail air international transportations analysis and forecasting. In this investigation the following problems are considered:

1. Analysis and *separately* forecasting of external and internal freight and mail international transportations by the member countries of the Europe Union.
2. Analysis and *total* forecasting of external and internal freight and mail international transportations by the member countries of the Europe Union.

For the first problem the multiply regression model is used and for the second problem the multivariate regression model. In this paper the used information base and the short theoretical description of the used regression models are considered. The best versions of elaborated models for forecasting air freight transportations are showed as well. The detailed analysis of the obtained results is given, i.e. the variance of total forecast for 2007 has been calculated and the corresponding confidence limits have been obtained.

2. Statistical Data

The data have been received from the electronic database "The Statistical Office of the European Communities" (EUROSTAT YEARBOOK 2006-07) [7]. Let us consider models for *transportation forecasting.* The main object of consideration, named *object,* is the member country of the Europe Union. We call as *observation* a data about object for fixed year, i.e. external and internal freight and mail international transportations expressed in thousands of tons [1, 2]. The following factors have been selected for the experiments:

t_1 – time factor (YEAR);
t_2 – trade integration of goods (TI) as a percentage of GDP;
t_3 – annual average EURO exchange rate versus national currencies (EURO rates);
t_4 – annual average inflation rate of change in Harmonized Indices of Consumer Prices (IR);
t_5 – prices on petroleum products (PP), euro per ton;
t_6 – GDP "per capita" in Purchasing Power Standards (GDP_PPS);
t_7 – gradation of countries under value of forecasted parameter (GradY);
t_8 – gradation of countries under population density (GradPD);
t_9 – gradation of countries under area (GradArea);
t_{10} – gradation of countries under the experience of membership in EU (GradMember);
t_{11} – internal freight and mail international transportations, in tons (FrMIntIntra);
t_{12} – external freight and mail international transportations, in tons (FrMIntExtra);
t_{13} – total population, in thousands (TotalPop).

Let us comment some of the described factors:

a) the volume index of GDP is expressed in relation to the European Union (EU-27) average set to equal 100. If the index of a country is higher than 100, this country's level of GDP per head is higher than the EU average and vice versa;
b) trade integration of goods as a percentage of GDP. Average of imports and exports of the item goods of the balance of payments divided by GDP. If the index increases over time it means that the country/zone is becoming more integrated within the international economy.

3. Structure of Models and Evaluation Procedure

In the current investigation two kinds of mathematical models are used. First of them is a well-known *multiple regression model* [5], there is one dependent variable. The corresponding model is as follows:

$$y = \mathrm{X}\beta + \varepsilon\,, \tag{1}$$

where y is the vector of observations on the dependent variable, X is the design matrix on the predictors, ε is the vector of random terms and β is the vector of unknown regression coefficients. *Multivariate regression model* [5, 6] usually is used when we have to forecast two or more strongly correlated variables, and includes simultaneously many dependent variables. There y, β and ε vectors become matrices, which we denote Y, B and E. So, multivariate regression model can be written in the following way:

$$\mathrm{Y} = \mathrm{XB} + \mathrm{E}\,, \tag{2}$$

$$\mathrm{Y} = \begin{pmatrix} y_{11} & y_{12} & \cdots & y_{1p} \\ y_{21} & y_{22} & \cdots & y_{2p} \\ \cdots & \cdots & \cdots & \cdots \\ y_{n1} & y_{n2} & \cdots & y_{np} \end{pmatrix},\ \mathrm{X} = \begin{pmatrix} 1 & x_{11} & \cdots & x_{1d} \\ 1 & x_{21} & \cdots & y_{2d} \\ \cdots & \cdots & \cdots & \cdots \\ 1 & x_{n1} & \cdots & x_{nd} \end{pmatrix},$$

$$\mathrm{B} = \begin{pmatrix} \beta_{01} & \beta_{02} & \cdots & \beta_{0p} \\ \beta_{11} & \beta_{12} & \cdots & \beta_{1p} \\ \cdots & \cdots & \cdots & \cdots \\ \beta_{d1} & \beta_{d2} & \cdots & \beta_{dp} \end{pmatrix},\ \mathrm{E} = \begin{pmatrix} \varepsilon_{11} & \varepsilon_{12} & \cdots & \varepsilon_{1p} \\ \varepsilon_{21} & \varepsilon_{22} & \cdots & \varepsilon_{2p} \\ \cdots & \cdots & \cdots & \cdots \\ \varepsilon_{n1} & \varepsilon_{n2} & \cdots & \varepsilon_{np} \end{pmatrix}.$$

Here Y is the matrix of observations on the dependent variables, X is the design matrix, B is the unknown matrix of regression coefficients, E is the matrix of random members. And p is the quantity of dependent variables, d is the quantity of predictors and n is the quantity of observations. Let us rewrite matrix E in the following way: $\mathrm{E} = (e_1, e_2, \ldots, e_n)'$. It is supposed that errors are independent:

$$\mathrm{E}(e_i) = 0\,;\ \mathrm{E}\left(e_i e_j{}'\right) = 0\,,\ i \neq j\,. \tag{3}$$

The matrix Σ of covariance of errors $\{e_i\}$ can be expressed as

$$Cov\left(e_i e_j{}'\right) = \Sigma = (\sigma_{kl}),\ k, l = 1, \ldots p\,,\ i, j = 1, \ldots, n. \tag{4}$$

The least squares estimates for the B coefficients are obtained by solving the normal equations like for multiple regression models:

$$\tilde{\beta}^{\langle k \rangle} = (X'X)^{-1} X'Y^{\langle k \rangle},\ \ k = 1, \ldots, p, \tag{5}$$

where $\tilde{\beta}^{\langle k \rangle}$ is the estimated vector of unknown coefficients for k-th dependent variable.

An unbiased estimate of Σ is given by

$$\tilde{\Sigma} = \frac{1}{n-d} Y'\left[I - X(X'X)^{-1} X'\right]Y. \tag{6}$$

Now we can express the covariance between two estimates of unknown vectors of regression coefficients:

$$Cov\left(\tilde{\beta}^{\langle k\rangle},\tilde{\beta}^{\langle l\rangle}\right)=\sigma_{kl}\left(X'X\right)^{-1},\ k,l=1,\dots p\,. \tag{7}$$

As the matrix Σ of covariance of errors is unknown, we can use its unbiased estimate $\tilde{\Sigma}$. So,

$$Cov^{*}\left(\tilde{\beta}^{\langle k\rangle},\tilde{\beta}^{\langle l\rangle}\right)=\tilde{\sigma}_{kl}\left(X'X\right)^{-1},\ k,l=1,\dots p\,. \tag{8}$$

In our case we have two dependent variables, i.e. internal and external freight and mail international transportations. First we construct two several multiple models for forecasting these values separately. Further we construct the multivariate model for their total forecasting. For total forecasting we have to obtain the covariance matrices of coefficients. These matrices allow us calculating the variance of sum of forecasts of analysed dependent variables. After that we are able to estimate the confidence limit for forecasted expectations of both transportations sum. Note, that if we consider multivariate model, it is necessary to calculate the joint covariance matrix for two estimated vectors of regression coefficients.

Let us obtain two several forecasts $\tilde{\mathrm{Y}}^{\langle 1\rangle}=\mathrm{X}\tilde{\beta}^{\langle 1\rangle}$ and $\tilde{\mathrm{Y}}^{\langle 2\rangle}=\mathrm{X}\tilde{\beta}^{\langle 2\rangle}$. Thus the sum of forecasts is as follows; $\tilde{S}=\tilde{\mathrm{Y}}^{\langle 1\rangle}+\tilde{\mathrm{Y}}^{\langle 2\rangle}$. The mean and the variance of this sum are:

$$E\left(\tilde{S}\right)=E\left(\tilde{\mathrm{Y}}^{\langle 1\rangle}\right)+E\left(\tilde{\mathrm{Y}}^{\langle 2\rangle}\right)=\mathrm{X}\left(\tilde{\beta}^{\langle 1\rangle}+\tilde{\beta}^{\langle 2\rangle}\right), \tag{9}$$

$$D\left(\tilde{S}\right)=D\left(\tilde{\mathrm{Y}}^{\langle 1\rangle}\right)+D\left(\tilde{\mathrm{Y}}^{\langle 2\rangle}\right)+2Cov\left(\tilde{\mathrm{Y}}^{\langle 1\rangle},\tilde{\mathrm{Y}}^{\langle 2\rangle}\right). \tag{10}$$

The first and the second terms of (10) are:

$$D\left(\tilde{\mathrm{Y}}^{\langle 1\rangle}\right)=D\left(\mathrm{X}\tilde{\beta}^{\langle 1\rangle}\right)=\mathrm{X}\cdot Cov\left(\tilde{\beta}^{\langle 1\rangle}\right)\cdot\mathrm{X}' \tag{11}$$

and

$$D\left(\tilde{\mathrm{Y}}^{\langle 2\rangle}\right)=D\left(\mathrm{X}\tilde{\beta}^{\langle 2\rangle}\right)=\mathrm{X}\cdot Cov\left(\tilde{\beta}^{\langle 2\rangle}\right)\cdot\mathrm{X}'\,, \tag{12}$$

where

$Cov\left(\tilde{\beta}^{\langle 1\rangle}\right)$ and $Cov\left(\tilde{\beta}^{\langle 2\rangle}\right)$ are covariance matrixes of vectors $\tilde{\beta}^{\langle 1\rangle}$ and $\tilde{\beta}^{\langle 2\rangle}$ respectively, calculated according formula (8). The third term of (10) can be defined as follows:

$$Cov\left(\tilde{\mathrm{Y}}^{\langle 1\rangle},\tilde{\mathrm{Y}}^{\langle 2\rangle}\right)=Cov\left(\mathrm{X}\tilde{\beta}^{\langle 1\rangle},\mathrm{X}\tilde{\beta}^{\langle 2\rangle}\right)=E\left(\left(\mathrm{X}\tilde{\beta}^{\langle 1\rangle}-\mathrm{X}\beta^{\langle 1\rangle}\right)'\cdot\left(\mathrm{X}\tilde{\beta}^{\langle 2\rangle}-\mathrm{X}\beta^{\langle 2\rangle}\right)\right)=$$

$$=\mathrm{X}\cdot E\left(\left(\tilde{\beta}^{\langle 1\rangle}-\beta^{\langle 1\rangle}\right)'\cdot\left(\tilde{\beta}^{\langle 2\rangle}-\beta^{\langle 2\rangle}\right)\right)\cdot\mathrm{X}'\,. \tag{13}$$

The member $E\left(\left(\tilde{\beta}^{\langle 1\rangle}-\beta^{\langle 1\rangle}\right)'\cdot\left(\tilde{\beta}^{\langle 2\rangle}-\beta^{\langle 2\rangle}\right)\right)$ is the joint covariance matrix of two vectors of coefficients, calculated by formula (8). On the base of all listed above the upper confidence limit for total forecast $\overline{E}\left(\overline{\tilde{S}}\right)=E(S)$ corresponding to confidence probability γ is $\left(0,\overline{S}_{\gamma}\right)$, and

$$\overline{S}_{\gamma}=E\left(\tilde{S}\right)+\sqrt{D\left(\tilde{S}\right)}\cdot\Phi^{-1}(\gamma), \tag{14}$$

where $\Phi^{-1}(\gamma)$ is γ –quantile of standard normal distribution.

Now we'll consider the analysed regression models for internal and external transportations forecasting.

4. Models for Transportations Forecasting

In this section we consider the models for internal and external air freight and mail international transportations forecasting. Note that the suggested models are the group models [2], i.e. we forecast the transportations for all the considered countries using the same sets of the explanatory variables and the same estimates of coefficients. The following countries were selected: Austria, Belgium, Czech Republic, Germany, Denmark, Estonia, Spain, Finland, France, Greece, Hungary, Ireland, Italy, Lithuania, Netherlands, Poland, Portugal, Slovakia and the United Kingdom. The analysed period is from 2001 to 2006.

The first and second models are multiple linear regression models (1). The dependent variable in the *first model* $Y^{(1)} = t_{11}/t_{13}$ is internal freight and mail international transportations in tons, divided by total population in thousands, i.e. *specific* transportations. Note, that superscript by Y is introduced just for identification of models. Explanatory variables are $x_1 = t_1$, $x_2 = t_2$, $x_3 = t_3$, $x_4 = t_4$, $x_5 = t_5$, $x_6 = t_6$, $x_7 = t_7$, $x_8 = t_8$, $x_9 = t_9$, $x_{10} = t_{10}$.

Notwithstanding that forecasted parameter is transportations in tons per thousand of inhabitants, it does not take into account such important characteristics, as area of countries, duration of membership in EU and so on. Let us explain the sense of the used gradations [4].

The first gradation t_7 is gradation of countries under value of forecasted parameter. It has been entered into the model with the purpose to allocate the countries with obviously larger specific transportations (such as Belgium, Germany and the United Kingdom) into separate group. Thus, we can hold corresponding observations in the model. It, in turn, enables us to predict large and small transportations using one and the same model. We are relieved of necessity of selection of two different models for large and small transportations [3]. t_7 is equal to 1 for countries with large specific transportations (equal or larger than 20 tons per thousand of inhabitants), and it is equal to 0 for countries with transportations smaller than 20 tons per thousand of inhabitants.

The second gradation t_8 is gradation of countries under population density. It is equal to 1 for countries with population density equal or larger than 200 inhabitants per km^2 (Belgium, Germany, the Netherlands and the United Kingdom), and it is equal to 0 for countries with population density smaller than 200 inhabitants per km^2 (i.e. for other considered countries).

The third gradation t_9 is gradation of countries under area. It is equal to 1 for countries with areas equal or less than 40 000 km^2, and it is equal to 0 for countries with areas larger than 40 000 km^2. This index is equal to 1 for Belgium.

The fourth gradation t_{10} is gradation of countries under the duration of membership in EU. This index is equal to 1 for new members of EU (Czech Republic, Estonia, Hungary, Lithuania, Poland and Slovakia) and is equal to 0 for former members of EU (i.e. for other considered countries).

The dependent variable in the *second model* $Y^{(2)} = t_{12}/t_{13}$ is external freight and mail international transportations in tons, divided by total population in thousands.

The *third model* is the multivariate one (2) and there are two dependent variables, which are described by the vector $Y^{(3)} = \left(Y_1^{(3)} = t_{11}/t_{13}, Y_2^{(3)} = t_{12}/t_{13} \right)$. The sets of explanatory variables for all the models coincide.

Further we perform two steps: considering of internal and external transportations. All experimental results are obtained by means of Statistica 6 and MathCad 13 packages.

5. Estimation of Internal Transportations

Let us present the results of the first model estimation. For the experiments data from 2001 till 2006 have been used. There are 94 observations that have been processed. The estimates of the coefficients and Student criterion values are resulted in Table 1. Here $\widetilde{\beta}_i^{\langle 1 \rangle}$ is an estimate of $\beta_i^{\langle 1 \rangle}$, $t(83)$ is the calculated value of Student criterion for 83 degrees of freedom, p-level is the error of second kind (or level of insignificance of variable). The theoretical value of Student criterion for 83 degrees of freedom and level of significance (or error of first kind) $\alpha = 45\%$ is equal to 0.76.

Taking into account the fact that the hypothesis of insignificance of explanatory variable is tested, we can see that most significant factors affected internal air transportations are all considered ones except GDP in PPS and gradation under population density. The signs by estimated coefficients correspond to the physical sense of considered factors. The coefficient R^2 for this model is equal to 0.82 and the Fisher criterion is 38.43. The theoretical value of Fisher criterion for 10 and 83 degrees of freedom and level of significance $\alpha = 5\%$ is equal to 1.95. Comparing the theoretical and calculated values of Fisher criterion we can conclude that first model cannot be recognized as insignificant. The residual sum of squares *RSS* [2,3,4,5] is equal to 2 202.

Table 1. Estimation results for the first model

Variable	Factor	$\widetilde{\beta}_i^{\langle 1\rangle}$	t(83)	p-level
	Intercept	-1192.52	-2.72571	0.007825
x_1	Year	0.60	2.74044	0.007511
x_2	TI	0.09	1.72584	0.088098
x_3	EuroRate	-0.00	-0.60482	0.546952
x_4	IR	-0.17	-0.62978	0.530567
x_5	PP	-0.01	-2.17534	0.032449
x_6	GDP_PPS	0.00	0.09337	0.925833
x_7	GradY	1.99	1.03248	0.304845
x_8	GradMember	-7.10	-2.51648	0.013779
x_9	GradArea	16.47	6.40056	0.000000
x_{10}	GradPD	-0.52	-0.30429	0.761669

The estimated model is the following:

$$\widetilde{E}\left(\mathrm{Y}^{\langle 1\rangle}(x)\right) = -1193 + 0.6x_1 + 0.09x_2 - 0.001x_3 - 0.17x_4 - 0.01x_5 + 1.99x_7 - 7.1x_8 + 16.47x_9 . \quad (15)$$

Note, that equation (15) contains most significant explanatory variables. At the Figure 1 we can visually evaluate, how estimated model smoothes experimental data. The observations are arranged in "country-year" order: every point corresponds to some country transportation during the analysed period from 2001 till 2006. Moreover, countries are sorted in alphabetical order. Horizontal axis reflects the observations, arranged in the above-mentioned order. Vertical axis reflects the corresponding transportations, expressed in thousands of tons. Notation: not all countries declared in the previous section have data about corresponding transportations for all six years. It is obvious the first model shows quite good smoothing.

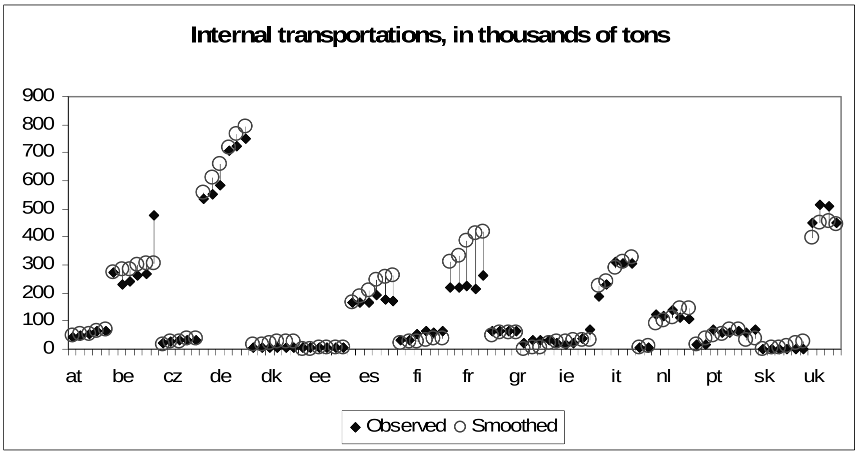

Fig. 1. Observed and smoothed internal transportations

We have estimated internal air freight and mail international transportations for several member countries of EU for 2007. Table 2 contains corresponding forecasted values of dependent variable $Y^{(1)}$ for 2007 (column 2007). Table contains *observed* data for 2006 (column 2006*) and forecasts for 2006 as well (column 2006) for seeing that there are no greater distinctions between 2007 and 2006, i.e. the tendency is kept. Note the fact that data in interesting affected parameters for 2007 were accessible only for these countries.

Table 2. Forecasted internal transportations, in thousands of tons

Country	2006*	2006	2007	Country	2006*	2006	2007
Austria	64.72	69.67	75.06	Ireland	70.41	32.57	39.44
Belgium	478.75	307.63	318.76	Italy	306.89	326.13	373.12
Czech	34.56	37.16	51.24	Lithuania	6.42	8.99	14.12
Denmark	5.69	26.46	31.03	Netherlands	105.44	146.01	156.53
Estonia	5.52	6.11	7.83	Poland	17.69	35.49	64.13
Finland	62.91	35.64	39.64	Slovakia	0.78	24.24	26.72
France	261.96	419.57	472.59	Spain	173.70	261.76	301.49
Germany	752.61	791.24	850.40	Sweden	68.67	38.05	46.30
Greece	61.67	60.48	72.59	UK	452.67	445.58	491.57
Hungary	22.97	24.32	37.16				

6. Estimation of External Transportations

Let us present the results of the second model estimation. For the experiments data from 2001 till 2006 have been used. 94 observations have been processed. The estimates of the coefficients and Student criterion values are resulted in Table 3. Here $\tilde{\beta}_i^{\langle 2 \rangle}$ is an estimate of $\beta_i^{\langle 2 \rangle}$, $t(83)$ is the calculated value of Student criteria for 83 degrees of freedom, *p*-level is the error of second kind (or level of insignificance of variable). The theoretical value of Student criterion for 83 degrees of freedom and level of significance (or error of first kind) $\alpha = 30\%$ is equal to 1.04.

Taking into account the fact that the hypothesis of insignificance of explanatory variable is tested, we can see most significant factors affected external air transportations are all considered ones except trade integration of goods and annual average inflation rate. The signs by estimated coefficients correspond to the physical sense of considered factors. The coefficient R^2 for this model is equal to 0.95 and the Fisher criterion is 150.88. The theoretical value of Fisher criterion for 10 and 83 degrees of freedom and level of significance $\alpha = 5\%$ is equal to 1.95. Comparing the theoretical and calculated values of Fisher criterion we can conclude that first model cannot be recognized as insignificant. The residual sum of squares (*RSS*) [2,3,4,5] is equal to 21 989.

Table 3. Estimation results for the second model

Variable	Factor	$\tilde{\beta}_i^{\langle 2 \rangle}$	*t*(83)	p-level
	Intercept	-3394.40	-4.9450	0.000004
x_1	Year	1.70	4.9551	0.000004
x_2	TI	0.05	0.6399	0.523968
x_3	Euro	-0.00	-1.0497	0.296887
x_4	IR	-0.13	-0.3002	0.764794
x_5	PP	-0.02	-3.5823	0.000573
x_6	GDP_PPS	0.07	1.3577	0.178251
x_7	GradY	-52.45	-17.3288	0.000000
x_8	GradMember	-6.55	-1.4802	0.142593
x_9	GradArea	9.65	2.3907	0.019081
x_{10}	GradPD	69.34	25.8921	0.000000

The estimated equation is as follows:

$$\tilde{E}\left(\mathrm{Y}^{\langle 2 \rangle}(x)\right) = -3394 + 1.7x_1 - 0.001x_3 - 0.02x_5 + 0.07x_6 - 52.5x_7 - 6.6x_8 + 9.7x_9 + 69.3x_{10}. \quad (16)$$

Note, that equation (16) contains most significant explanatory variables. At the Figure 2 we can visually evaluate, how estimated model smoothes experimental data. Every point corresponds to transportations of some country during the analysed period from 2001 till 2006.

We have estimated external air freight and mail international transportations for several member countries of EU for 2007. Table 4 contains corresponding forecasts for 2006 and 2007 (columns 2006 and 2007 respectively) and observed external transportations for 2006 as well (column 2006*).

Note the fact that the model describing external transportations gives a larger absolute error (i.e. *RSS* = 21 989) in comparison with the model describing internal transportations (*RSS* = 2 202). Therefore the forecasts of external transportations, received for 2007, for certain are less exact. Here we can draw the following conclusion that all chosen explanatory factors in a greater measure approach for the description and forecasting of internal transportations, rather than external.

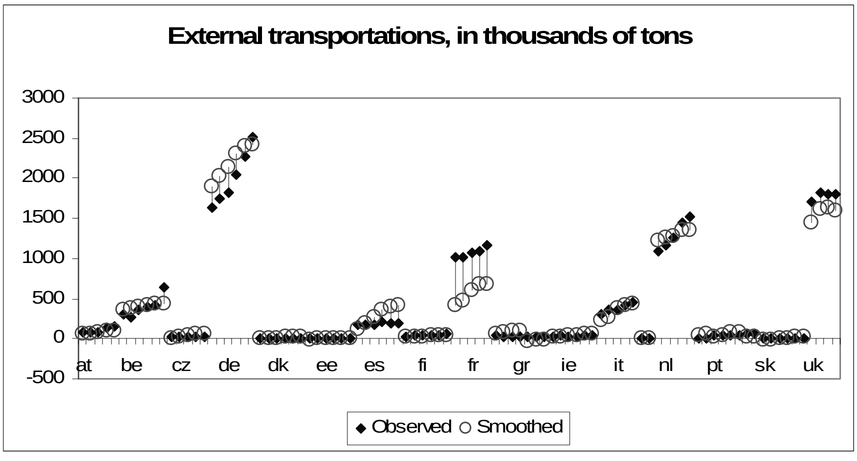

***Fig. 2.** Observed and smoothed external transportations*

Table 4. Forecasted external transportations, in thousands of tons

Country	2006*	2006	2007	Country	2006*	2006	2007
Austria	163.21	104.55	121.53	Ireland	41.23	53.70	69.99
Belgium	646.20	435.33	463.03	Italy	457.16	436.58	576.77
Czech	22.49	53.47	81.97	Lithuania	6.25	12.95	22.01
Denmark	1.00	29.25	41.61	Netherlands	1 515.91	1 353.06	1 388.74
Estonia	4.53	9.97	13.13	Poland	14.33	69.14	129.74
Finland	55.51	48.53	61.52	Slovakia	4.58	26.20	34.65
France	1 159.45	686.25	834.43	Spain	195.27	411.72	509.72
Germany	2 515.30	2 422.69	2 593.99	Sweden	70.81	28.36	48.37
Greece	28.93	94.46	125.31	UK	1 795.74	1 599.09	1 756.88
Hungary	41.92	16.90	44.73				

The next steps of our investigation are considering of forecasted expectations of both transportations sums for each country for 2007 and calculating of upper confidence limits for corresponding forecasts.

7. Estimation of Total Transportations

For further analysis we shall use the results obtained above (see Tables 1-4). Additionally, we must estimate the covariance matrix of errors (6). It has been calculated on the base of data from the considered period from 2001 till 2006:

$$\tilde{\Sigma} = \begin{bmatrix} 9.12 & 5.24 \\ 5.24 & 22.45 \end{bmatrix}. \tag{17}$$

Then on the base of (17) we are able to obtain covariance matrices of each estimated vectors of regression coefficients $\tilde{\beta}^{\langle 1 \rangle}$ and $\tilde{\beta}^{\langle 2 \rangle}$, and the joint covariance matrix of both vectors of coefficients, calculated by (8).

Table 5 contains obtained on the base of covariance matrices (8) variances of forecasted internal and external transportations, calculated by (11) and (12) (columns Var_1 and Var_2 respectively), the joint covariance (13) of forecasted sum of both transportations (columns Cov) and the variance of total forecast of both transportations (10) for 2007 (column Var_{2007}). Note that values contained in Table 5 are given for specific transportations.

Table 5. Variance of the forecasted total specific transportations for 2007

Country	Var_1	Var_2	Cov	Var_{2007}	Country	Var_1	Var_2	Cov	Var_{2007}
Austria	1.05	2.58	0.60	4.84	Ireland	1.75	4.32	1.01	8.08
Belgium	2.07	5.08	1.19	9.53	Italy	2.08	5.13	1.20	9.61
Czech	1.67	4.11	0.96	7.69	Lithuania	1.39	3.41	0.80	6.39
Denmark	1.27	3.12	0.73	5.84	Netherlands	2.32	5.72	1.34	10.72
Estonia	1.28	3.16	0.74	5.91	Poland	2.44	6.02	1.41	11.27
Finland	0.64	1.58	0.37	2.95	Slovakia	1.37	3.36	0.79	6.30
France	0.98	2.41	0.56	4.51	Spain	1.10	2.70	0.63	5.06
Germany	1.47	3.63	0.85	6.79	Sweden	2.10	5.18	1.21	9.70
Greece	1.05	2.58	0.60	4.84	UK	1.62	3.99	0.93	7.47
Hungary	1.02	2.50	0.58	4.68					

Table 6. Total forecasted transportations, in thousands of tons

Country	2006*	2006	2007	Upper conf. limits			
				60%	70%	80%	90%
Austria	227.92	174.21	196.60	204.77	213.73	224.09	238.47
Belgium	1 124.95	742.95	781.78	788.92	796.75	805.80	818.35
Czech	57.05	90.63	133.22	136.51	140.12	144.30	150.09
Germany	3 267.91	3 213.93	3 444.38	3 498.02	3 556.90	3 624.96	3 719.35
Denmark	6.69	55.71	72.64	73.45	74.35	75.38	76.82
Estonia	10.05	16.08	20.96	24.03	27.40	31.29	36.69

Country	2006*	2006	2007	Upper conf. limits			
				60%	70%	80%	90%
Spain	368.97	673.48	811.22	817.36	824.11	831.91	842.72
Finland	118.43	84.17	101.16	126.16	153.60	185.32	229.31
France	1 421.41	1 105.82	1 307.02	1 340.66	1 377.59	1 420.27	1 479.47
Greece	90.60	154.94	197.92	243.74	294.03	352.17	432.80
Hungary	64.89	41.21	81.90	84.04	86.39	89.10	92.87
Ireland	111.64	86.28	109.44	114.88	120.86	127.77	137.35
Italy	764.05	762.71	949.88	963.28	977.97	994.96	1 018.52
Lithuania	12.67	21.94	36.12	40.69	45.70	51.49	59.53
Netherlands	1 621.35	1 499.07	1 545.27	1 577.28	1 612.40	1 653.00	1 709.32
Poland	32.02	104.63	193.87	197.25	200.97	205.26	211.22
Sweden	139.48	66.41	94.66	96.93	99.41	102.29	106.28
Slovakia	5.36	50.44	61.37	68.46	76.25	85.25	97.74
UK	2 248.41	2 044.67	2 248.39	2 289.97	2 335.60	2 388.36	2 461.52

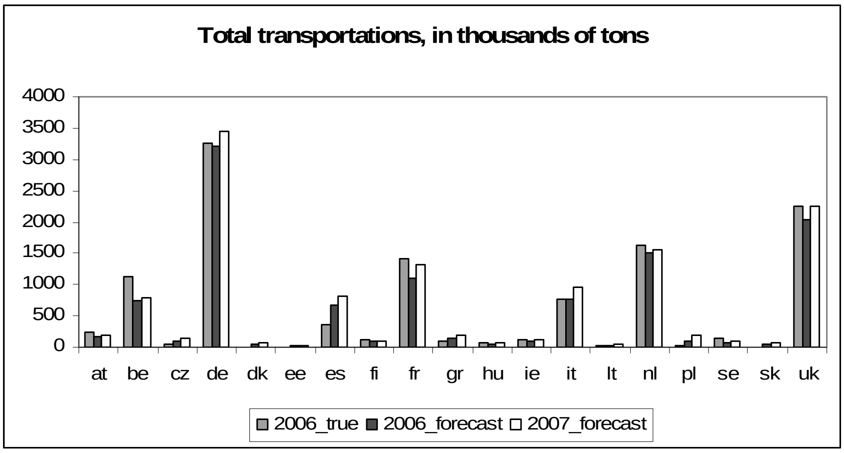

Fig. 3. *Total observed and forecasted transportations*

Table 6 contains the mentioned above total forecasts for 2007 and for 2006 (9) (columns 2007 and 2006 respectively) and sums of observed internal and external transportations for 2006 (column 2006*). We have calculated the upper confidence limits (14) with different confidence probabilities γ for obtained total forecasts for 2007 for each country. The corresponding data are presented in Table 6 as well. At Figure 3 we can see represented in Table 6 observed and forecasted transportations.

8. Conclusions

The given paper is purposed to elaboration of regression models for member countries of the Europe Union freight and mail air international transportations analysis and forecasting. The air international transportations are divided into two parts, thus there are two forecasted variables: internal and external transportations in relation to the EU. Two types of models have been investigated: multiple regression and multivariate regression. The last model is applied for forecasting of sums of the mentioned transportations by means of confidence intervals.

The results obtained in the research show conclusive evidence of the proposed approach. However, individual models would be preferable for forecasts obtaining for several countries, and stated approach also could be applied for forecasting of factors of interest for the EU in totals. It is especially important, when there are no observations on the separate countries for certain years. It is not recommended to forecast total transportations on the basis of time series of transportations sums. In this connection it is supposed to improve the present research.

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