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TOWARDS APPLYING A THEORY OF CHAINS AND SIGNALS FOR SOLVING ENGINEERING TASKS

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1. Introduction

In order to solve complex engineering tasks, the development of modelling systems and methods is often necessary. In particular, in engineering design, it is ordinarily to consider any product component as the behavioural model described by the equation set or a certain algorithm. If the interaction among components is realised through common variables, then a total behavioural model of the product can be created by using a theory of chains and signals [1]. The aim of this paper is to describe a general theoretical background of such approach and to present some examples of equivalent chains for modelling behaviours of multi-component devices and processes of different physic nature.

2. Theoretical background

In general form, a chain *C* is an arbitrary set of components, whose behaviours are described by a set *V* of decision variables. The calculated values of decision variables specify the solution of chain *C*.

The component *M* with decision variables V_M is associated with the graph $\Gamma_M = (V_M, S_M, E_M)$ shown in Fig.1, where S_M is a set of component links, and E_M is a mapping of V_M on S_M. The mapping E_M involves a mathematical model of component *M* (in the form of equations or algorithm) and method of solving that model. This definition of a component is used in different chains for supporting calculations at topological, numerical and physical levels of simulation modelling.

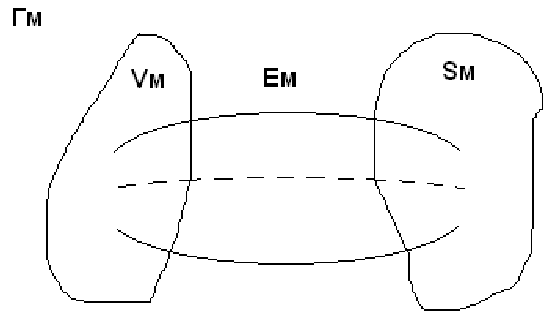

Figure 1. General model of a chain component

At the topological level, as shown in Fig.2, a chain C can be described as the graph $\Gamma_C = (N, S, B)$, where N is a set of chain nodes, $S = \cup S_M$ is a totality of all component links with decision variables, B is a set of chain branches representing links of nodes with variables in a set N. The chain branches can be both directed and undirected ones.

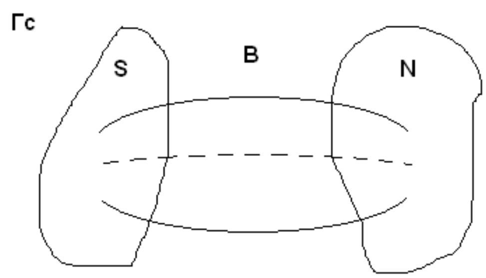

Figure 2.Topological model of a chain

At the numerical level (see Fig.3), a chain C should be viewed as the graph $G_C = (V, \{N, B\}, E_C)$, where V is a set of decision variables, $\{N, B\}$ is a totality of chain nodes and branches, E_C is a mapping that involves mathematical models of components, nodal topological laws and method of solving the integrated mathematical model.

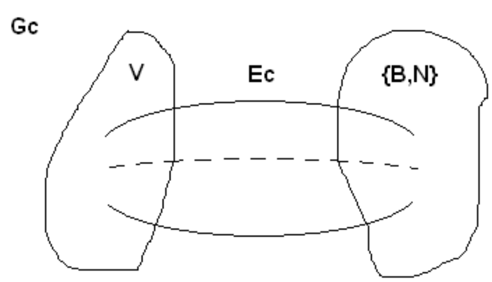

Figure 3. Numerical model of a chain

The given definition of a chain is a recursive because of the compatibility of graphs Γ_C and G_C. It means that viewing nodes and branches of graph G_C by way of chain links, we enable using a chain C as a component of other chains. Hence, a hierarchical representation of complex system including a hierarchical organisation of numerical process within behavioural modelling is realised. In that case, we should call the graph G_C a sub-chain or macro component. Also, we should distinguish on a set $\{N, B\}$ its subsets $\{N_\Gamma , B_\Gamma\}$, whose elements result in a set S_M of the so-called output nodes and branches of graph Γ_C.

In the process of behavioural modelling, the configuration of component models and overall number L of decision variables characterising the dimension of chain C can be changed. This can be caused by modification of the system structure or by a change of the system states depending on a change of parameter values. Correspondingly, the configuration of $\{N, B\}$ and method of solving a chain C as a whole (believing that $E_M \subseteq E_C$) is also changed. Thus, in final form, the component model M can be represented by the six sets as follows:

$$M = (M_R, M_m, M_B, M_\Gamma, M_A, P_m), \text{ where} \tag{1}$$

M_R is topological data about configuration and variable links of component M;
M_m is a totality of mathematical models in accordance with different applications of that component;
M_B is a totality of numerical methods used in given mathematical models;
M_Γ is a totality of possible graphical images of component M including the representation of its dynamic characteristics specified by current values of variables V_M;
M_A is a totality of criteria used for choosing elements from M_m, M_B and M_Q;
P_m denotes component parameters.

The introduced definitions and matched software support are sufficient to describe a wide class of multi-component devices and systems for modelling their behaviours. The process of modelling is fulfilled subject to the division of set V into subsets V_p and V_b of potential and flow decision variables relatively. The process consists of the following stages:

1. Inquiry of component models M_R of the investigated chain to construct the incidence matrix regarding V_p and V_b. Indexing variables V_M to represent the solution of chain C so as $V = \cup V_M$.
2. Generating the system of linear algebraical equations to set nodal topological laws like the law of those variables V_p equality, which have the same name and are incidental to the same node. As a result, the general index is selected for them. Example is Kirchhoff's law for electric currents. Variables V_b are involved with description of components used for the representation of nodal laws in the form of graphs of links. Equations for them are not constructed.
3. Inquiry of mathematical models M_m to construct the system of linearised algebraical equations by formulas of numerical integration for regular differential equations, by means of the finite-difference approximation for equations with partial derivatives or by means of iteration methods for nonlinear equations.
4. Solution of the aggregate system of linearised algebraical equations to work out the solution V of investigated chain for each time step or iteration step.
5. Visualisation of solution V in accordance with models M_Γ of investigated chain.

A choice of visualisation mode and modelling accuracy along with other information is given before the system initialisation. For modelling dynamic processes, the time lag $[T_{min}, T_{max}]$ is also assigned.

3. Modelling the production process

In developing models of production process, different factors are taken into account. First of all, they are the lead time needed for making products in each production sector, constraints by raw materials, stocks used, and others. Production sectors play the role of basic components of the chain. Models of components are algorithms that calculate inputs and outputs of production sectors. The interaction between components is provided with the help of oriental chain links. On these links, the variables operate to describe types and volumes of output products. In that case, all variables are interpreted as potential ones. Therefore, it is not required to construct and to solve the equations set. It is sufficient to have in the system a library of necessary components (unique ones for each production) to solve real problems with using a graphic shell and tool applied for drawing component models. In order to evaluate a guaranteed production output the method of statistical tests is applied.

Flow variables are introduced allowing for other features of production, such as water supply, power supply, a compressed air delivery, and so on. An example of water supply distribution network used for production needs is shown in Fig.4.

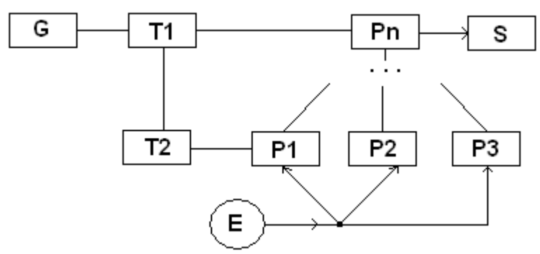

Figure 4. A simplified scheme of production with water supply

Here, the component *G* simulates the operation of pumping station as a source of water pressure; the component *E* is a source of raw materials and component parts; the component *S* is storage of finished products; $P_1, P_2, \dots, P_n$ are production sectors including blocks for monitoring water pressure; T_1 and T_2 are local networks of water supply.

In contrast to production sectors, components T_1 and T_2 are described by equations for calculating parameters of chain flow (for different pipe lengths) like:

$$V_1 - V_2 - R \cdot Q = 0, \quad (2)$$

where V_1 and V_2 are pressures on pipe ends; *Q* is flow quantity; *R* is a laminar water resistance.

These equations are involved in a common model of production process accounting the cost of water supply. It is also possible to solve an inverse problem, when input is warehouse *E* specifying the production requirements.

4. Equivalent models of electrical chains

Sometimes, components with using potential and flaw variables are introduced in order to replace combinatorial methods of optimisation by equivalent models of electrical chains. Simplest examples are problems of optimal sharing resources such as delivery of goods and goods dynamic consuming.

4.1. Delivery of goods

Let some goods should be delivered from a point *A* to a point *B* with minimal transport costs. A generalised scheme of transport network is shown in Fig.5a. Analysing possible delivery routes we can represent a characteristic segment of that transport network in form of equivalent electrical chain shown in Fig.5b. Here, components R_i correspond to transport costs, a total volume of goods is modelled by potential source *Z*, and terminal *B* is modelled by an earth of electrical chain.

We can see that one delivery route is to create parallel transport streams passing through components R_1, R_2,…, R_n, and other route is to provide a single transport stream through components R_{n+1} and R_{n+2}.

Let *J* be flow variable (an electrical current) that simulates a volume of goods passing through component *R*. Then, a model of this component can be described as the linear equation like (3).

$$U_1 - U_2 - R \cdot J = 0, \qquad (3)$$

where U_1, U_2 are potentials on *R* ends.

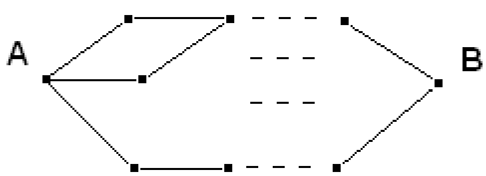

Figure 5a. Scheme of transport network

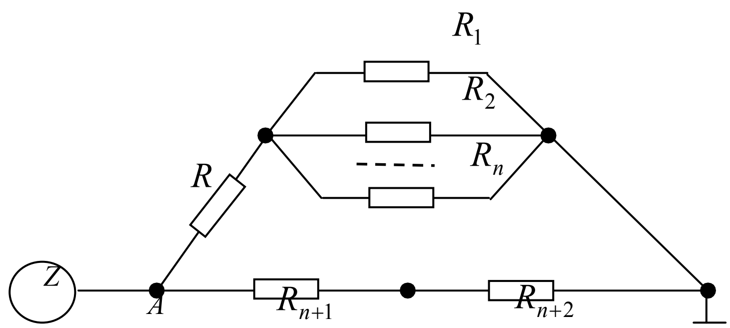

Figure 5b. Equivalent model of electrical chain

It is evident that a choice of optimal route based on minimisation of current *J* can result in mistake, because the decrease in current *J* leads to the decrease in parallel currents passing through components $R_1, R_2,…, R_n$ (and vice versa). Accordingly, total transport costs will increase. Therefore, under large *n*, the optimal route can pass through R_{n+1} and R_{n+2}. By reason of this situation, a nonlinear character of *R* in equation (3) is assigned.

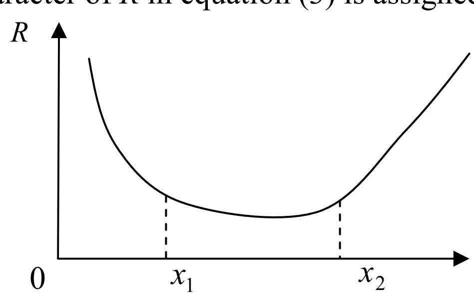

Figure 6. A change of electrical resistance *R* in dependence on current *J*

A change of *R* values in dependence on volume *J* of goods is shown in Fig.6, where the curve line [0, x_1] characterises a trend to the constancy of transport costs for small volumes of goods. For example, a cost of trucking one match equals to a cost of trucking one box of matches. The curve line [x_2, >x_2] indicates restrictions by the currying capacity of a given path line.

Thus, in choosing the correct steepness of initial path line for *R*, the optimal or near-optimal distribution of streams in the transport network is achieved. In case of need, transport costs can also involve the costs concerned with a time of passing certain length of path.

4.2. Goods dynamic consuming

This problem differs from the previous one in that it considers the effectiveness of consuming resources for different objects in dependence on time. For example, income *S* from seasonal sales of some goods can be represented by the curve in Fig.7, where variable *Z* denotes the volume of some goods being distributed.

The curve line [0, x_1] corresponds to the first emergence of these goods into the market when income *S* increases progressively. Further, the demand saturation [x_1, x_2] occurs, and income S doesn't increase. At last, the period [x_2, >x_2] arises inevitably when income *S* begins to drop because of the marketable surpluses and unrealised stock of goods.

Equivalent model of electrical chain for modelling this task is shown in Fig.8. Here, income *S* is inversely proportional to costs *R* concerned with goods consuming.

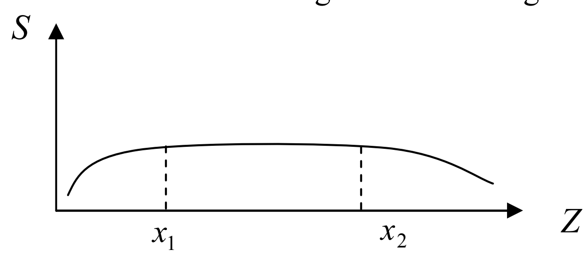

Figure 7. A graphic chart among income S and a volume *Z* of goods

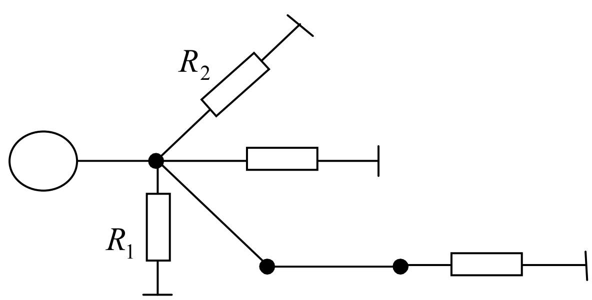

Figure 8. Electrical chain for modelling resources being consumed

Also, the components for indicating transport costs can be used in links of the chain. Besides, parallel with R_i, additional components can be used to model the distribution loss caused by reduction of range of goods.

5. Conclusion

The paper describes the method and some examples of behavioural modelling based on using a theory of chains and signals. It is indicated that equivalent models of electrical chains enable the engineer to receive solutions close to optimal ones. In conditions of uncertainty, these models can be used for finding white solutions to avoid errors in making fundamental technical and economical decisions.

References

1. Арайс Е.А., Дмитриев В.М. Моделирование неоднородных цепей и систем на ЭВМ. М., Радио и связь, 1982, 160с.
2. Арайс Е.А., Дмитриев В.М.Автоматизация моделирования многосвязных механических систем. М.,Машиностроение, 1987, 240 с.
3. Arajs, J. Napalkov, E**.** Zars,V. *Integrated System for Simulation of Multicomponent Engineering Products.* Proceedings of RTU 45. International Conference "Information technologies and management", October, 2004, Riga, P. 106-113.

Arais Jevgenijs, Napalkovs Eduards. Ķēžu un signālu teorijas pielietošana inženiertehnisko uzdevumu risināšanā

Dotajā darbā ir aprakstīta izstrādātā sistēma, kas ir izmantota daudz komponenšu, fiziski heterogēnu tehnisko ierīču un procesu modelēšanai. Ir sniegts teorētiskais pamatojums tāda veida ierīču un procesu modelēšanai. Šis teorētiskais pamatojums balstās uz elektrisko ķēžu un signālu teoriju. Elektriskās ķēdes var būt pielietotas mehānisko, hidraulisko un elektronisko ierīču modelēšanai. Dotajā rakstā kā piemēri ir izskatītas izstrādātās sistēmas pielietošanas iespējas ražošanas procesu, transporta plūsmu un resursu sadalīšanas modelēšanai. Izstrādātās sistēmas ietvaros ķēde tiek attēlota kā komponenšu kopa. Šajā kopā katrs komponents tiek aprakstīts ar vienādojumu sistēmu vai algoritmu. Ja komponenšu modeli ir nodefinēti, tad tiek veidots tehnisko ierīču un procesu pilnais skaitliskais modelis. Šinī gadījumā tiek izmantota specifiskā tehnoloģija. Šajā darbā ir aprakstīta secība, kurā jāpārveido ķēdes topoloģijas modelis skaitliskajā modelī. Saites starp komponentēm tiek attēlotas kā koku zari, kuriem tiek piešķirti potenciālie un plūsmas mainīgie. Piemērā ir parādīts kā var aprakstīt incidenču attiecības starp komponentēm un koku zariem, pielietojot potenciālos un plūsmas mainīgos.

Arais Jugenij, Napalkov Eduard. Towards applying a theory of chains and signals for solving engineering tasks

The paper contains generic information about the system developed for behavioural modelling of multi-component and physically heterogeneous technical devices and processes. It includes a theoretical background of modelling system based on construction of electrical chains and signals. Enabling the designer to make decisions in the process of performing different experiments these chains can be used for behavioural modelling of mechanic, electronic, hydraulic and other devices. So, the different applications of the developed system are analysed including the cases study of production process, approximate solutions of some transportation problems and the problem of resources dynamic allocation. The basis of the developed system is representation of any chain in the form of a set of components whose behaviours are described by mathematical equations or certain algorithms. If the component models are known, then a specific technique is used for building a total numerical model of investigated device or process. In the paper, a sequence of transforming the topologic model into the numeric model of a device or process is described including the visualisation of its behaviour. Also, it is indicated that links between components can be viewed as chain branches, where potential and flow variables can operate. Accordingly, the cases of its operation are presented in describing the relationships of incidence between components and branches.

Арайс Евгений, Напалков Эдуард. К применению теории цепей и сигналов для решения инженерно-технических задач

В статье представлены общие сведения о разработанной системе поведенческого моделирования многокомпонентных и физически разнородных технических устройств и процессов. Статья включает теоретические основы моделирующей системы, определяемые теорией построения электрических цепей и сигналов. Эти цепи могут быть использованы для моделирования механических, гидравлических, электронных и других устройств. Рассматриваются различные приложения разработанной системы на примерах построения эквивалентных цепей для моделирования производственных процессов, распределения транспортных потоков в зависимости от объёма перевозимого груза, решения задачи динамического распределения ресурсов. В основе разработанной системы лежит представление о цепи как о множестве компонентов, поведение каждого из которых описывается системой уравнений или определенным *алгоритмом. Если модели этих компонентов заданы, то используется технология, по которой строится общая вычислительная модель технического устройства или процесса. В статье в общем виде представлена последовательность преобразования топологической модели цепи в вычислительную модель, включая визуализацию поведения. Показано, что связи между компонентами удобно рассматривать как ветви цепи, на которых могут действовать потенциальные и потоковые переменные. Соответственно, приведены примеры использования этих переменных для описания отношений инцидентности между компонентами и ветвями цепи.*