RĪGAS TEHNISKĀ UNIVERSITĀTE
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Matemātiskās modelēšanas doktora programmas doktorants

# SIENAS NEGLUDUMA IETEKME UZ MHD PLŪSMAS STRUKTŪRU UN SEKLA ŪDENS PLŪSMAS STABILITĀTI

# THE EFFECT OF SURFACE ROUGHNESS ON THE STRUCTURE OF MAGNETOHYDRODYNAMIC FLOWS AND STABILITY OF SHALLOW WATER FLOWS

Promocijas darba kopsavilkums
Summary of Ph.D thesis

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**PROMOCIJAS DARBS**
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APSTIPRINĀJUMS

Es apstiprinu, ka esmu izstrādājis doto promocijas darbu, kurš iesniegts izskatīšanai Latvijas Universitātē matemātikas doktora grāda iegūšanai. Promocijas darbs nav iesniegts nevienā citā universitātē zinātniskā grāda iegūšanai.

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Promocijas darbs ir uzrakstīts angļu valodā, satur 5 nodaļas, literatūras sarakstu, 16 zīmējumus un ilustrācijas, 4 tabulas, kopā 92 lappuses. Literatūras sarakstā ir 75 nosaukumi.

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## IEVADS

Promocijas darbs ir veltīts tādu faktoru analīzei, kuri ietekmē magnetohidrodinamisko (MHD) plūsmu un sekla ūdens plūsmu struktūru un stabilitāti. Jo īpaši sienas pretestības ietekmi uz plūsmu var raksturot lokāli (ņemot vērā robežas negludumu) vai globāli (izmantojot pusempīriskas formulas, kas apraksta iekšējās berzes ietekmi).
Robežas negludums var rasties korozijas dēļ. Eksperimentālie rezultāti parādīja nozīmīgu magnētiskā lauka ietekmi uz korozijas procesiem – gan uz korozijas intensitāti, gan tās īpašībām. Tādēļ no praktiskās puses ir būtiski analizēt negluduma ietekmi uz MHD plūsmu struktūru. Šī ietekme darbā tiek novērtēta, analītiski atrisinot magnētiskās hidrodinamikas vienādojumu sistēmu (izmantojot Furjē transformācijas). Darbā ir skatītas vairākas virsmas negluduma formas. Analītiskie risinājumi ir iegūti un ātruma sadalījums ir skaitliski analizēts dažādiem Hartmana skaitļiem. Tāpat ir iegūts asimptotiskais risinājums lieliem Hartmana skaitļiem. Risinājumi ir iegūti kā integrāļi, kas iekļauj oscilējošas funkcijas. Darbā šie integrāļi tiek pārveidoti par integrāļiem no neoscilējošām funkcijām.
Iekšējās berzes globālo ietekmi visbiežāk ņem vērā, izmantojot empīriskās pretestības formulas, piemēram, Čezi formulu, lai prognozētu turbulentu plūsmu "koncentrācijas" ietekmi, lai izskaitļotu plūsmas ātrumu un zudumus kanālos vai caurulēs un atvērto kanālu konstrukciju. Šīs formulas satur empīriskas berzes koeficientus, kas ir tieši saistīti ar plūsmas Reinoldsa skaitli un robežvirsmas negludumu. Tiek uzskatīts, ka saistītās struktūras plūsmās aiz šķēršļiem parādās kā plūsmas hidrodinamiskās nestabilitātes galaprodukts. Iepriekš dažādām plūsmām tika izmantotas vāji nelineārās stabilitātes teorijas metodes, kas parasti noved pie amplitūdas evolūcijas vienādojumiem visnestabilākajā režīmā. Viens no šādiem vienādojumiem ir kompleksais Ginzburga-Landau vienādojums. Izmantojot vāji nelineāro teoriju kvazi-divdimensionālām plūsmām ar Releja berzi (tiek pieņemts, ka iekšējā berze ir lineāri saistīta ar ātruma sadalījumu), tika secināts, ka ātrums ir ļoti atkarīgs no bāzes plūsmas profila. Literatūras apskatā tika secināts, ka vāji nelineāros modeļus nevar izmantot šādiem gadījumiem, jo nav iespējams eksperimentāli noteikt ar augstu precizitāti bāzes plūsmas ātruma sadalījumu un tādēļ nav iespējams izmantot ticamas Ginzburga-Landau vienādojuma koeficientu vērtības. Darbā ir parādīts, ka ja nelineāru funkciju izmanto grunts berzes modelēšanai, tad Ginzburga-Landau vienādojuma koeficienti nav jutīgi pret bāzes plūsmas ātruma sadalījumu.

## PĒTĪJUMA AKTUALITĀTE

Ir būtiski pētīt un analizēt MHD plūsmas cauruļvados vai kanālos magnētiskā lauka klātbūtnē tādiem pielietojumiem kā MHD ģeneratoru un sūkņu projektēšana un analīze. Vadošā metāla virsmas negluduma ietekme uz MHD plūsmu ir noderīga procesos, kas notiek *Tokamak* reaktora dzesēšanas sistēmās. Turklāt šādu reaktoru

projektēšanā un veiktspējas analīzē ir jāņem vērā korozija. Ir zināms, ka magnētiskais lauks ietekmē gan korozijas pakāpi, gan korozijas īpašības. Tādēļ ir būtiski analizēt korozijas ietekmi uz MHD plūsmu struktūru.
Stabilitātes īpašības seklām plūsmām aiz šķēršļiem ir nozīmīgas no vides viedokļa. Vāja ūdens cirkulācija reģionos aiz salām var izraisīt ūdens kvalitātes pasliktināšanos un dažos gadījumos pat zivju bojāeju. Tādēļ ir būtiski zināt seklu plūsmu struktūru reģionos aiz šķēršļiem.

## DARBA MĒRĶIS

Darba mērķis ir analizēt virsmas negluduma ietekmi uz MHD plūsmu struktūru un seklu plūsmu stabilitāti. Virsmas negludums rodas no vadošā šķidruma izraisītās korozijas magnētiskā laukā. Iegūtos analītiskos risinājumus var izmantot korozijas ietekmes prognozēšanai šādos gadījumos.
Lineāras un vāji nelineāras stabilitātes analīze seklām plūsmām var noderēt, nosakot ūdens paraugus reģionos aiz tādiem šķēršļiem kā salas. Darbā iegūtos rezultātus var izmantot seklu plūsmu vides novērtēšanai.

## PĒTĪJUMA METODES

Šajā darbā analizētie matemātiskie modeļi ir bāzēti uz magnētiskās hidrodinamikas vienādojumiem un sekla ūdens vienādojumiem. MHD plūsmu analīzei pār negludiem elementiem tiek izmantotas šādas metodes:

1. magnētiskās hidrodinamikas vienādojumu transformācijas, lai iegūtu inducētā magnētiskā lauka veidu plūsmas reģionā, ja ir doti negluduma elementi;
2. Furjē kosinusa un sinusa transformācijas MHD plūsmu pār negluduma elementiem problēmu analītiskajam risinājumam;
3. rezidiju teorēmu izmanto integrāļu, kuru zemintegrāļu funkcijas ir oscilējošas, pārveidošanai par monotonu funkciju integrāļiem;
4. neīstu integrāļu skaitliskais novērtējums ar "Mathematica".

Sekla ūdens plūsmas modelis ir izmantots lineāras un vāji nelineāras nestabilitātes analīzei vienai no plūsmu klasēm. Analīzes metodes iekļauj:

1. lineārās stabilitātes analīzi kustības vienādojumiem;
2. kolokācijas metodi, kuras pamatā ir Čebiševa polinomi, stabilitātes robežas aprēķināšanai;
3. asimptotiskos izvirzījumus kritiskā punkta apkaimē, lai veiktu vāji nelineāras stabilitātes analīzi;
4. skaitliskās metodes robežvērtības problēmu atrisināšanai vienkāršiem diferenciālvienādojumiem, lai aprēķinātu amplitūdas evolūcijas vienādojuma koeficientus.

## ZINĀTNISKĀ NOVITĀTE UN GALVENIE REZULTĀTI

Ir iegūts magnētiskā lauka veids un MHD vienādojumi pilnībā attīstītai plūsmai, ko izraisījis robežvirsmas negludums.
Ar analītisko metožu palīdzību ir analizēta virsmas negluduma ietekme uz MHD plūsmu struktūru pustelpā. Virsmas negludums tiek pieņemts kā konstante dotajā intervālā.
Iegūtais asimptotiskais atrisinājums tiek analizēts lielu Hartmana skaitļu gadījumam.
Neīstie integrāļi, kas satur oscilējošas funkcijas, kuras radušās izmantojot MHD plūsmas pār negluduma elementiem, tiek pārveidoti par integrāļiem, kas satur monotonas funkcijas, kuras ir skaitliskiem aprēķiniem.
Ir veikta teorētisko rezultātu analīze, kas attiecas uz eksperimentiem par korozijas ietekmi magnētiskā laukā.
Lineāra un vāji nelineāra sekla ūdens plūsmu analīze ir veikta vienai plūsmu klasei. Pētījumā ir izmantots nelineārs Čezi modelis, lai atveidotu grunts berzi. Ir parādīts, ka Ginzburga-Landau vienādojuma koeficienti nav jutīgi pret bāzes plūsmu profila formu pretēji iepriekšējiem pētījumiem, kur berze tika modelēta ar lineāru ātruma funkciju.

## PIELIETOJUMI

Iepriekšējie eksperimentālie pētījumi Fizikas institūtā Salaspilī (Latvijā) ir parādījuši, ka dažu apstākļu ietekmē virsmas, kas pakļautas magnētiskam laukam, veido kaut kādu regulāru viļņveidīgu struktūru . Vispārējos vilcienos – magnētiskais lauks ietekmē ne tikai korozijas pakāpi, bet arī korozijas raksturu . Tādēļ jāņem vērā virsmas negludums, skatot reālus modeļus šķidra metāla plūsmām reaktorā. Darbā redzamos teorētiskos rezultātus var izmantot, analizējot korozijas procesu magnētiskā laukā.
Lineāras un vāji nelineāras stabilitātes metodes bieži izmanto seklu plūsmu struktūras analīzē. Darbā grunts berze ir modelēta ar nelineāru ātruma funkciju. Darbā ir parādīts, ka Ginzburga-Landau vienādojuma, kas apraksta plūsmu nestabilitātes attīstību virs sliekšņa, koeficienti nav jutīgi pret bāzes plūsmas profila variācijām. Šis rezultāts ir pretrunā iepriekšējiem pētījumiem, kad iekšējā berze tika modelēta ar lineāru ātruma funkciju. Tādēļ ir ticams pieņēmums, ka vienkāršotus modeļus, kas bāzēti uz tik sarežģītiem amplitūdas evolūcijas vienādojumiem kā Ginzburga-Landau vienādojums, var izmantot apgabala virs sliekšņa visnestabilākā režīma uzvedības analīzei.

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## ĪSS SATURS UN DARBA STRUKTŪRA

Darbā ir piecas nodaļas, ievads un secinājumi. Tajā ir 92 lappuses, 16 attēli un četras tabulas. Literatūras sarakstā ir 75 izmantotās literatūras darbu nosaukumi. Darbs ir uzrakstīts angļu valodā.

# 1. nodaļa. Ievads.

Ievadā ir skatīts literatūras apskats. Papildus tam ir apspriesta darba struktūra un galvenie rezultāti.

Promocijas darba galvenais temats ir tādu faktoru analīze, kas ietekmē MHD plūsmu un sekla ūdens plūsmu struktūru un stabilitāti. Galvenokārt koncentrēsimies uz robežvirsmas negluduma ietekmi. Analīzes metodes ir pamatotas uz analītiskiem risinājumiem, kas atrasti dažām MHD plūsmām pār negluduma elementiem spēcīgos magnētiskos laukos taisnstūrveida kanālos. MHD risinājumi, kas aprakstīti darbā, atvieglo šķidruma analīzi apgabalos, kur ir spēcīgs magnētiskais lauks (Hartmana skaitlis ir liels). MHD plūsmu uzvedības analīze ar lieliem Hartmana skaitļiem ir temats, par kuru ir pieaugoša interese, jo to izmanto galvenokārt MHD iekārtās, piemēram, sūkņos, un MHD ģeneratoros. MHD šķidruma-metāla plūsmu ar lielu Hartmana skaitli galvenās īpašības ir: „homogēns” ātruma profils kanāla kodolā un plāni robežas slāņi tuvu robežvirsmām. Elektriskā strāva, kas inducēta šķidrumā, maina plūsmas lauku. Zinot šīs strāvas ceļu ir iespējams paredzēt plūsmas struktūru.

Hidraulikas inženieri efektīvi izmanto Čezi formulu „lempīguma” efekta enerģijas zudumu prognozēšanai turbulentās plūsmās plūsmas ātruma un zaudējumu aprēķināšanai kanālos vai caurulēs. Par robežvirsmas negludumu ir gādāts, izmantojot empīriskus berzes koeficientus. Šie koeficienti ir saistīti ar vairākām empīriskām formulām pie plūsmas Reinoldsa skaitļa, kā arī pie robežvirsmas negluduma. Tiek uzskatīts, ka līdzplūsmās saistītās struktūras parādās kā plūsmas hidrodinamiskās nestabilitātes gala produkts. Vāji nelineārās teorijas metodes ir aplūkotas literatūrā dažādām plūsmām un visbiežāk noved pie amplitūdas evolūcijas visnestabilākajā režīmā. Viens no šādiem vienādojumiem ir kompleksais Ginzburga-Landau vienādojums. Vāji nelineārā teorija ir izmantota kvazi-divdimensiju plūsmām [22] ar Releja berzi (iekšējo berzi pieņem kā lineāri saistītu ar ātruma sadalījumu). Kā redzams [22], Ginzburga-Landau vienādojuma koeficienti gadījumam, kad iekšējo berzi attēlo ar lineāru ātruma funkciju, ir ļoti atkarīgi no bāzes plūsmas profila. Tādēļ [22] ir secināts, ka šādos gadījumos nevar izmantot vāji nelineārus modeļus, jo nav iespējams eksperimentāli noteikt bāzes plūsmas ātruma sadalījumu ar augstu precizitāti un tādēļ analīzē nav iespējams izmantot ticamas vērtības Ginzburga-Landau vienādojuma koeficientiem. Tomēr šā darba 5. nodaļā ir parādīts, ka nelielas lineāras stabilitātes īpašību variācijas neizraisa lielas Landau konstantes izmaiņas (Landau konstante ir reālā daļa vienam no Ginzburga-Landau vienādojuma koeficientiem ), kad grunts berzes modelēšanai ir izmantota nelineāra Čezi formula.

2., 3. un 5. nodaļas ir teorētiskas, 4. nodaļa ir praktiska, kur skatīta *EUROFER* tērauda korozija Pb17Li plūsmā un tās pielietojums D-T (deitērija-tritija) plazmas ierobežojumam reaktorā.

2. nodaļā ir noteikti MHD plūsmu principi, izklāstīta virsmas negluduma ietekme uz vadošā metāla MHD plūsmu un noteikti galvenie vienādojumi. Tā kā MHD plūsmu problēmas ir plaši pētītas dažādu formu kanālos un ar dažādiem robežu nosacījumiem, šādu pētījumu rezultātiem ir tiešs pielietojums dažādās magnētiskās hidrodinamikas nozarēs [29], [38] un [58]. Tā kā magnētiskā hidrodinamika pēta elektrību vadošu šķidrumu kustību magnētisko lauku klātbūtnē, ir acīmredzami, ka magnētiskie lauki ietekmē šķidrumu kustību. Visbiežāk MHD problēmās elektromagnētiskais spēks tiek pievienots kustības vienādojumam, un magnētiskais

lauks (izmantojot Oma likumu) maina šķidruma kustību. Ir aprakstītas dažas MHD plūsmas problēmas kanālos pāri negluduma elementiem stiprā magnētiskā laukā, un šādu problēmu analītiskie risinājumi tiek iegūti, izmantojot Diraka deltas funkciju (skatīt [6], [7], [12], [13], [17], [18]).

Šo problēmu asimptotiskā analīze veikta spēcīgu magnētisko lauku gadījumam, un strāvas z-komponenšu diagrammas attēlotas dažādiem Hartmana skaitļiem. Lauka ātrumam un strāvas z-komponentēm pie lieliem Hartmana skaitļiem analizēti dažādi robežvirsmas slāņi. MHD problēma pilnībā attīstītai plūsmai ir atrisināta homogēna un nehomogēna ārējā magnētiskā lauka gadījumam, kad ir ņemts vērā virsmas negludums. Šķidruma ātruma sadalījums, inducētā strāva ar tās potenciālu un ārējais magnētiskais lauks ir iegūti (skatīt turpmākās atsauces par līdzīgu problēmu analīzi [2], [5], [11]-[13], [17], [18], [21], [30], [31], [42], [50], [53], [54], [57], [59], [65], [69]).

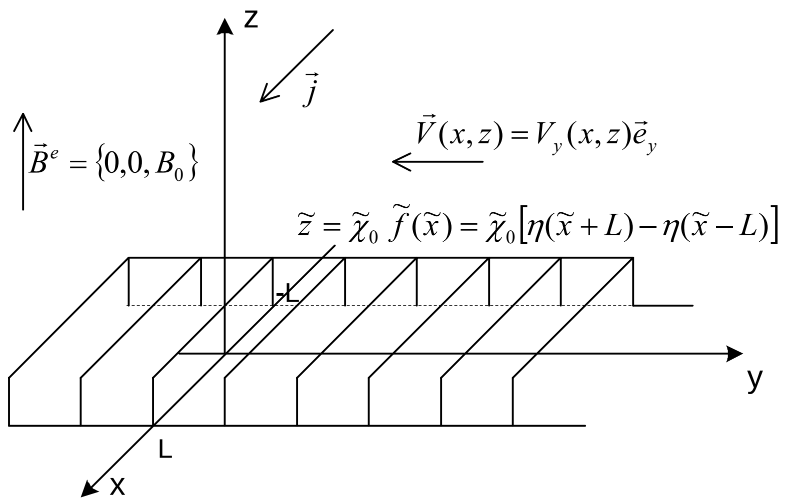
3. nodaļa ir veltīta dažu neīstu oscilējošu integrāļu klašu aprēķināšanai. Ir parādīts, ka dažos gadījumos oscilējošus integrāļus var transformēt par neoscilējošu funkciju integrāļiem. Šādiem integrāļiem ir tiešs pielietojums MHD plūsmām, kas analizētas darbā. Šie rezultāti tiek izmantoti, lai transformētu dažu MHD problēmu risinājumu pustelpā $z \geq 0$ kā rezultātu virsmas negludumam z=0 dažādiem robežslāņiem (skatīt [3], [4], [6], [7], [17], [21] , [72], [74]).

Septiņu gadu laikā, esot Rīgā, Latvijā (viens no galvenajiem MHD pielietojuma centriem Eiropā), man bija iespēja apmeklēt dažas interesantas ar MHD pētniecību saistītas vietas kā, piemēram, Fizikas institūtu Salaspilī, kur esmu redzējis trīs nesen plānotas eksperimentu sesijas (katra 2000 stundas gara), kas ir sekmīgi beigušās. Šajos pētījumos iegūtie rezultāti parādīja magnētiskā lauka nozīmīgu ietekmi uz korozijas procesiem gan korozijas intensitātē, gan tās būtībā. Jauni rezultāti saistībā ar korozijas profilu ir iegūti [55] un [56]. Šādiem pētījumiem ir nozīmīga ietekme uz to, kā ierobežot un kontrolēt D-T plazmas dedzināšanu ar spēcīgu magnētiska lauka slogu reaktora iekšienē (skatīt [1], [9], [55], [56], [70], [73]). Papildus man bija iespēja piedalīties dažās *PAMIR* MHD starptautiskajās konferencēs (4., 5., un 7. *PAMIR* starptautiskā konference). Saistībā ar šīm aktivitātēm ir uzrakstīta promocijas darba 4. nodaļa, kurā izklāstīti praktiskie aspekti saistībā ar virsmas negluduma ietekmi uz MHD plūsmām ([1], [9], [32]-[37], [39], [40], [48], [49], [55]-[57], [60], [64], [68], [70] un [73]).

5. nodaļa ir veltīta sekla ūdens plūsmas analīzei vāji nelineārā režīmā, izmantojot komplekso Ginzburga-Landau vienādojumu (KGLV). Iepriekšējie ar vāji nelineāru kvazi-divdimensiju plūsmu analīzi saistītie pētījumi [22] (sekla ūdens plūsma ir viens no piemēriem, kas apskatīti [22]) ir parādījuši, ka Landau konstantes vērtības atšķiras par koeficientu trīs diviem dažādiem ātruma profiliem, kuru lineārās stabilitātes raksturotāji atšķiras ne vairāk par 20%. Citādi sakot, ir atklāts, ka Landau konstante ir ļoti jutīga pret bāzes plūsmas profila formu. 5.nodaļā grunts berze ir modelēta ar nelineāru Čezi formulu [64]. Datu analīze, kas redzama 1. un 2. tabulā, parāda, ka seklu līdzplūsmu viena parametra saimei izmaiņas lineārās stabilitātes īpašībās izraisa pat mazākas izmaiņas KGLV koeficientos. Tādēļ ir ticams, ka komplekso Ginzburga-Landau vienādojumu var izmantot sekla ūdens plūsmu analīzei vāji nelineārā režīmā (skatīt [8], [10], [14]-[16], [19], [22], [26], [43]-[47] un [67]) izmantošanai vāji nelineāriem modeļiem dažādām plūsmām šķidruma mehānikā.

## 2. nodaļa. Plūsma pār negluduma elementiem stipros magnētiskos laukos.

2. nodaļa ir veltīta MHD plūsmu struktūras analīzei pār dažādu formu negluduma elementiem. Ir iegūti analītiski risinājumi atbilstošajai MHD vienādojumu sistēmai. Ir parādīti skaitlisko aprēķinu un asimptotiskās analīzes rezultāti lieliem Hartmana skaitļiem.
Galvenie MHD principi ir formulēti un skatīti sadaļā 2.1. Tāda magnētiskā lauka forma un MHD vienādojumi pilnībā attīstītai plūsmai, ko izraisījis robežvirsmas negludums, ir iegūti sadaļā 2.2.
Tiek skatīta problēma par MHD plūsmu pustelpā $\tilde{z} \geq 0$, ko izraisījis robežvirsmas negludums $\tilde{z} = 0$ . Pretēji tam, kas paveikts monogrāfijā [75], vispirms tiek pieņemts, ka inducētajam magnētiskajam laukam $\vec{B}^i$ ir $x$, $y$ un $z$ komponentes. Pēc tam tiek izmantota plūsmas simetrija un pierādīts, ka inducētajam magnētiskajam laukam ir viena $y$-komponente. Tiek skatīts viendabīgs magnētiskais lauks sadaļā 2.2.1 un neviendabīgs magnētiskais lauks sadaļā 2.2.2. Plūsmas ģeometrija ir redzama 1. att.




1. attēls. Plūsmas ģeometrija.

Vadošais šķidrums ir pustelpā $\tilde{z} > 0$ , $-\infty < \tilde{x}, \tilde{y} < +\infty$ . Ārējā magnētiskā lauka forma ir

$$\vec{B}^e = B_0 \vec{e}_z \qquad (1)$$

Nepārtraukta strāva plūst ar blīvumu $\vec{j}_0 = j_0 \vec{e}_x$ $x$-ass virzienā. Ja virsma $\tilde{z} = 0$ ir ideāli gluda, tad plūsmas nav, jo elektromagnētiskais spēks $\vec{F} = \vec{j} \times \vec{B}^e$ ir konstants un $\operatorname{rot} \vec{F} = 0$ . Turpmāk pieņemsim, ka uz virsmas $\tilde{z} = 0$ negluduma forma ir :

$$\tilde{z} = \begin{cases} \tilde{f}(\tilde{x}), -L \leq \tilde{x} \leq L, -\infty < \tilde{y} < +\infty, \\ 0, \tilde{x} \notin (-L, L). \end{cases} \qquad (2)$$

Šajā gadījumā pilna strāva ir vienāda ar $\vec{j} = \vec{j}_0 + \vec{j}(\tilde{x}, \tilde{z})$ un šķidruma plūsma ar ātrumu

$$\vec{V}_y = \tilde{V}_y(\tilde{y},\tilde{z})\vec{e}_y \tag{3}$$

plūst $\tilde{y}$ -ass pretējā virzienā (skatīt 1.att.)

Ir pierādīts, ka inducētais magnētiskais lauks $\overset{\rightarrow}{B^i}$ šajā gadījumā ir

$$\tilde{\vec{B}}^i = \tilde{B}^i(\tilde{x},\tilde{z})\vec{e}_y \tag{4}$$

un MHD vienādojumi šķidruma ātrumam $V_y(y,z)$ un inducētās strāvas potenciālam $\Phi(y,z)$, izmantojot bezdimensiju mainīgos, ir

$$\Delta V_y - Ha^2 V_y + Ha\frac{\partial \Phi}{\partial x} = 0, \tag{5}$$

$$\Delta \Phi = Ha\frac{\partial V_y}{\partial x}, \tag{6}$$

kur $\Delta = \partial^2/\partial x^2 + \partial^2/\partial z^2$, $Ha = B_0 L\sqrt{\sigma/\rho\nu}$ ir Hartmana skaitlis un $\sigma,\rho,\nu$ ir attiecīgi šķidruma vadītspēja, blīvums un viskozitāte.
Tiek izmantots nesaspiežama šķidruma MHD vienādojums un Oma likums (skatīt [29], [50] un [58]) :

$$\left(\tilde{\vec{V}}\nabla\right)\tilde{\vec{V}} = -\frac{1}{\rho}grad\tilde{P} + \nu\Delta\tilde{\vec{V}} + \frac{1}{\rho}\left(\tilde{\vec{j}}\times\tilde{\vec{B}}\right), \tag{7}$$

$$\tilde{\vec{j}} = \sigma\left(\tilde{\vec{E}} + \tilde{\vec{V}}\times\tilde{\vec{B}}\right) = \sigma\left(-grad\tilde{\Phi} + \tilde{\vec{V}}\times\tilde{\vec{B}}\right), \tag{8}$$

kur $\Delta = \frac{\partial^2}{\partial\tilde{x}^2} + \frac{\partial^2}{\partial\tilde{y}^2} + \frac{\partial^2}{\partial\tilde{z}^2}$, $\quad \tilde{\vec{V}}\nabla = V_x\frac{\partial}{\partial\tilde{x}} + V_y\frac{\partial}{\partial\tilde{y}} + V_z\frac{\partial}{\partial\tilde{z}}$.

Šajā gadījumā

$$\tilde{\vec{V}} = \tilde{V}_y(x,z)\vec{e}_y, \tag{9}$$

$$\vec{B} = \tilde{\vec{B}}^i(\tilde{x},\tilde{z}) + \vec{B}^e, \tag{10}$$

kur $\tilde{\vec{B}}^i$ ir inducētais magnētiskais lauks.

Vispirms tiek pierādīts, ka

$$\tilde{\vec{B}}^i(\tilde{x},\tilde{z}) = B^i(\tilde{x},\tilde{z})\vec{e}_y \tag{11}$$

ar nosacījumu, ka inducētās strāvas vektora forma ir

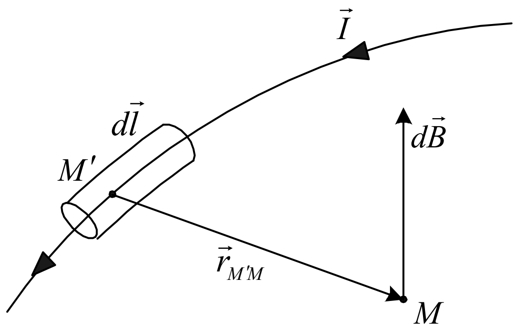
$$\tilde{\vec{j}}(\tilde{x},\tilde{z}) = j_x(\tilde{x},\tilde{z})\vec{e}_x + j_z(\tilde{x},\tilde{z})\vec{e}_z \tag{12}$$

un kā secinājums parādīts, ka šķidruma ātruma vektors ir dots ar (9). Šā mērķa dēļ tiks izmantots Bio-Savāra likums, saskaņā ar kuru inducētā magnētiskā lauka vektors $d\vec{B}$, ko veidojis elements $d\vec{l}$ no bezgalīgi tievas stieples, kas virzīta strāvas $\vec{I}$ virzienā, ir vienāds ar

$$d\tilde{\vec{B}} = I\frac{d\vec{l} \times \vec{r}_{M'M}}{\left|\vec{r}_{M'M}\right|^3} \ , \tag{13}$$

kur $\vec{r}_{M'M}$ ir rādiusa vektors, kas savieno punktu $M'(\tilde{x}', \tilde{y}', \tilde{z}') \in d\vec{l}$ un novērošanas punktu $M(\tilde{x}, \tilde{y}, \tilde{z})$ (skatīt 2. att.):

$$\vec{r}_{M'M} = (\tilde{x} - \tilde{x}')\vec{e}_x + (\tilde{y} - \tilde{y}')\vec{e}_y + (\tilde{z} - \tilde{z}')\vec{e}_z \, . \tag{14}$$




2. attēls. Magnētiskā indukcija $d\vec{B}$, ko izraisījusi elementāra strāva $Id\vec{l}$ .

Nezaudējot vispārinājumu, varam izvēlēties novērošanas punktu *M (0, 0, 0)* koordinātu sākumpunktā. Katram punktam $M'(\tilde{x}', \tilde{y}', \tilde{z}')$ šķidrumā vienmēr var izvēlēties simetrisku punktu $N'(\tilde{x}', -\tilde{y}', \tilde{z}')$ attiecībā pret punktu *M (0, 0, 0).* Tiek apskatīta magnētiskā indukcija $d\vec{B}$, ko izraisījusi elementārā strāva $Id\vec{l}$ , izejot caur punktu $M'(\tilde{x}', \tilde{y}', \tilde{z}')$ un elementārā strāva $I_1 d\vec{l}$ , izejot cauri simetriskajam punktam $N'(\tilde{x}', -\tilde{y}', \tilde{z}')$. (skatīt 3. attēlu) $\vec{I}$ un $\vec{I}_1$ ir strāvas ar blīvumu $\tilde{\vec{j}}(\tilde{x}, \tilde{z})$, kas dots formulā (12).

Tā kā vektors $\tilde{\vec{j}}(\tilde{x}, \tilde{z})$ nav atkarīgs no mainīgā $\tilde{y}$, šajā gadījumā $\vec{I}_1 = \vec{I}$

Tad saskaņā ar formulu (13) tiek iegūts

$$d\vec{B}\Big|_M = Dd\vec{l} \times \left(\vec{r}_{M'M} + \vec{r}_{N'M}\right) \ , \tag{15}$$

kur $D = I\left|r_{MM'}\right|^{-3}$, $d\vec{l} = dl_x\vec{e}_x + dl_z\vec{e}_z$, (16)

$$\vec{r}_{M'M} = -\left(\tilde{x}'\vec{e}_x + \tilde{y}'\vec{e}_y + \tilde{z}'\vec{e}_z\right), \ \vec{r}_{N'M} = -\left(\tilde{x}'\vec{e}_x - \tilde{y}'\vec{e}_y + \tilde{z}'\vec{e}_z\right). \tag{17}$$

Aizvietojot (16) un (17) formulā (15), tiek iegūts:

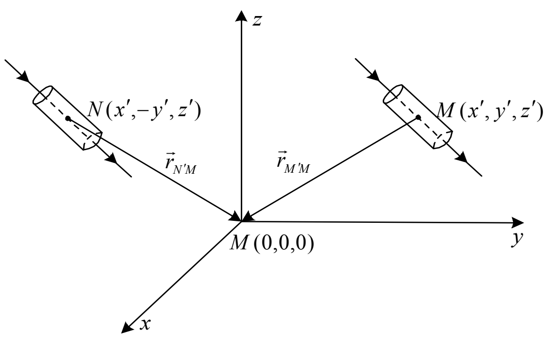
$$d\vec{B} = D\left(2\tilde{z}'dl_x - 2\tilde{x}'dl_z\right)\vec{e}_y \, . \tag{18}$$

Summmējot formulā (18) pa visiem elementiem $d\vec{l}$ šķidrumā, tiek iegūta formula (11), kas pabeidz pierādījumu.

Lai iegūtu vienādojumus (5), (6), tiek aizvietoti vektori $\tilde{\vec{V}}$ un $\tilde{\vec{B}}^i$ no (9), (10) un (11) vienādojumos (7) un (8). Pēc dažiem algebriskiem aprēķiniem tiek iegūts

$$\tilde{\vec{j}} \times \tilde{\vec{B}} = \sigma \left\{ B_0 \frac{\partial \tilde{\Phi}}{\partial \tilde{x}} \vec{e}_y - \frac{\partial \tilde{\Phi}}{\partial \tilde{x}} \tilde{B}^i \vec{e}_z + \frac{\partial \tilde{\Phi}}{\partial \tilde{z}} \tilde{B}^i \vec{e}_x - B_0{}^2 \tilde{V}_y \vec{e}_y + B_0 \tilde{V}_y \tilde{B}^i \vec{e}_z \right\} \tag{19}$$

$$\left( \tilde{\vec{V}} \nabla \right) \tilde{\vec{V}} = 0 \, . \tag{20}$$




3. attēls. Simetrisks attēlojums, kas nepieciešams formulas (18) pierādījumam.

Aizvietojot (19) un (20) vienādojumā (7) un, veicot dažas algebriskas transformācijas, tiek iegūta šāda saistība

$$\nu \left( \frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{z}^2} \right) \tilde{V}(\tilde{x}, \tilde{z}) + \frac{\sigma}{\rho} \left[ B_0 \frac{\partial \tilde{\Phi}(\tilde{x}, \tilde{z})}{\partial \tilde{x}} - B_0{}^2 \tilde{V}_y(\tilde{x}, \tilde{z}) \right] = 0.$$

Tagad tiek izmantoti bezdimensiju lielumi, ņemot vērtības $L, \nu / L, B_0, \nu\sqrt{\rho\nu/\sigma}, \nu\sqrt{\rho\nu/\sigma} / L^2$ attiecīgi kā garuma, ātruma, magnētiskā lauka, potenciāla un strāvas apmērus.

Lai iegūtu vienādojumu (6), tiek izmantoti diverģences operācija vienādojumam (8) un nepārtrauktības nosacījums $div \, \tilde{\vec{j}} = 0$:

$$0 = -\Delta \tilde{\Phi} + B_0 div \tilde{V}_y(\tilde{x}, \tilde{z}) \vec{e}_y \tag{21}$$

Izmantojot formulu (21) bezdimensiju mainīgajiem, tiek iegūts vienādojums (6).

Līdzīga analīze ir veikta sadaļā 2.2, kur ir pieņemts, ka ārējam magnētiskajam laukam un dotajai ārējai strāvai ir tikai $x$ un $z$ komponentes, kas nav atkarīgas no $y$ mainīgā.

MHD problēmas pār negluduma elementiem homogēnā ārējā magnētiskā laukā analītiskais risinājums ir iegūts sadaļā 2.3.

Vadošais šķidrums ir pustelpā $\tilde{z} > 0,\ -\infty < \tilde{x}, \tilde{y} < +\infty$. Ārējā magnētiskā lauka forma ir (1).

Robežvirsma $\tilde{z} = 0$ nevada strāvu. Nepārtraukta strāva plūst ar blīvumu $\vec{\tilde{j}} = j_0 \vec{e}_x$ $x$-ass virzienā. Ja virsma $\tilde{z} = 0$ ir ideāli gluda, tad plūsmas nav, jo elektromagnētiskais spēks $\vec{F} = \vec{\tilde{j}} \times \vec{B}$ ir konstants un $rot\vec{F} = 0$. Pieņemsim, ka uz virsmas $\tilde{z} = 0$ ir taisnstūrveida formas negludums (skatīt 1. att.):

$$\tilde{z} = \tilde{\chi}_0\, \tilde{f}(\tilde{x}) = \tilde{\chi}_0 \left[\eta(\tilde{x} + L) - \eta(\tilde{x} - L)\right] = \begin{cases} \tilde{\chi}_0, & -L < \tilde{x} < L, \\ 0, & |\tilde{x}| > L, \end{cases} \tag{22}$$

kur $\eta(\tilde{x})$ ir Hevisaida funkcija:

$$\eta(\tilde{x}) = \begin{cases} 0, \tilde{x} < 0, \\ 1, \tilde{x} > 0. \end{cases} \tag{23}$$

Šajā gadījumā pilna strāva ir vienāda ar $\vec{j} = \vec{j}_0 + \vec{j}(\tilde{x}, \tilde{z})$ un šķidruma plūsma ar ātrumu $\vec{V} = \tilde{V}_y(\tilde{y}, \tilde{z})\vec{e}_y$ plūst $\tilde{y}$ asij pretējā virzienā (1. att.).

Tiek iegūti robežnosacījumi elektriskā lauka potenciālam $\tilde{\Phi}(\tilde{x}, \tilde{y})$ uz virsmas $\tilde{z} = \tilde{\chi}_0 \tilde{f}(\tilde{x})$. Strāvas normālajai komponentei uz šīs virsmas jābūt vienādai ar nulli, jo robežvirsma $\tilde{z} = \tilde{\chi}_0 \tilde{f}(\tilde{x})$ nevada strāvu, tas ir, tai ir jābūt $\vec{j} \cdot \vec{n} = 0$ uz virsmas ($\vec{n}$ ir virsmas normālā vienība).

Izmantojot formulu $\vec{n} = grad[\tilde{z} - \tilde{\chi}_0 \tilde{f}(\tilde{x})] / \sqrt{1 + \tilde{\chi}_0^{\,2} \tilde{f}'^{2}(\tilde{x})}$, tiek iegūts

$$\vec{n} = \left[-\tilde{\chi}_0 \tilde{f}'(\tilde{x}) \vec{e}_x + \vec{e}_z\right] / \sqrt{1 + \tilde{\chi}_0^{\,2} \tilde{f}'^{2}(\tilde{x})}\,, \tag{24}$$

kur

$$\tilde{f}'(\tilde{x}) = \left[\delta(\tilde{x} + L) - \delta(\tilde{x} - L)\right],$$

$\delta(\tilde{x})$ ir Dīraka delta funkcija.

Aizvietojot $\vec{n}$ no (24) un $\vec{\tilde{j}} = \left(j_0 + \tilde{j}_x(\tilde{x}, \tilde{z})\right)\vec{e}_x + \tilde{j}_z(\tilde{x}, \tilde{z})\vec{e}_z$ par $\vec{\tilde{j}} \cdot \vec{n} = 0$ un izmantojot formulu $\vec{\tilde{j}} = \sigma\left[-grad\tilde{\mathbf{\Phi}} + \vec{\tilde{V}} \times \vec{\tilde{B}}\right]$, t. i., $\tilde{j}_x = -\sigma \partial\tilde{\Phi} / \partial\tilde{x}$, $\tilde{j}_z = -\sigma \partial\tilde{\Phi} / \partial\tilde{z}$ uz virsmu, kur $\vec{\tilde{V}} = 0$, tiek iegūts robežnosacījums potenciālam $\tilde{\Phi}(\tilde{x}, \tilde{z})$:

$$\tilde{z} = \tilde{\chi}_0 \tilde{f}(\tilde{x}):\ -\sigma \frac{\partial\tilde{\Phi}}{\partial\tilde{z}} = \tilde{\chi}_0 \left[ j_0 \tilde{f}'(\tilde{x})\ - \sigma \frac{\partial\Phi}{\partial\tilde{x}} \tilde{f}'(\tilde{x}) \right]. \tag{25}$$

Vienīgā šajā nodaļā veiktā aproksimācija ir: robežnosacījums tiek transformēts no virsmas $\tilde{z} = \tilde{\chi}_0 \tilde{f}(\tilde{x})$ uz plakni $\tilde{z} = 0$, t. i., tiek vienkārši pieņemts, ka $\tilde{\chi}_0 \left|\tilde{f}(\tilde{x})\right|$ vērtība ir maza. Tā rezultātā tiek iegūts robežnosacījums potenciālam formā

$$\tilde{z} = 0 : \partial\tilde{\Phi} / \partial\tilde{z} = \tilde{\chi}_0 \left[-j_0 \sigma^{-1} + \partial\tilde{\Phi} / \partial\tilde{x}\right] \cdot \left[\delta(\tilde{x} + L) - \delta(\tilde{x} - L)\right]. \tag{26}$$

Tiek izmantoti šādi bezdimensiju lielumi, izmantojot vērtības L, $\nu / L$, $B_0$, $\nu\sqrt{\rho\nu / \sigma} / L$, $\nu\sqrt{\rho\nu\sigma} / L^2$ attiecīgi kā garuma, ātruma, magnētiskā lauka, potenciāla un strāvas apmēri. Kur σ, ρ, ν ir attiecīgi šķidruma vadītspēja, blīvums un viskozitāte. Tad MHD vienādojumu un robežnosacījumu forma ir (skatīt [28]):

$$\Delta V_y - Ha^2 V_y + Ha \cdot \partial\Phi / \partial x = 0\,, \quad \Delta\Phi = Ha \cdot \partial V_y / \partial x \,, \qquad (27),(28)$$

$$z = 0: V_y = 0\,, \partial\Phi / \partial z = \chi_0 \left[ -A + F(x,0) \right] \cdot \left[ \delta(x+1) - \delta(x-1) \right], \qquad (29),(30)$$

$$\sqrt{x^2 + z^2} \to \infty: \quad V_y \to 0, \Phi \to 0\,, \qquad (31)$$

kur $\Delta = \partial^2 / \partial x^2 + \partial^2 / \partial z^2$, $Ha = B_0 L \sqrt{\sigma / \rho\nu}$ ir Hartmana skaitlis, $A = j_0 L^2 / (\nu\sqrt{\rho\nu\sigma})$, $\chi_0 = \tilde{\chi}_0 / L$ un $F(x,0) = \left.\frac{\partial\Phi}{\partial x}\right|_{z=0}$.

Lai atrisinātu problēmu (27)-(31), tiek izmantota šīs problēmas simetrija attiecībā pret $x$: funkcija $V_y(x,z)$ ir pāra funkcija, $\Phi(x,z)$ ir nepāra funkcija attiecībā pret $x$. Tas nozīmē, ka funkcijas $V_y(x,z)$ un $\Phi(x,z)$ apmierina papildu robežu nosacījumus:

$$z = 0: \quad \frac{\partial V_y}{\partial x} = 0, \Phi(x,0) = 0\,. \qquad (32)$$

Tādēļ problēmu (27)-(31) var atrisināt ar Furjē kosinusa un Furjē sinusa transformācijas palīdzību (skatīt [3], [4]). Vārdu sakot, tiek izmantota Furjē kosinusa transformācija pret $x$ vienādojumu (27) un $V_y$ robežu nosacījumā (29), un Furjē sinusa transformācija vienādojumam (28) un $\partial\Phi / \partial z$ robežu nosacījumā (30), tas nozīmē, aizvietojot:

$$V_y^{\,c}(\lambda,z) = \sqrt{\frac{2}{\pi}} \int_0^\infty V_y(x,z) \cos\lambda x dx\,, \Phi^s(\lambda,z) = \sqrt{\frac{2}{\pi}} \int_0^\infty \Phi(x,z) \sin\lambda x dx\,. \qquad (33)$$

Tiek iegūta šāda vienkārša diferenciālvienādojumu sistēma nezināmām funkcijām $V_y^{\,c}(\lambda,z)$, $\Phi^s(\lambda,z)$:

$$-\lambda^2 V_y + \frac{d^2 V_y^{\,c}}{dz^2} - Ha^2 V_y^{\,c} + Ha\lambda\Phi^s = 0\,, \qquad (34)$$

$$-\lambda^2 \Phi^s + \frac{d^2\Phi^s}{dz^2} + Ha\lambda V_y^{\,c} = 0\,. \qquad (35)$$

Tiek izmantotas arī transformācijas (33) robežu nosacījumiem:

$$z = 0: \quad V_y^{\,c} = 0, \quad \frac{d\Phi^s}{dz} = \chi_0 \left[ A - F(1,0) \right] \sqrt{\frac{2}{\pi}} \sin\lambda\,; \qquad (36)$$

$$z \to \infty: V_y^{\,c}, \Phi^s \to 0, \qquad (37)$$

kur $F(1,0) = \frac{\partial\Phi}{\partial x}$ pie $x = 1,\ z = 0$ ir nezināma konstante. Problēmas (34)-(37) risinājuma forma ir:

$$\Phi^s(\lambda,z) = \chi_0 \sqrt{\frac{2}{\pi}} \left[ -F(1,0) + A \right] \frac{\sin\lambda}{2\lambda^2} \left( k_1 e^{k_2 z} + k_2 e^{k_1 z} \right)\,, \qquad (38)$$

$$V^c(\lambda, z) = \chi_0 \sqrt{\frac{2}{\pi}} [- F(1{,}0) + A] \frac{\sin \lambda}{2\lambda} \left( e^{k_1 z} - e^{k_2 z} \right), \tag{39}$$

kur $k_1 = -(\sqrt{\lambda^2 + \mu^2} + \mu)$, $k_2 = -(\sqrt{\lambda^2 + \mu^2} - \mu)$, $2\mu = Ha$.

Izmantojot inversās Furjē sinusa un kosinusa transformācijas formulām (38), (39), tiek iegūts risinājums problēmai (27)-(31), kas satur nezināmu konstanti $F(1{,}0)$:

$$\Phi(x, z) = \frac{\chi_0}{\pi} [- F(1{,}0) + A] \int_0^\infty \left( k_1 e^{k_2 z} + k_2 e^{k_1 z} \right) \frac{\sin \lambda}{\lambda^2} \sin \lambda x d\lambda, \tag{40}$$

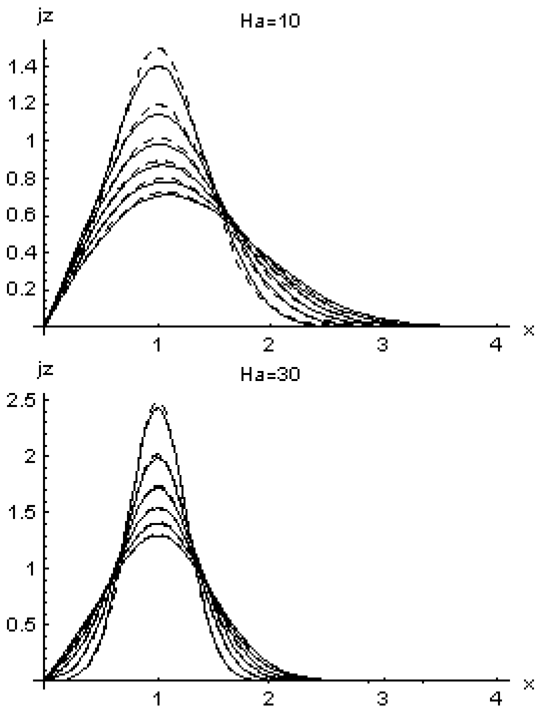
$$V_y(x, z) = \frac{\chi_0}{\pi} [- F(1{,}0) + A] \int_0^\infty \left( e^{k_1 z} - e^{k_2 z} \right) \frac{\sin \lambda}{\lambda} \cos \lambda x d\lambda . \tag{41}$$

Izmantojot komponentes $j_x$ un $j_z$ no inducētās strāvas blīvuma un veicot dažas risinājuma transformācijas, tiek iegūta formula nezināmai konstantei $F(1{,}0)$:

$$F(1{,}0) = -\frac{\chi_0}{2\pi} A \frac{1}{1 - \frac{\chi_0}{2\pi}} . \tag{42}$$

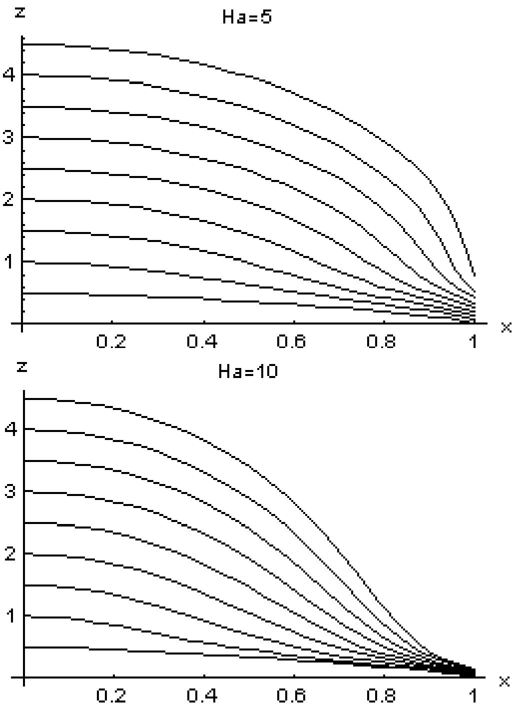
Risinājuma asimptotiska analīze gadījumam $Ha \to \infty$ ir veikta arī sadaļā 2.3. Ir atrasti vairāki plūsmas apgabali lieliem Hartmana skaitļiem, vārdu sakot, Hartmana robežas slānis, plūsmas kodols un attāla līdzplūsma.

Diagrammas strāvas $z$-komponentei $j_z(x, z)$ ir redzamas 4. att. Aprēķini ir veikti ar „Matemātika *(Mathematica)*".




4. att. Diagrammas strāvas $z$-komponentei ar precīzu formulu (---) un ar aproksimētu formulu ( ______ ) no $z = 1$ (divas augšējās līnijas) līdz $z = 3.5$ (divas apakšējās līnijas) ar soli $\Delta z = 0.5$. Funkcija $j_z(x, z)$ ir nepāra attiecībā pret $x$.

Strāvas pludlīnijas $\vec{j}(x, z)$ apgabalā $0 \le x \le 1$ ir redzamas 5. att. divām $Ha$ vērtībām.




5.att. Strāvas pludlīnijas $\vec{j}(x,z)$ apgabalā $0 \le x \le 1$, ja $Ha = 5$ un, ja $Ha = 10$ .

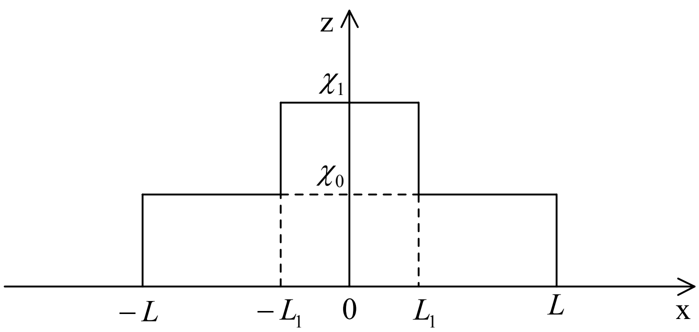
Turpmākos secinājumus var izsecināt no problēmas risinājuma:

1. analītiskais risinājums ir iegūts tikai ar vienu aproksimācijas pieņēmumu, ka negluduma augstums ir mazs. Risinājumi šķidruma ātruma $y$ komponentei un inducētās strāvas $x$ komponentei ir iegūti neīstu vienkāršu funkciju integrāļu formā. Turpretim inducētās strāvas $z$ komponente ir izteikta ar Beseļa funkciju;
2. asimptotiskais risinājums problēmai, ja Hartmana skaitlis $Ha \to \infty$, ir iegūts elementāru funkciju formā. Hartmana skaitļiem $Ha \ge 10$ tiešais un asimptotiskais risinājums praktiski sakrīt;
3. ir atrasti vairāki robežslāņi šķidruma ātrumam un strāvas $x$ un $z$ komponentēm ar lieliem Hartmana skaitļiem.

sadaļā 2.3 aprakstītais analītiskais risinājums ir vispārināts sadaļā 1.4 gadījumam ar negludumu formā

$$\tilde{z} = \tilde{F}(\tilde{x}) = \begin{cases} \tilde{\chi}_1, & |\tilde{x}| < L_1 \\ \tilde{\chi}_0, & L_1 < |\tilde{x}| < L \\ 0, & |\tilde{x}| > L \end{cases}.$$

Šīs funkcijas grafiks ir redzams zemāk.




6. Att. Plūsmas ģeometrija.

## 3. nodaļa. Neīstā integrāļa aprēķināšana.

Šajā nodaļā integrāli formā

$$\int_0^\infty \frac{P_n(\lambda^2)}{Q_m(\lambda^2)} e^{-a\sqrt{\lambda^2+b^2}} \frac{\cos\lambda\cos\lambda x}{\lambda^2-\frac{\pi^2}{4}} d\lambda\,, \tag{43}$$

kur $P_n(\lambda^2)$, $Q_m(\lambda^2)$ ir attiecīgi $n$ un $m$ pakāpes polinomi un $m \geq n,\; a>0,\quad b>0$, $x>0$ ir pozitīvi parametri, transformēsim integrāļos ar monotonām funkcijām, izmantojot konvolūcijas teorēmu divu Furjē transformāciju reizinājumam.

Pieņemsim, ka visas polinoma $Q(\lambda^2)$ nulles ir vienkāršas un ir izteiktas kā: $\lambda_k^{\;2} = -a_k^{\;2}$, $k = 1, 2, \cdots 3, ..., n$.

Pieņemsim, ka

$$F_c(\lambda) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x)\cos\lambda x dx \tag{44}$$

ir Furjē kosinusa transformācija funkcijai $f(x)$.

Izmantosim teorēmu (skatīt [23]).

Ja $F_c(\lambda)$ un $\Phi_c(\lambda)$ ir Furjē kosinusa transformācijas attiecīgi funkcijām $f(x)$ un $\varphi(x)$, tad

$$\int_0^\infty F_c(\lambda)\Phi_c(\lambda)\cos\lambda x d\lambda = \frac{1}{2}\int_0^\infty \varphi(\xi)\left[f(|x-\xi|)+f(x+\xi)\right]d\xi\,. \tag{45}$$

Pieņemsim, ka $\dfrac{P_n(\lambda^2)}{Q_m(\lambda^2)} \dfrac{\cos\lambda}{\lambda^2-\frac{\pi^2}{4}} = \Phi_c(\lambda),\, e^{-a\sqrt{\lambda^2+b^2}} = F_c(\lambda)$. (46)

Lai iegūtu funkcijas $\varphi(x)$, $f(x)$, ir jānovērtē integrāļi:

$$I_1 = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{P_n(\lambda^2)}{Q_m(\lambda^2)} \frac{\cos\lambda\cos\lambda x\, d\lambda}{\lambda^2-\frac{\pi^2}{4}} = \varphi(x),\; I_2 = \sqrt{\frac{2}{\pi}}\int_0^\infty e^{-a\sqrt{\lambda^2+b^2}} \cos\lambda x\, d\lambda = f(x)$$

Lai izteiktu $I_2$, izmantojam literatūrā zināmu integrāli:

$$\int_0^\infty \frac{e^{-a\sqrt{\lambda^2+b^2}}}{\sqrt{\lambda^2+b^2}} \cos\lambda x\, d\lambda = K_0(b\sqrt{a^2+x^2})\,, \tag{47}$$

kur $K_0(z)$ ir modificēta Beseļa otrā veida funkcija ar kārtu 0.

Diferencējot formulu (47) pret $a$, tiek izteikts integrālis $I_2$:

$$I_2 = \sqrt{\frac{2}{\pi}}\int_0^\infty e^{-a\sqrt{\lambda^2+b^2}} \cos\lambda x\, d\lambda = \sqrt{\frac{2}{\pi}} \frac{K_1(b\sqrt{a^2+x^2})}{\sqrt{a^2+x^2}} = f(x) \tag{48}$$

kur $K_1(z)$ ir modificēta Beseļa otrā veida pirmās kārtas funkcija.

Lai izteiktu integrāli $I_1$, izmantojam rezidiju metodi (no [6]):

$$I_1 = \sqrt{\frac{2}{\pi}}\frac{1}{2}\mathrm{Re}\left\{\left(2\pi i\sum_{k=1}^{m}\underset{a_k i}{\mathrm{Res}}+\pi i\underset{\pi/2}{\mathrm{Res}}\right)\frac{P_n(z^2)}{Q_m(z^2)}\left[e^{iz(1-x)}+e^{iz(1+x)}\right]\right\}, (49)$$

kur $\underset{z_0}{\mathrm{Res}}\frac{\varphi(z)}{\psi(z)}=\frac{\varphi(z_0)}{\psi'(z_0)}$,

ar nosacījumu, ka $\varphi(z)$ un $\psi(z)$ ir analītiskas funkcijas pie $z_0$ un mazā apkaimē, kur $\psi(z_0)=0$, $\psi'(z_0)\neq 0$. No (50) var secināt, ka

$$I_1 = \sqrt{\frac{2}{\pi}}\frac{\pi}{2}\sum_{k=1}^{m}\frac{P_n(-a_k^2)}{Q_m'(-a_k^2)}\left[e^{-a_k|1-x|}\,\mathrm{sign}(1-x)+e^{-a_k(1+x)}\right]+$$

$$+\frac{1}{2}\frac{P_n\left(\frac{\pi^2}{4}\right)}{Q_m'\left(\frac{\pi^2}{4}\right)}\left[\sin\left(\frac{\pi}{2}|1-x|\right)sign(1-x)+\sin\left(\frac{\pi}{2}|1+x|\right)\right]=\varphi(x), (50)$$

kur $sign(1-x)$ norāda $(1-x)$ zīmi.

Izmantojot (43), (46) un (51) un ņemot vērā, ka $|1-x|^2=(1-x)^2$, tiek transformēts integrālis (43) par integrāli ar neoscilējošu funkciju:

$$\int_0^\infty \frac{P_n(\lambda^2)}{Q_m(\lambda^2)}e^{-a\sqrt{\lambda^2+b^2}}\cos\lambda\cos\lambda x d\lambda =$$

$$=\frac{ab}{\pi}\int_0^\infty \varphi(\xi)\left[\frac{K_1\left(b\sqrt{(x-\xi)^2+a^2}\right)}{b\sqrt{(x-\xi)^2+a^2}}+\frac{K_1\left(b\sqrt{(x+\xi)^2+a^2}\right)}{b\sqrt{(x+\xi)^2+a^2}}\right]d\xi, \qquad (51)$$

kur $\varphi(\xi)$ ir dots formulā (50).

Līdzīgi var norādīt, ka

$$\int_0^\infty \frac{e^{-z\sqrt{\lambda^2+\mu^2}}}{\frac{\pi^2}{4}-\lambda^2}\cos\lambda\cos\lambda x d\lambda =$$

$$=\frac{2\mu z}{\pi}\int_0^1\left[\frac{K_1\left(\mu\sqrt{z^2+(x-\xi)^2}\right)}{\sqrt{z^2+(x-\xi)^2}}+\frac{K_1\left(\mu\sqrt{z^2+(x+\xi)^2}\right)}{\sqrt{z^2+(x+\xi)^2}}\right]\cos\frac{\pi}{2}\xi\, d\xi. \quad (52)$$

Integrāli (53) var viegli novērtēt, izmantojot paketi “Mathematica” visām parametru vērtībām $x\geq 0$ un $z\geq 0$. Kā var redzēt formulā (53), transformācijas priekšrocības ir:

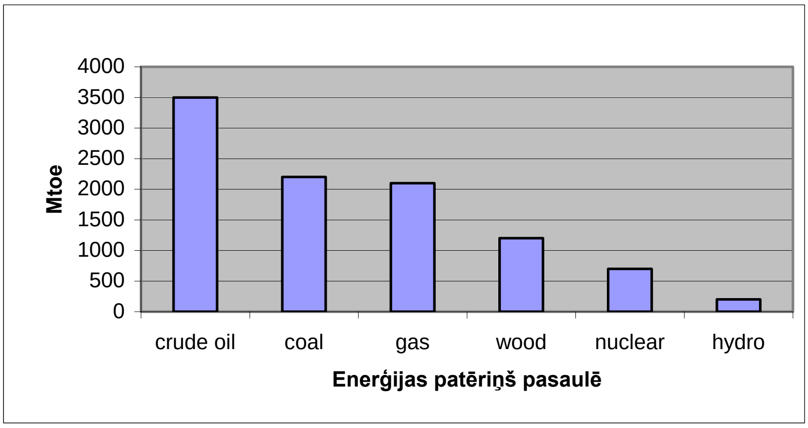
1. parametrs $x$ pāriet no oscilējošas funkcijas kosinusa argumenta uz monotonas Beseļa funkcijas argumentu $K_1$;
2. integrēšanas robežas tiek nomainītas uz ierobežotu apgabalu $0\leq\xi\leq 1$.

Integrāļi (52) un (53) tiek izmantoti MHD plūsmu problēmu, kas rodas virsmas negluduma dēļ, risinājuma novērtēšanai vai transformēšanai (skatīt [2], [13]). Viens šāds piemērs ir dots sadaļā 4.2.

**4. nodaļa. *EUROFER* tērauda korozija un plazmas magnētiskais ierobežojums reaktoros.**

**4.1 *Deitērija-Tritija* reakcija un tās lietojums reaktoros.**

Šā gadsimta laikā līdz 2050. gadam pasaules iedzīvotāju skaits palielināsies no sešiem miljardiem līdz desmit miljardiem cilvēku. Tomēr lielāka nozīme ir tam, ka nākotnē tiks patērēts būtiski vairāk enerģijas nekā pašlaik. Gadsimta vidū patērētās enerģijas daudzums, iespējams, būs divas reizes lielāks, turklāt vislielākais patēriņa palielinājums būs vērojams elektroenerģijas rādītājos.




*Crude oil* – jēlnafta
*Coal* – ogles
*Gas* – gāze
*Wood* – koksne
*Nuklear* – kodolenerģija
*Hydro* – hidroelektroenerģija

*Mtoe* - Mt

1.tabula. Enerģijas patēriņš 2007.gadā [Mt (miljoni tonnu eļļas ekvivalents)]
(Precīzās vērtības ir attiecīgi – 3500, 2200, 2100, 1200, 700 un 200.)

Zinātnieki no visām Eiropas dalībvalstīm un G8 valstīm, kas saistītas ar EURATOM kodolenerģijas programmu, ir centušies mazināt *Deitērija-Tritija* plazmas reakcijas procesu uz Zemes reaktora iekšienes virspusē:

$$ {}_1^2\mathrm{H} + {}_1^3\mathrm{H} \rightarrow {}_2^4\mathrm{He} + {}_0^1\mathrm{n} + 17.6\ \mathrm{MeV} \quad (53)$$

(skatīt [1], [32], [33], [57], [49]).

Kodolenerģija ( 17,6 MeV ) parādās gan kā neitronu kinētiskā enerģija (14,1 MeV), kas jāuzglabā reaktora iekšpusē, izmantojot svinu, gan arī kā alfa daļiņas (3,5MeV), kas tiek izmestas no konkrēta reaktora caur skursteni kopā ar pelniem [1], [36], [37] un [64].
*Deitērijs* galvenokārt atrodams jūras ūdenī, bet *tritijs* ir radioaktīvs elements, kas uz Zemes dabiskā veidā ir reti sastopams. Tomēr to iespējams radīt reaktorā, izmantojot neitronu reakciju virspusē, izmantojot litiju − bagātīgu, vieglu metālu, kas sastopams dabā kā:

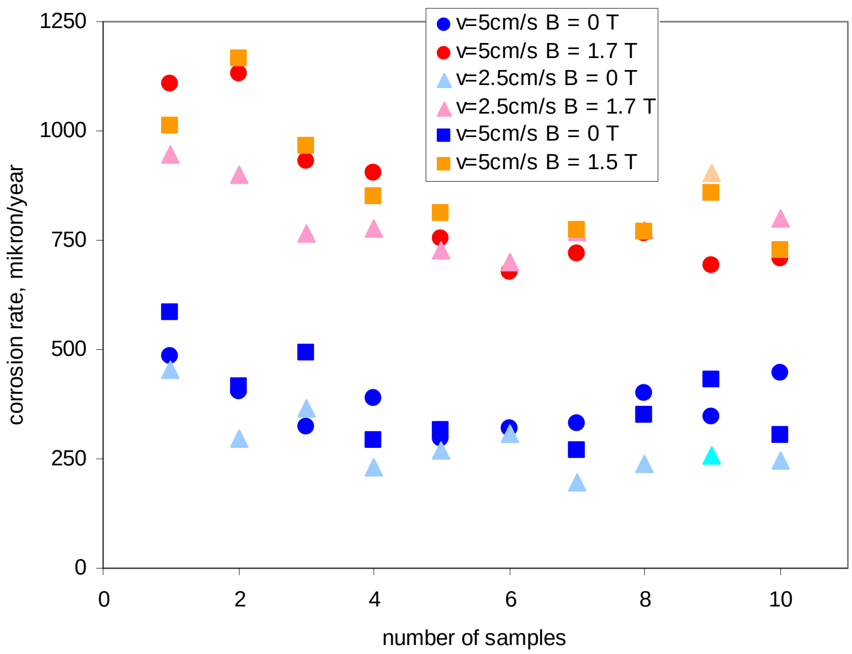
Li6 + n ---- T + He4 + 4,8 MeV

Li7 + n ---- T + He4 – 2,5 MeV

Desmit grami *deitērija*, ko iespējams iegūt no 500 litriem ūdens un 15 g *tritija*, ko savukārt iegūst no 30 g litija, nodrošina viena cilvēka mūžam pietiekami daudz elektrības industriāli attīstītā vidē (skatīt [1], [32], [39], [35], [49], [55], [56], [64]).

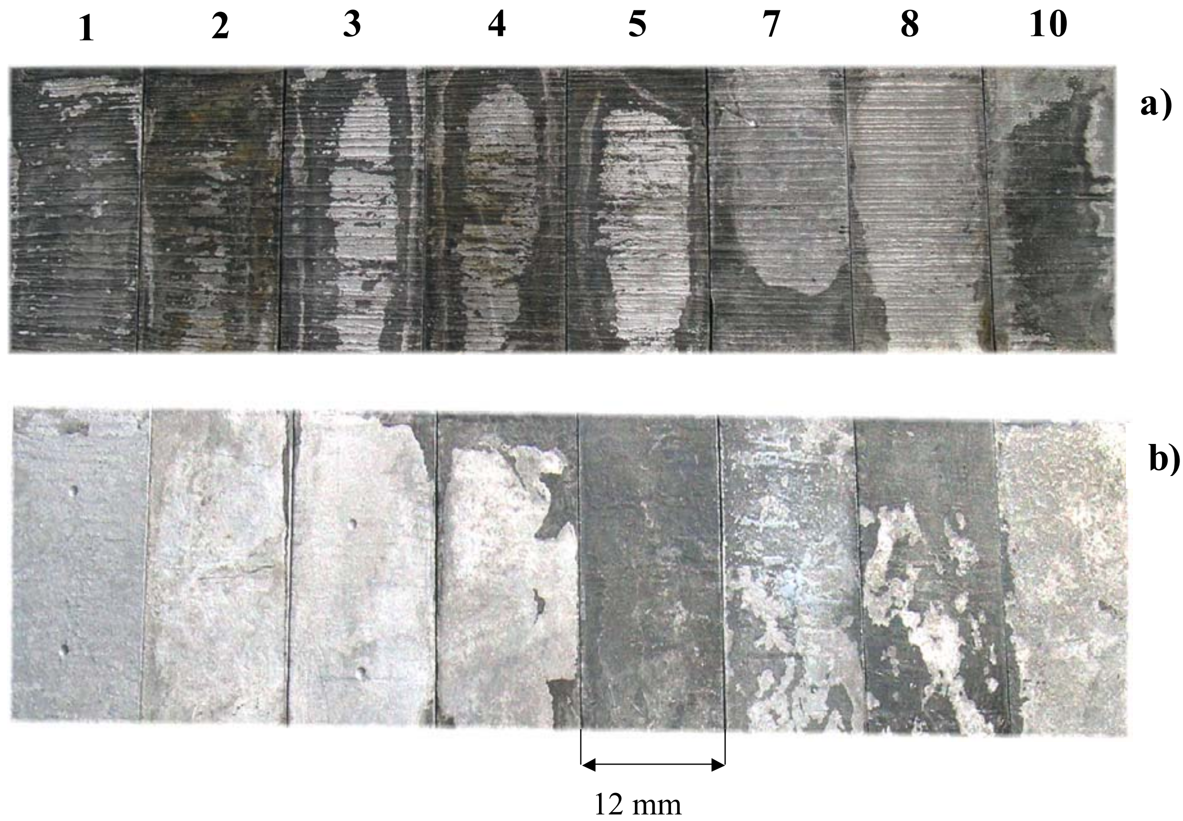
D-T plazmas ierobežojums ir apspriests jau sākot ar ***JET*** projektu *(Joint European Torus)*, panākot *ITER (International Thermonuclear Experimental Reactor Starptautiskais Termonukleārais eksperimentālais reaktors)*, kas nodrošinās tehniskos datus, kuri nepieciešami turpmāko lēmumu pieņemšanai par citiem reaktoriem, piemēram, DEMO un PROTO, kas var kļūt par pirmajiem spēkstaciju prototipiem ar pilnīgām reaktoru sistēmām un papildsistēmām, kuras ražos elektrību komerciālā mērogā (pieņemot, ka tā būvniecība tiks uzsākta šā gadsimta piecdesmitajos gados, bet darbu tas uzsāks šā gadsimta septiņdesmitajos gados) [1], [33], [35], [49], [57], [64].

**4.2 MHD fenomena ietekme uz *EUROFER* tērauda koroziju Pb-17Li plūsmā.**
***(Šeit MHD plūsma ar virsmas negludumiem kā vadošā šķidruma*** $\tilde{z} = \tilde{\chi}_0 \cos(\pi\tilde{x}/2L)$ ***ir pustelpā un tiek pieņemts,)*** Aplūkosim MHD plūsmu ar negludumu veidā $\tilde{z} = \tilde{\chi}_0 \cos(\pi\tilde{x}/2L)$ pustelpā $\tilde{z} > 0, -\infty < \tilde{x}, \tilde{y} < +\infty$. Ārējais magnētiskais lauks ir $B^c = B_0 e_z$. *EUROFER* tērauda korozija Pb-17Li plūsmā var tikt uzskatīta par negludumu radītām sekām, kur Hartmana virsmas plūsmas arī ir perpendikulāri plūsmai. Neņemot vērā to, ka tērauda koroziju Pb-17Li plūsmā ir neliela, tomēr nozīmīga reaktora darbības daļa, tiek informēts par tās svarīgumu un jaunākajiem rezultātiem, kas iegūti, pētot korozijas procesu Fizikas institūtā Latvijā [55], [56]. Šis eksperiments tika veikts ar dažādiem paraugiem, kuru plūsmas ātrums bija 2,5 cm/s un 5 cm/s, bet magnētiskā strāva bija 0, 1,5 un 1,7 T. Rezultāti, kas tika iegūti šajos pētījumos, pierādīja, ka magnētiskajam laukam ir liela ietekme uz korozijas procesiem gan korozijas intensitātes, gan tās būtības ziņā (7. attēls).




7. attēls. *EUROFER* paraugu korozijas ātruma salīdzinājums magnētiskā laukā un bez magnētiskā lauka.
*Corrosion rate* – korozijas ātrums
*Mikron/year* – mikroni/gadā *Number of samples* – paraugu skaits

Cits eksperiments pierādīja, ka korozijas virsmas ir vērstas plūsmas virzienā (8. attēls).




8. attēls. *EUROFER* paraugu virsmas reljefs, kas pakļauts korozijai Pb17Li 2000 stundu laikposmā.

Cerams, ka līdz šā gadsimta beigām zinātnieki, izmantojot pieejamās tehnoloģijas un pētījumus, spēs gūt panākumus *ITER* projektā, kas nodrošinās fizikas un tehnoloģiju pamatu tādas elektriski ģenerējamas spēkstacijas demonstrācijas būvniecībai kā, piemēram, DEMO un PROTO . Pēc tam jauns, tīrs un lēts enerģijas avots kļūtu par daļu no cilvēku dzīves (skatīt [1], [9], [20], [28], [32]-[37] , [40] , [49] , [51], [55], [56], [62], [70] un [73]).

## 5. nodaļa. Ginzburga-Landau vienādojums sekla ūdens stabilitātes analīzei vāji nelineārā režīmā.

Hidraulikā zaudējumi, kas rodas turbulentas berzes dēļ, bieži tiek attēloti empīrisku (vai daļēji empīrisku) formulu, piemēram, Čezi vai Manninga formulu [22] veidā. Īpaši bieži tiek lietota Čezi formula, lai attēlotu berzes spēku $\vec{F}$ formulā

$$\vec{F} = \frac{\rho g A c_f}{h} \vec{v} |\vec{v}|, \qquad (55)$$

kur $\rho$ šķidruma blīvums, $g$ ir smaguma spēka paātrinājums, $A$ ir šķērsgriezuma laukums, $h$ ir ūdens dziļums, $c_f$ ir berzes (jeb grumbuļainuma) koeficients, $\vec{v}$ ir ātruma vektors un $\vec{F}$ berzes spēks. Koeficients $c_f$ tiek aprēķināts ar vairāku empīrisko formulu palīdzību, kas atrodamas literatūrā. Viens piemērs ir Kolbruka formula [67], kas attiecas uz $c_f$ Reinoldsa plūsmas skaitli.

Pēc tam tiek ņemta vērā formas bāzes plūsma

$$\vec{U} = (U(y),0) \quad , \tag{56}$$

kur

$$U(y) = 1 - \frac{2R}{1-R}\frac{1}{\cosh^2(\alpha y)} . \tag{57}$$

Ieteikumi par bāzes plūsmu (57) doti [19], pēc rūpīgas pieejamo eksperimentu datu analīzes par dziļajām ūdens plūsmām aiz apļveida cilindriem. Profils (55) arī ir pielāgots esošajam pētījumam. Parametrs $R$ ir ātruma koeficients: $R = (U_m - U_a)/(U_m + U_a)$, kur $U_m$ ir seklā ūdens plūsmas ātrums un $U_a$ ir vides ātrums, bet $\alpha = \sinh^{-1}(1)$. Rakstā [44] redzams, ka seklā ūdens lineāro stabilitāti raksturo šāda problēma:

$$\varphi_1''(U - c + SU) + SU_y \varphi_1' + \left( k^2 - U_{yy} - k^2 U - \frac{S}{2} kU \right) \varphi_1 = 0 \tag{56}$$

$$\varphi_1(\pm\infty) = 0, \tag{57}$$

kur straumei ir plūsmas funkcija, $\psi(x,y,t)$, pieņem kā

$$\psi(x,y,t) = \varphi_1(y)\exp[ik(x-ct)] + c.c. \tag{58}$$

Šeit $\varphi_1(y)$ ir perturbācijas amplitūda, $k$ ir viļņu skaitlis, $c$ ir perturbācijas viļņu ātrums, bet $c.c.$ nozīmē "kompleksais konjugāts". Bāzes plūsmas lineāro stabilitāti (55) nosaka pēc īpašvērtību problēmas (56), (57) īpašvērtībām, $c_m = c_{rm} + ic_{im}$, $m = 1,2,...$. Plūsma (55) ir lineāri stabila, ja $c_{im} < 0$ visiem $m$, bet tā ir lineāri nestabila, ja $c_{im} > 0$ vismaz vienai $m$ vērtībai.

Lineārās stabilitātes problēma (56), (57) tiek risināta ar pseidospektrālā izvietojuma metodi, kas pamatota uz Čebiševa polinomiem. Aprēķina procedūra īsumā ir izklāstīta turpmāk (papildu informāciju par skaitļu metodēm meklējiet [44]). Intervāls $-\infty < y < +\infty$ ir kartēts uz intervālu $-1 < r < 1$, veicot transformācijas $r = \frac{2}{\pi}\arctan y$.

Risinājums (56), (57) tiek meklēts

$$\varphi_1(r) = \sum_{k=0}^{N} a_k (1-r^2) T_k(r), \tag{59}$$

kur $T_k(r)$ ir Čebiševa polinoms $k$ pakāpē. Izvietojuma punkti $r_j$ ir

$$r_j = \cos\frac{\pi j}{N}, \; j = 0,1,...,N. \tag{60}$$

Stabilitātes parametra $S$ kritiskās vērtības ir redzamas 2. tabulā.

Ņemot vērā [67], tiek veikta vāji nelineārā stabilitātes analīze kritiskā punkta apkārtnē.

| $R$ | $k$ | $S_c$ | $c$ |
|---|---|---|---|
| -0.3 | 0.892 | 0.11819 | 0.69814 |
| -0.4 | 0.909 | 0.15689 | 0.65964 |
| -0.5 | 0.926 | 0.19548 | 0.62394 |
| -0.6 | 0.944 | 0.23409 | 0.59083 |
| -0.7 | 0.962 | 0.27286 | 0.55925 |
| -0.8 | 0.980 | 0.31189 | 0.52882 |

2. tabula. Stabilitātes parametra $S$ kritiskās vērtības.

| $R$ | $\sigma$ | $\delta$ | $\mu$ |
|---|---|---|---|
| -0,3 | 0,063 + 0,004i | 0,060 – 0.206i | 4,673 + 13,294i |
| -0,4 | 0,078 + 0,003i | 0,090 – 0.195i | 3,796 + 10,938i |
| -0,5 | 0,090 + 0,000i | 0,115 – 0.184i | 3,895 + 10,119i |
| -0,6 | 0,100 – 0,003i | 0,136 – 0.172i | 4,375 + 10,109i |
| -0,7 | 0,109 – 0,007i | 0,153 – 0.161i | 5,149 + 10,590i |
| -0,8 | 0,116 – 0,012i | 0,167 – 0.152i | 6,302 + 11,448i |

3. TABULA
CGLE koeficienti (61)

Amplitūdas evolūcijas vienādojums (kompleksais Ginzburga-Landau vienādojums), lai novērtētu visnestabilāko režīmu sekla ūdens plūsmās aiz šķēršļiem, tiek iegūts [44], visas formulas Ginzburga-Landau vienādojuma koeficientu aprēķināšanai arī sniegtas [44]. Kompleksajam Ginzburga-Landau vienādojumam ir šāds pieraksts:

$$\frac{\partial A}{\partial \tau} = \sigma A + \delta \frac{\partial^2 A}{\partial \xi^2} - \mu |A|^2 A \qquad (61)$$

Izmantojot [44] aprakstīto metodi, tiek aprēķināti CGLE (61) koeficienti dažādām $R$ vērtībām. Rezultāti ir apkopoti 3. tabulā.

Viens no galvenajiem secinājumiem, kas izdarīts pēc vāji nelineārās stabilitātes analīzes, kas piemērota kvazi-divdimensiju plūsmām [22], bija Landau konstantes $\mu_r$ spēcīgā atkarība no bāzes plūsmas profila formas. Aprēķini, kas sniegti [22], pierāda, ka Landau konstantes vērtības atšķiras par koeficientu trīs diviem bāzes plūsmas ātruma profiliem, kuru lineārās stabilitātes raksturotāji atšķīrās tikai par 20%. Tādēļ [22] tika secināts, ka vāji nelineārās teorijas metodes nav iespējams piemērot praksē, jo eksperimentu gaitā nav iespējas precīzi noteikt bāzes plūsmas profilu. Citiem vārdiem sakot, [22] tika secināts, ka Landau konstantes noteikšanas problēma ar vāji nelineārās teorijas palīdzību ir jutīga attiecībā pret bāzes profila izmaiņām, jo nelielas novirzes no bāzes plūsmas profila rada vērā ņemamas izmaiņas Landau konstantē.

Aprēķini, kas doti 2. un 3. tabulā, pierāda, ka KGLV koeficienti nav tik jutīgi pret izmaiņām parametros $R$ bāzes plūsmas profilā (55), kā tas tika atzīts [22]. Faktiski, ne vien Landau konstante nav jutīga pret izmaiņām profilā (55), bet visi CGLE koeficienti ir samērā nemainīgi.

## SECINĀJUMI

Darbs ir veltīts to faktoru analīzei, kas ietekmē MHD plūsmu un sekla ūdens plūsmu struktūru un stabilitāti. Jo īpaši sienas pretestības ietekmi uz plūsmu var aprakstīt lokāli (ņemot vērā robežas negludumu) vai globāli (izmantojot pusempīriskas formulas, kas apraksta iekšējās berzes ietekmi).

Robežvirsmas negludums var rasties korozijas dēļ. Eksperimentālie rezultāti parādīja nozīmīgu magnētiskā lauka ietekmi uz korozijas procesiem – gan uz korozijas intensitāti, gan tās īpašībām. Tādēļ no praktiskās puses ir būtiski analizēt negluduma ietekmi uz MHD plūsmu struktūru. Šī ietekme darbā tiek novērtēta, analītiski atrisinot magnētiskās hidrodinamikas vienādojumu sistēmu (izmantojot Furjē transformācijas). Darbā ir skatītas vairākas virsmas negluduma formas. Analītiskie risinājumi ir iegūti, un ātruma sadalījums ir skaitliski analizēts dažādiem Hartmana skaitļiem. Tāpat ir iegūts asimptotiskais risinājums lieliem Hartmana skaitļiem. Risinājumi ir iegūti kā integrāļi, kas iekļauj oscilējošas funkcijas. Darbā šie integrāļi tiek pārveidoti par integrāļiem no neoscilējošām funkcijām.

Turklāt tiek sniegta informācija par jaunākajiem rezultātiem, kas iegūti trijās eksperimentu sesijās (katra 2000 stundu ilga), kuras sekmīgi veiktas Salaspilī, Latvijā. Šajos pētījumos gūtie rezultāti pierādīja vērā ņemamu magnētiskā lauka ietekmi uz

korozijas procesiem gan korozijas intensitātes, gan tās būtības ziņā. Turklāt tika iegūti jauni rezultāti, kas saistīti ar korozijas profilu [56]. *EUROFER* korozijas paraugu izpētes process pierādīja, ka ir pietiekama atšķirība starp korozijas procesiem paraugos, kas izvietoti ārpus magnētiskā lauka (***B*** = 0), un paraugos, kas izvietoti magnētiskā lauka zonā (***B*** = 1,7 T). Šādi pētījumi tiek veikti, lai nodrošinātu kodolenerģijas kontroli reaktoros. Viens no šīs programmas pamatjautājumiem ir šķidro metālu virskārtas problēma. Virskārtas strukturālajam materiālam jāatbilst augstām prasībām ekstremālo darbības apstākļu dēļ. Tāpēc ir nepieciešamas zināšanas par metālu plūsmas ātruma, temperatūras, kā arī par neitronu starojuma un magnētiskā lauka ietekmi uz korozijas procesiem. Pašlaik par vispiemērotāko reaktora tritija virskārtas materiālu tiek uzskatīts eutektiskais svins – litijs (Pb-17Li) [1], [55] un [56].
Tiek pētīta sekla ūdens plūsmu stabilitātes analīze vāji nelineārā režīmā, izmantojot Ginzburga-Landau vienādojuma komplekso formu, lai iegūtu aprēķinus, kas norāda, ka Landau konstanšu vērtības [22] ir salīdzinoši jutīgas pret bāzes plūsmas profila formu. Darbā ir pierādīts, ka berzes spēku modelē ar nelineāru Čezi formulu [66]. KGLV koeficienti nerada īpašu ietekmi uz intervālu $-0.8 \le R \le -0.3$. Šis $R$ vērtību intervāls atbilst konvektīvi nestabilajam režīmam [44]. Tādēļ jāsecina, ka kompleksais Ginzburga-Landau vienādojums ir izmantojams seklu ķīļūdens plūsmu analīzei vāji nelineārā režīmā.

Pirmās divas nodaļas ir veltītas negluduma elementu ietekmei uz MHD plūsmām stiprā magnētiskajā laukā. Ir iegūti atbilstošo problēmu analītiskie atrisinājumi ar Furjē transformācijas metodi. Sadaļā 2.3.1. tika pierādīts, ka divdimensiju MHD plūsma rodas $y$ asij pretējā virzienā tikai tad, ja robežas ir nelīdzenas. $y$ komponentes risinājumi šķidruma plūsmas ātrumam un $x$ komponentes risinājumi inducētajai strāvai tiek iegūti kā elementārfunkciju neīstie integrāļi. Toties inducētās strāvas $z$ komponente ir izteikta ar Beseļa funkciju. Problēmas asimptotiskais risinājums, ja Hartmana skaitlis ir $Ha \to \infty$, tiek iegūts elementārfunkciju veidā. Ja Hartmana skaitlis ir $Ha \ge 10$, precīzie un asimptotiskie risinājumi praktiski sakrīt.
Turklāt problēmā, kas izvirzīta šajā sadaļā, ir pierādīts, ka inducētajam magnētiskajam laukam ir tikai $y$ komponente. Tiek iegūti MHD vienādojumu sistēmas risinājumi par šķidruma plūsmas ātrumu un inducētās strāvas potenciālu. Tiek iegūti arī spiediena gradienta $x$ un $z$ komponenšu vienādojumi. Šķidruma plūsmas ātrums centrālajā plūsmā ir konstants, ja Hartmana skaitlis ir liels. Tas nozīmē, ka tas nav atkarīgs no $Ha$. Palielinoties Hartmana skaitlim, palielinās vienīgi centrālā apgabala augstums. MHD atrisinājumi, kas izklāstīti šajā darbā, atvieglo šķidruma izplatīšanās izpēti apgabalā ar stipru magnētisko lauku (Hartmana skaitlis ir liels). Šie secinājumi ir nozīmīgi, un tie var palīdzēt citu ar elektrību vadošu šķidrumu problēmu risināšanā dažādās tehnoloģiju un inženierzinātņu jomās, piemēram, MHD plūsmas mērierīcēs, MHD sūkņos utt.

Trešā nodaļa galvenokārt ir veltīta jautājumam, ka noteiktu problēmu par MHD vadoša šķidruma plūsmu pustelpā risinājumi ir izteikti dažu meromorfu funkciju un funkcijas $\exp(-a\sqrt{\lambda^2+b^2})\cos\lambda\cos\lambda x$ reizinājuma neīsto integrāļu veidā, kur $a > 0$ un $b > 0$ ir parametri, $x > 0$ ir $x$ koordināta Dekarta koordinātu sistēmā. Tomēr šīs funkcijas ir ļoti oscilējošas, ja $x$ ir liels, tādējādi sarežģījot šo integrāļu aprēķināšanu ar paketes "Mathematica" starpniecību. Šajā daļā minētie integrāļi tiek pārveidoti par monotonās funkcijas integrāļiem, izmantojot konvolūcijas teorēmu par divu Furjē kosinusa transformāciju reizinājumu. Iegūtie rezultāti atvieglo atsevišķu problēmu novērtēšanu matemātikas, inženiertehnisko zinātņu un inženiertehniskās matemātikas jomās.

Ceturtā nodaļa ir veltīta *EUROFER* korozijas Pb17Li plūsmā praktiskajam pētījumam, kur iegūti triju nesen plānoto un sekmīgi pabeigto eksperimentālo sesiju rezultāti. Rezultāti, kas iegūti šajos pētījumos, pierāda magnētiskā lauka ievērojamo ietekmi uz korozijas procesiem gan attiecībā uz korozijas intensitāti, gan tās būtību. Turklāt tiek iegūti jauni rezultāti par korozijas profilu [56]. *EUROFER* korozijas paraugu izpētes process parādīja, ka magnētiskais lauks rada pietiekamu ietekmi uz koroziju: vizuāla testēšanas paraugu novērošana, kas iegūti no testēšanas sekcijas pēc eksperimentiem, uzrādīja atšķirības starp korozijas procesiem paraugos, kas izvietoti ārpus magnētiskā lauka (***B*** = 0), un paraugos, kas izvietoti magnētiskā lauka zonā (***B*** = 1,7 T). Jaunu enerģijas avotu meklējumi arvien lielāku uzmanību piesaista reaktoru izmantošanai. Zinātnieki, kas iesaistīti EUROATOM programmā, cenšas izprast veidu, kā kodolreaktori spēj darboties, izmantojot D-T plazmas koncepciju. JET, DEMO un PROTO ir projekti, kuru īstenošana var palīdzet ražot pilnībā kontrolētu enerģiju, kas tiks tieši pievienota elektrības tīkliem. Turklāt tā ir viena no nedaudzajām iespējām, kas potenciāli ir pieņemamas gan no apkārtējās vides drošības (pilnīgi bez $CO_2$ izmetes), gan no ekonomiskā viedokļa.

Piektā nodaļa ir veltīta seklo ūdens plūsmu stabilitātes analīzei vāji nelineārā režīmā, izmantojot Ginzburga-Landau komplekso vienādojumu rezultātu iegūšanai un pētījuma secinājumu izdarīšanai. Aprēķini, kas sniegti [22], liecina, ka Landau konstantes vērtības atšķiras par koeficientu trīs diviem dažādiem ātruma profiliem ar lineārās stabilitāttes raksturotājiem, kas atšķiras ne vairāk par 20%. Citiem vārdiem sakot, tika secināts, ka Landau konstante [22] ir samērā jutīga pret bāzes plūsmas profila formu. Promocijas darbā berzes spēku modelē ar nelineāro Čezi formulu [67]. 1. tabulas un 2. tabulas datu analīze liecina, ka seklās ķīļūdens plūsmām lineārās stabilitātes raksturojuma izmaiņu dēļ radās mazākas izmaiņas KGLV koeficientos. Tādēļ var secināt, ka komplekso Ginzburga-Landau vienādojumu ir iespējams izmantot, lai veiktu sekla ķīļūdens analīzi vāji nelineārā režīmā.

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## INTRODUCTION

The thesis is devoted to the analysis of factors that influence the structure and stability of magnetohydrodynamic (MHD) flows and shallow water flows. In particular, the effects of wall resistance on the flow can be described locally (taking into account roughness of the boundary) or globally (using semi-empirical formulas describing the effect of internal friction).

Roughness of the boundary can occur as a result of corrosion. Experimental results demonstrated essential influence of magnetic field on the corrosion processes both in the intensity of corrosion and its character. Therefore, is it important from a practical point of view to analyze the effect of roughness on the structure of magnetohydrodynamic flows. This effect is evaluated in the thesis by solving the system of magnetohydrodynamic equations analytically (using the Fourier transform). Several forms of surface roughness are considered in the thesis. Analytical solutions are found and velocity distribution is analyzed numerically for different Hartmann numbers. Asymptotical solution for high Hartmann numbers is also found. The solutions are found in terms of integrals containing oscillatory functions. These integrals are transformed in the thesis to integrals containing non-oscillatory functions.

Global effect of internal friction is usually taken into account by using empirical resistance formulas like Chezy formula to estimate the “lumped” effect of friction in turbulent flows for the computation of flow rate and losses in channels or pipes and design of open channels. These formulas contain empirical friction coefficients that are directly related to the Reynolds number of the flow and the roughness of the boundary. The coherent structures in wake flows behind obstacles are believed to appear as a final product of hydrodynamic instability of the flow. Methods of weakly nonlinear stability theory have been applied in the past to different flows and usually lead to amplitude evolution equations for the most unstable mode. One of such equations is the complex Ginzburg-Landau equation. Weakly nonlinear theory applied to quasi-two-dimensional flows with Rayleigh friction (internal friction is assumed to be linearly related to the velocity distribution) led to the conclusion that the coefficients of the amplitude evolution equation (Ginzburg-Landau equation) for the most unstable mode strongly depend on the shape of the base flow profile. As a result it was concluded in the literature that weakly nonlinear models cannot be used for such cases since it is impossible to determine experimentally the base flow velocity distribution with high accuracy and, therefore, one cannot use reliable values of the coefficients of the Ginzburg- Landau equation. It is shown in the thesis that if a nonlinear formula is used to model bottom friction then the coefficients of the Ginzburg-Landau equation are not sensitive to the base flow velocity distribution.

## IMPORTANCE OF THE RESEARCH

Study and analysis of magnetohydrodynamic flows in ducts or channels in the presence of a magnetic field is important in applications such as design and analysis of MHD generators or pumps. The influence of the surface roughness on the MHD flow of a conducting metal is useful for the techniques used to set up the cooling system of the Tokamak reactor. In addition, corrosion has to be taken into account in design and analysis of the performance of such reactors. It is known that magnetic

field affects both the rate of corrosion and the character of corrosion. It is, therefore, important to analyze the effect of corrosion on the structure of magnetohydrodynamic flows.
The stability characteristics of shallow flows behind obstacles are important from environmental point of view. Poor water circulation in the regions behind islands can lead to deterioration of water quality and, in some cases, even to fish mortality. As a result, it is important to know the structure of shallow water flows in the regions behind obstacles.

## SCOPE AND GOALS

The objective of the thesis is to analyze the effect of surface roughness on the structure of magnetohydrodynamic flows and stability of shallow flows. Surface roughness appears as a result of corrosion of a conducting fluid in a magnetic field. The obtained analytical solutions can be used to estimate the effect of corrosion in such cases.
Linear and weakly nonlinear stability analysis of shallow flows can be useful in determining water patterns in regions behind obstacles such as islands. The results obtained in the thesis may be used for environmental assessment of shallow flows.

## METHODS OF RESEARCH

Mathematical models that are analyzed in the thesis are based on equations of magnetohydrodynamics and shallow water equations. The following methods are used for the analysis of magnetohydrodynamic flows over roughness elements:

5. Transformations of the equations of magnetohydrodynamics in order to derive the form of the induced magnetic field in the flow region when roughness elements are present;
6. Fourier cosine and sine transforms for the analytical solution of the problems of MHD flows over roughness elements;
7. The residue theorem is used to transform integrals containing oscillatory integrands to integrals of monotonic functions;
8. Numerical evaluation of improper integrals with "Mathematica".

Shallow water flow model is used to analyze linear and weakly nonlinear instability of one class of wake flows. The methods of analysis include:

5. Linear stability analysis of the equations of motion;
6. Collocation method based on Chebyshev polynomials to calculate the stability boundary;
7. Asymptotic expansions in the neighborhood of the critical point to perform weakly nonlinear stability analysis;
8. Numerical methods for solution of boundary value problems for ordinary differential equations in order to calculate the coefficients of the amplitude evolution equation.

## SCIENTIFIC NOVELTY AND MAIN RESULTS

The form of the magnetic field and MHD equations for fully developed flow caused by the roughness of the boundary is obtained.
The effect of surface roughness on the structure of magnetohydrodynamic flows in half-space is analyzed by means of analytical methods. Surface roughness is assumed to be of the form of a piecewise constant function.
Asymptotic of the obtained solution is analyzed for the case of high Hartmann numbers.
Improper integrals containing oscillatory functions that arise in applications of MHD flows over roughness elements are transformed to integrals containing monotonic functions which are more suitable for numerical calculations.
Analysis of theoretical results that are relevant to experiments on the effect of corrosion in the magnetic field is performed.
Linear and weakly nonlinear analysis of shallow water flows is performed for one class of wake flows. A nonlinear Chezy model is used in our study to represent bottom friction. It is shown that the coefficients of the Ginzburg-Landau equation are not sensitive to the shape of the base flow profile in contrast with previous studies where the friction was modeled by a linear function of the velocity.

## APPLICATIONS

Previous experimental studies in the Institute of Physics in Salaspils (Latvia) have shown that under some conditions surfaces exposed to the magnetic field resemble some sort of a regular wave-like pattern. In general, the magnetic field affects not only corrosion rate, but also corrosion pattern. As a result, surface roughness has to be taken into account when realistic models of liquid metal flows in reactors are considered. Theoretical results presented in the thesis can be used in the analysis of the corrosion process in magnetic field.
Methods of linear and weakly nonlinear stability are often used to analyze the structure of shallow wake flows. Bottom friction is modeled in the thesis by a nonlinear function of the velocity. It is shown in the thesis that the coefficients of the Ginzburg-Landau equation which describes the development of flow instability above the threshold are not so sensitive to the variation of the base flow profile. This result contradicts the previous studies where the internal friction was modeled by a linear function of the velocity. As a result, it is plausible to assume that simplified models based on amplitude evolution equations such as complex Ginzburg-Landau equation can be used to analyze the behavior of the most unstable mode in the region above the threshold.

## List of publications

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5. Chaddad I. A., Kolyshkin A. Ginzburg-Landau model: an amplitude evolution equation for shallow wake flows, International Journal of Mathematical, Physical and Engineering Sciences, vol. 2, no. 3, pp. 126 – 130, (2008).
6. Chaddad I. A., Kolyshkin A.A. Ginzburg-Landau model: an amplitude evolution equation for shallow wake flows", Proceedings of World Academy of Science, Engineering and Technology, vol. 28, pp. 1 – 5, (April 2008).
7. Kolyshkin A.A. Chaddad I.A. Ginzburg- Landau equation for stability of water flow in a weakly nonlinear regime, Proceedings of Riga Technical University, 5th series: Computer Science,49th thematic issue, pp. 58-64, Riga, RTU (2007).

## Presentations at conferences

1. Antimirov M.Ya., Chaddad I.A. On new class of integrals used in some MHD problems, RTU 44$^{th}$ International Scientific Conference, October 9 – 11, RTU, Riga (2003).
2. Antimirov M.Ya., Chaddad I.A. Solution of a problem to the MHD flow of a conducting fluid over roughness elements of rectangular form in a magnetic field, RTU 45$^{th}$ International Scientific Conference, October 14 – 16, RTU, Riga (2004).
3. Antimirov M.Ya., Chaddad I.A. On the conditions of coincidence of MHD problems at the exact statement and at the inductionless approximation, RTU 45$^{th}$ International Scientific Conference, October 14 – 16, RTU, Riga (2004).
4. Antimirov M.Ya., Chaddad I.A. On the MHD flows arising at the interacting of magnetic field and given current, RTU 45$^{th}$ International Scientific Conference, October 14 – 16, RTU, Riga (2004).
5. Chaddad I.A., Antimirov M.Ya. Solution of a problem to the flow over the roughness elements of a special form in a strong magnetic field , 5$^{th}$ Pamir International conference, Fundamental and applied MHD, Poster session 1, June 27 – July 1, Riga, Latvia (2005).
6. Chaddad I.A. Influence of wall roughness on the structure of fluid flow in a magnetic field, RTU 48$^{th}$ International Scientific Conference, October 11 – 13, RTU, Riga (2007).

7. Chaddad, I. A, Kolyshkin A. A  Ginzburg-Landau model: an amplitude evolution equation for shallow wake flows, International conference on Mathematics and Statistics, Rome, Italy, April 25 – 27, 2008.

**BRIEF CONTENT AND STRUCTURE OF THE THESIS**

The thesis contains 5 chapters, list of references, 16 figures, 4 tables, 92  pages. List of references contains 75 titles. The thesis is written in English.

**Chapter 1. Introduction.**

Literature review is presented in the Introduction. In addition, the structure of the thesis and the main results are discussed.

The main topic of the PhD thesis is the analysis of the factors that influence the structure and stability of magnetohydrodynamic (MHD) flows and shallow water flows. In particular, we shall concentrate of the effect of roughness of the boundary. Methods of analysis are based on analytical solutions which are found for some MHD flows over roughness elements in strong magnetic fields in rectangular ducts. The MHD solutions described in our work facilitate the investigation of the redistribution of the fluid in a region where the magnetic field is strong (the Hartmann number is large). The analysis of the behavior of MHD flows at high Hartmann numbers is a topic of increasing interest since it is mainly applicable to MHD devices such as pumps, and MHD generators. The main features of MHD liquid-metal flows at large Hartmann number are as follows: A ‘ flat’ velocity profile in the core of a channel and thin boundary layers near the boundaries. Electric currents induced in the fluid modify the field of the flow. Knowing the path of these currents then it is possible to predict the flow structure.

Hydraulic engineers are effectively using Chezy formula to estimate the “lumped” effect of energy losses in turbulent flows for the computation of flow rate and losses in channels or pipes. Roughness of the boundary is taken care of by using empirical friction coefficients. These coefficients are related by several empirical formulas to the Reynolds number of the flow as well as to the roughness of the boundary. The coherent structures in wake flows are believed to appear as a final product of hydrodynamic instability of the flow. Methods of weakly nonlinear theory have been applied in the past to different flows and usually lead to amplitude evolution equations for the most unstable mode. One of such equations is the complex Ginzburg-Landau equation. Weakly nonlinear theory is applied to quasi-two-dimensional flows in [22] with Rayleigh friction (internal friction is assumed to be linearly related to the velocity distribution). It is shown in [22] that the coefficients of the Ginzburg-Landau equation for the case where the internal friction is represented by a linear function of the velocity strongly depend on the shape of the base flow profile. As a result it was concluded in [22] that weakly nonlinear models cannot be used for such cases since it is impossible to determine experimentally the base flow velocity distribution with high accuracy and, therefore, one cannot use reliable values of the coefficients of the Ginzburg-Landau equation in the analysis. However, in Chapter 5 of our work we show that small variations of linear stability characteristics do not lead to large changes in the Landau constant (the Landau constant is the real part of one of the coefficients of the Ginzburg-Landau equation) when a nonlinear Chezy formula is used to model bottom friction.

Chapters 2, 3 and 5 are theoretical while Chapter 4 is practical dealing with corrosion of EUROFER steel in the Pb17Li flow and its application to D-T ( Deutrium-tritium ) plasma confinement in a reactor.

In Chapter 2 we state the principles of MHD flows and then we describe the influence of the surface roughness on the MHD flow of a conducting metal and state the governing equations. Since MHD flow problems are widely studied in channels of various forms and different boundary conditions, the results of such studies have direct applications in different fields of magnetohydrodynamics [29], [38], and [58]. Since magnetohydrodynamics studies the motion of electrically conducting fluids in the presence of magnetic fields, it is obvious that the magnetic field influences the fluid motion. Usually in MHD problems electromagnetic force is added to the equation of motion and the magnetic field (through Ohm's law) changes the fluid motion. We describe some MHD flow problems in ducts over the roughness elements in a strong magnetic field and analytical solutions of such problems are obtained using of the Dirac delta function (see [6], [7], [12], [13], [17], [18]).

Asymptotic analysis of these problems is performed for the case of strong magnetic fields and graphs of the z-components of the current are shown for different Hartmann numbers. Different boundary layers for the field velocity and for the z-components of the currents at large Hartmann numbers are analyzed. The MHD problem for fully developed flow is solved for the cases of a uniform and non-uniform external magnetic field where the surface roughness is taken into account. The distribution of fluid velocity, induced current with its potential and external magnetic field are derived (see the following references for the analysis of similar problems [2], [5], [11]-[13], [17], [18], [21], [30], [31], [42], [50], [53], [54], [57], [59], [65], [69]).

Chapter 3 is devoted to the calculation of some classes of improper oscillatory integrals. It is shown that oscillatory integrals in some cases can be transformed to integrals of non-oscillating functions. Such integrals have direct applications to MHD flows analyzed in the thesis. These results are applied in order to transform the solution of some MHD problems arising in half space $z \geq 0$ as a result of roughness of the surface $z = 0$ for various boundary layers (see [3], [4], [6], [7], [17], [72], [74]).

During my seven year stay in Riga, Latvia (one the main MHD application centers in Europe), I had the opportunity to visit some interesting sites related to MHD study such as the Physics Institute in Salaspils where I have seen the three recently planned experimental sessions (each 2000 hours long) which have been successfully completed. Results gained in these investigations demonstrated essential influence of magnetic field on the corrosion processes both in the intensity of corrosion and its character. New results concerning the profile of corrosion are obtained in [55] and [56]. Such studies have an important implication on how to confine and control the burning D-T plasmas by a strong drag of magnetic fields inside a reactor (see [1], [9], [55], [56], [70] and [73]). In addition, I had the opportunity to participate in some PAMIR MHD International Conferences ($4^{th}$, $5^{th}$ and $7^{th}$ PAMIR International Conferences) . As a result of these activities Chapter 4 of the thesis describing practical aspects related to the effect of surface roughness on MHD flows ([1], [9], [32]-[37], [39], [40], [48], [49], [55]-[57], [60], [64], [68], [70] and [73]) was written.

Chapter 5 is devoted to the analysis of shallow water flow in a weakly nonlinear regime using the complex Ginzburg-Landau equation (CGLE). It is shown in the previous studies [22] related to weakly nonlinear analysis of quasi-two-dimensional flows (shallow water flow is one of the examples considered in [22]) that the values of the Landau's constant differ by a factor of 3 for two different velocity profiles with linear stability characteristics (differing by not more that 20%). In other words, the Landau's constant was found to be quite sensitive to the shape of the base flow profile. In Chapter 5 the bottom friction is modeled by a nonlinear Chezy formula [66]. The analysis of data presented in Table 3 and Table 4 shows that for a one-parametric family of shallow wake flows the changes in the linear stability characteristics resulted in even smaller changes in the coefficients of the CGLE. As a result, it is plausible to conclude that the complex Ginzburg-Landau equation can be used for the analysis of shallow wake flows in a weakly nonlinear regime (see [8], [10], [14]-[16], [19], [22], [26], [43]-[47], and [67]) for the application of weakly nonlinear models to different flows in fluid mechanics.

**Chapter 2. Flow over the roughness elements in strong magnetic fields.**

Chapter 2 is devoted to the analysis of the structure of MHD flows over roughness elements of different shape. Analytical solutions of the corresponding system of MHD equations are obtained. Results of numerical computations and asymptotic analysis for high Hartmann numbers are presented.

Main principles of MHD flows are formulated and discussed in Section 1. The form of the magnetic field and MHD equations for fully developed flow caused by the roughness of the boundary are derived in Section 2.

We consider a problem about the MHD flow in half-space $\tilde{z} \geq 0$ caused by the roughness of the boundary $\tilde{z} = 0$. In contrast to what is done in monograph [75] it is assumed here at the first that the induced magnetic field $\vec{B}^i$ has the $x$, $y$ and $z$ components. After that the symmetry of the flow is used and it is proved that the induced magnetic field has a single $y$-component, i.e. it has the form (4). We consider uniform external magnetic field in subsection 1.2.1 and non uniform magnetic field in subsection 1.2.2. The geometry of the flow is given on Fig. 1.

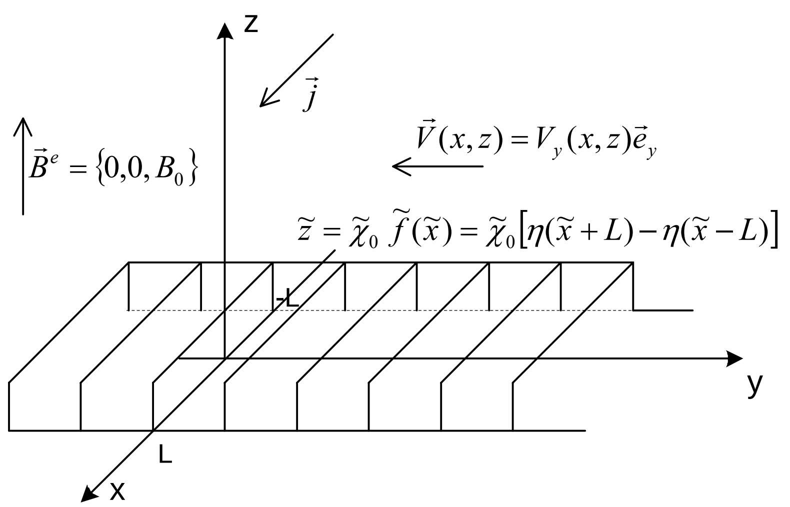


Figure 1. The geometry of the flow.

Conducting fluid is located in the half-space $\tilde{z} > 0$ , $-\infty < \tilde{x}, \tilde{y} < +\infty$. The external magnetic field is of the form

$$\vec{B}^e = B_0 \vec{e}_z \tag{1}$$

A steady current flows with the density $\vec{j}_0 = j_0 \vec{e}_x$ in the direction of the x-axis. If the surface $\tilde{z} = 0$ is ideally smooth, then the flow is absent because the electromagnetic force $\vec{F} = \vec{j} \times \vec{B}^e$ is constant and rot $\vec{F} = 0$. Suppose further that on the surface $\tilde{z} = 0$ the roughness is of the form :

$$\tilde{z} = \begin{cases} \tilde{f}(\tilde{x}), -L \le \tilde{x} \le L, -\infty < \tilde{y} < +\infty, \\ 0, \tilde{x} \notin (-L, L). \end{cases} \tag{2}$$

In this case the full current is equal to $\vec{j} = \vec{j}_0 + \vec{j}(\tilde{x}, \tilde{z})$ and the flow of the fluid with velocity

$$\vec{V}_y = \tilde{V}_y(\tilde{y}, \tilde{z}) \vec{e}_y \tag{3}$$

arises in the direction opposite to the $\tilde{y}$ -axis (see Fig.1)

It is proved that the induced magnetic field $\vec{B}^i$ in this case has the form

$$\tilde{\vec{B}}^i = \tilde{B}^i(\tilde{x}, \tilde{z}) \vec{e}_y \tag{4}$$

and the MHD equations for the fluid velocity $V_y(y, z)$ and for the potential of the induced current $\Phi(y, z)$ can be written in the following dimensionless form

$$\Delta V_y - Ha^2 V_y + Ha \frac{\partial \Phi}{\partial x} = 0, \tag{5}$$

$$\Delta \Phi = Ha \frac{\partial V_y}{\partial x}, \tag{6}$$

where $\Delta = \partial^2 / \partial x^2 + \partial^2 / \partial z^2$, $Ha = B_0 L \sqrt{\sigma / \rho \nu}$ is the Hartmann number and $\sigma, \rho, \nu$ are, respectively, the conductivity, the density and the viscosity of the fluid.
We use the MHD equations of incompressible fluid and the Ohm's law (see[29], [50] and [58]) :

$$\left(\tilde{\vec{V}}\nabla\right)\tilde{\vec{V}} = -\frac{1}{\rho} grad\tilde{P} + \nu\Delta\tilde{\vec{V}} + \frac{1}{\rho}\left(\tilde{\vec{j}} \times \tilde{\vec{B}}\right), \tag{7}$$

$$\tilde{\vec{j}} = \sigma\left(\tilde{\vec{E}} + \tilde{\vec{V}} \times \tilde{\vec{B}}\right) = \sigma\left(-grad\tilde{\Phi} + \tilde{\vec{V}} \times \tilde{\vec{B}}\right), \tag{8}$$

where $\Delta = \frac{\partial^2}{\partial\tilde{x}^2} + \frac{\partial^2}{\partial\tilde{y}^2} + \frac{\partial^2}{\partial\tilde{z}^2}$, $\quad \tilde{\vec{V}}\nabla = V_x \frac{\partial}{\partial\tilde{x}} + V_y \frac{\partial}{\partial\tilde{y}} + V_z \frac{\partial}{\partial\tilde{z}}$.

In our case

$$\tilde{\vec{V}} = \tilde{V}_y(x,z)\vec{e}_y, \tag{9}$$

$$\vec{B} = \tilde{\vec{B}}^i(\tilde{x},\tilde{z}) + \vec{B}^e, \tag{10}$$

where $\tilde{\vec{B}}^i$ is the induced magnetic field.

First, we prove that

$$\tilde{\vec{B}}^i(\tilde{x},\tilde{z}) = B^i(\tilde{x},\tilde{z})\vec{e}_y \tag{11}$$

at the condition that the vector of the induced current has the form

$$\tilde{\vec{j}}(\tilde{x},\tilde{z}) = j_x(\tilde{x},\tilde{z})\vec{e}_x + j_z(\tilde{x},\tilde{z})\vec{e}_z \tag{12}$$

and it will be shown as the corollary that the vector of fluid velocity is given by (9). For this purpose we use the Bio-Savare's law, according to which the induced magnetic field vector $d\vec{B}$ created by an element $d\vec{l}$ of infinitely thin wire directed along the current $\vec{I}$ is equal to

$$d\tilde{\vec{B}} = I\frac{d\vec{l} \times \vec{r}_{M'M}}{\left|\vec{r}_{M'M}\right|^3} \tag{13}$$

where $\vec{r}_{M'M}$ is a radius vector connecting the point $M'(\tilde{x}',\tilde{y}',\tilde{z}') \in d\vec{l}$ and the point of observation $M(\tilde{x},\tilde{y},\tilde{z})$ (see Fig. 2):

$$\vec{r}_{M'M} = (\tilde{x} - \tilde{x}')\vec{e}_x + (\tilde{y} - \tilde{y}')\vec{e}_y + (\tilde{z} - \tilde{z}')\vec{e}_z. \tag{14}$$

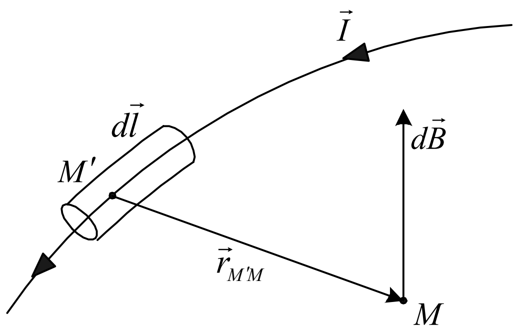


Figure 2. Magnetic induction $d\vec{B}$ caused by elementary current $Id\vec{l}$ .

Without loss of generality we can choose the point of observation *M (0, 0, 0)* in the origin. For each point $M'(\tilde{x}',\tilde{y}',\tilde{z}')$ in the fluid we always can choose the symmetric point $N'(\tilde{x}',-\tilde{y}',\tilde{z}')$ with respect to point *M (0, 0, 0).* We consider the magnetic induction $d\vec{B}$ caused by elementary current $Id\vec{l}$ passing through the point $M'(\tilde{x}',\tilde{y}',\tilde{z}')$ and by elementary current $I_1 d\vec{l}$ passing through the symmetric point $N'(\tilde{x}',-\tilde{y}',\tilde{z}')$ (see Fig. 3). Here $\vec{I}$ and $\vec{I}_1$ are the currents with density $\tilde{\vec{j}}(\tilde{x},\tilde{z})$ given by formula (12).

Since vector $\tilde{\vec{j}}(\tilde{x},\tilde{z})$ doesn't depend of variable $\tilde{y}$ we have in our case that $\vec{I}_1 = \vec{I}$

Then, according to formula (13), we have

$$d\vec{B}\Big|_M = Dd\vec{l} \times \left(\vec{r}_{M'M} + \vec{r}_{N'M}\right) \tag{15}$$

where $D = I\left|r_{MM'}\right|^{-3}$, $d\vec{l} = dl_x\vec{e}_x + dl_z\vec{e}_z$, (16)

$$\vec{r}_{M'M} = -\left(\tilde{x}'\vec{e}_x + \tilde{y}'\vec{e}_y + \tilde{z}'\vec{e}_z\right),\ \vec{r}_{N'M} = -\left(\tilde{x}'\vec{e}_x - \tilde{y}'\vec{e}_y + \tilde{z}'\vec{e}_z\right). \tag{17}$$

Substituting (16) and (17) into (15) we obtain:

$$d\vec{B} = D\left(2\tilde{z}'dl_x - 2\tilde{x}'dl_z\right)\vec{e}_y . \tag{18}$$

Summing formula (18) over the whole elements $d\vec{l}$ in the fluid we obtain formula (11), which completes the proof.

In order to obtain equations (5), (6) we substitute vectors $\vec{\tilde{V}}$ and $\vec{\tilde{B}}^i$ from (9), (10) and (11) into equations (7) and (8). After some algebraic manipulations we obtain

$$\vec{\tilde{j}} \times \vec{\tilde{B}} = \sigma \left\{ B_0 \frac{\partial \tilde{\Phi}}{\partial \tilde{x}} \vec{e}_y - \frac{\partial \tilde{\Phi}}{\partial \tilde{x}} \tilde{B}^i \vec{e}_z + \frac{\partial \tilde{\Phi}}{\partial \tilde{z}} \tilde{B}^i \vec{e}_x - B_0^2 \tilde{V}_y \vec{e}_y + B_0 \tilde{V}_y \tilde{B}^i \vec{e}_z \right\} \quad (19)$$

$$\left(\vec{\tilde{V}} \nabla\right) \vec{\tilde{V}} = 0 \, . \quad (20)$$

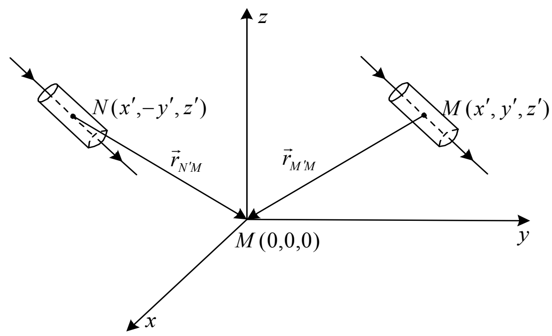


Figure 3. Symmetric representation needed to the proof of formula (18).

Substituting (19) and (20) into (7) and performing some algebraic transformations we obtain the following relationship

$$\nu \left( \frac{\partial^2}{\partial \tilde{x}^2} + \frac{\partial^2}{\partial \tilde{z}^2} \right) \tilde{V}(\tilde{x}, \tilde{z}) + \frac{\sigma}{\rho} \left[ B_0 \frac{\partial \tilde{\Phi}(\tilde{x}, \tilde{z})}{\partial \tilde{x}} - B_0^2 \tilde{V}_y(\tilde{x}, \tilde{z}) \right] = 0.$$

We use the dimensionless quantities by taking the values $L, \nu / L, B_0, \nu \sqrt{\rho \nu / \sigma}, \nu \sqrt{\rho \nu / \sigma} / L^2$ as the scales of length, velocity, magnetic field, potential and current, respectively.

To obtain equation (6), we apply operation of divergence to equation (8) and use the equation of continuity $div \, \vec{\tilde{j}} = 0$:

$$0 = -\Delta \tilde{\Phi} + B_0 div \tilde{V}_y(\tilde{x}, \tilde{z}) \vec{e}_y \quad (21)$$

Passing in formulae (21) to the dimensionless variables, we obtain equation (6).

Similar analysis is performed in Section 1.2 where it is assumed that the external magnetic field and the given external current has only $x$ and $z$ components, which do not depend on the variable $y$.

Analytical solution to the MHD problem over roughness elements in a uniform external magnetic field is obtained in Section 1.3.

The conducting fluid is located in the half space $\tilde{z} > 0,\ -\infty < \tilde{x}, \tilde{y} < +\infty$. The external magnetic field has the form (1).

The boundary $\tilde{z} = 0$ is not conducting. A steady current flows with the density $\tilde{\vec{j}} = j_0 \vec{e}_x$ in the direction of the $x$-axis. If the surface $\tilde{z} = 0$ is ideally smooth then the flow is absent because the electromagnetic force $\vec{F} = \tilde{\vec{j}} \times \vec{B}$ is constant and $rot\vec{F} = 0$. Suppose that on the surface $\tilde{z} = 0$ there is roughness of the rectangular form (see Fig.1):

$$\tilde{z} = \tilde{\chi}_0\, \tilde{f}(\tilde{x}) = \tilde{\chi}_0 \left[\eta(\tilde{x} + L) - \eta(\tilde{x} - L)\right] = \begin{cases} \tilde{\chi}_0, & -L < \tilde{x} < L, \\ 0, & |\tilde{x}| > L, \end{cases} \tag{22}$$

where $\eta(\tilde{x})$ is the Heaviside step function: $\eta(\tilde{x}) = \begin{cases} 0, & \tilde{x} < 0, \\ 1, & \tilde{x} > 0. \end{cases}$ (23)

In this case the full current is equal to $\vec{j} = \vec{j}_0 + \vec{j}(\tilde{x}, \tilde{z})$ and the flow of the fluid with the velocity $\vec{V} = \tilde{V}_y(\tilde{y}, \tilde{z})\vec{e}_y$ arises in the direction opposite to the $\tilde{y}$ axis (Fig.1).

We will derive the boundary condition for the potential $\tilde{\Phi}(\tilde{x}, \tilde{y})$ of an electrical field on the surface $\tilde{z} = \tilde{\chi}_0 \tilde{f}(\tilde{x})$. The normal component of the current on this surface must be equal to zero because the boundary $\tilde{z} = \tilde{\chi}_0 \tilde{f}(\tilde{x})$ is not conducting, i.e. it must be $\vec{j} \cdot \vec{n} = 0$ on the surface ($\vec{n}$ is the unit normal to the surface).

Using formula $\vec{n} = grad[\tilde{z} - \tilde{\chi}_0 \tilde{f}(\tilde{x})] / \sqrt{1 + \tilde{\chi}_0^{\,2} \tilde{f}'^2(\tilde{x})}$ we obtain

$$\vec{n} = \left[-\tilde{\chi}_0 \tilde{f}'(\tilde{x})\vec{e}_x + \vec{e}_z\right] / \sqrt{1 + \tilde{\chi}_0^{\,2} \tilde{f}'^2(\tilde{x})}\,, \tag{24}$$

where

$$\tilde{f}'(\tilde{x}) = \left[\delta(\tilde{x} + L) - \delta(\tilde{x} - L)\right],$$

$\delta(\tilde{x})$ is the Dirac delta function.

Substituting $\vec{n}$ from (24) and $\vec{\tilde{j}} = (j_0 + \tilde{j}_x(\tilde{x},\tilde{z}))\vec{e}_x + \tilde{j}_z(\tilde{x},\tilde{z})\vec{e}_z$ into $\vec{\tilde{j}} \cdot \vec{n} = 0$ and using formula $\vec{\tilde{j}} = \sigma\left[-grad\Phi + \vec{\tilde{V}} \times \vec{\tilde{B}}\right]$, i.e. $\tilde{j}_x = -\sigma\,\partial\tilde{\Phi}/\partial\tilde{x}$, $\tilde{j}_z = -\sigma\,\partial\tilde{\Phi}/\partial\tilde{z}$ on the surface, where $\vec{\tilde{V}} = 0$, we obtain the boundary condition for the potential $\tilde{\Phi}(\tilde{x},\tilde{z})$:

$$\tilde{z} = \tilde{\chi}_0 \tilde{f}(\tilde{x}) :\ -\sigma\frac{\partial\tilde{\Phi}}{\partial\tilde{z}} = \tilde{\chi}_0\left[\ j_0 \tilde{f}'(\tilde{x})\ \ -\sigma\frac{\partial\Phi}{\partial\tilde{x}}\tilde{f}'(\tilde{x})\right]. \qquad (25)$$

The only approximation which is made in this section is the following: we transfer the boundary condition from the surface $\tilde{z} = \tilde{\chi}_0\tilde{f}(\tilde{x})$ to the plane $\tilde{z} = 0$, i.e. we only assume that the value $\tilde{\chi}_0\left|\tilde{f}(\tilde{x})\right|$ is small. As a result, we obtain the boundary condition for the potential in the form

$$\tilde{z} = 0 : \partial\tilde{\Phi}/\partial\tilde{z} = \tilde{\chi}_0\left[-j_0\sigma^{-1} + \partial\tilde{\Phi}/\partial\tilde{x}\right]\cdot\left[\delta(\tilde{x}+L) - \delta(\tilde{x}-L)\right]. \qquad (26)$$

We use the following dimensionless quantities using the values L, $\nu/L$, $B_0$, $\nu\sqrt{\rho\nu/\sigma}/L$, $\nu\sqrt{\rho\nu\sigma}/L^2$ as scales of length, velocity, magnetic field, potential and current, respectively. Here $\sigma$, $\rho$, $\nu$ are, respectively, the conductivity, the density and the viscosity of the fluid. Then the MHD equations and the boundary conditions have the form (see [28]):

$$\Delta V_y - Ha^2 V_y + Ha\cdot\partial\Phi/\partial x = 0,\quad \Delta\Phi = Ha\cdot\partial V_y/\partial x\ , \qquad (27),(28)$$

$$z = 0 : V_y = 0\,, \partial\Phi/\partial z = \chi_0\left[-A + F(x,0)\right]\cdot\left[\delta(x+1) - \delta(x-1)\right], \qquad (29),(30)$$

$$\sqrt{x^2 + z^2} \to \infty :\ \ V_y \to 0, \Phi \to 0\,, \qquad (31)$$

where $\Delta = \partial^2/\partial x^2 + \partial^2/\partial z^2$, $Ha = B_0 L\sqrt{\sigma/\rho\nu}$ is the Hartmann number, $A = j_0 L^2/(\nu\sqrt{\rho\nu\sigma})$, $\chi_0 = \tilde{\chi}_0/L$ and $F(x,0) = \left.\frac{\partial\Phi}{\partial x}\right|_{z=0}$.

In order to solve problem (27)-(31) we use the symmetry of this problem with respect to $x$: the function $V_y(x,z)$ is an even function, $\Phi(x,z)$ is an odd function with respect to $x$. This means that the functions $V_y(x,z)$ and $\Phi(x,z)$ satisfy additional boundary conditions:

$$z = 0 :\ \ \frac{\partial V_y}{\partial x} = 0, \Phi(x,0) = 0\,. \qquad (32)$$

Therefore, problem (27)-(31) can be solved by means of Fourier cosine and Fourier sine transforms (see [3], [4]). Namely, we apply the Fourier cosine transform with respect to $x$ to equation (27) and to $V_y$ in boundary condition (29) and the Fourier sine transform to equation (28) and to $\partial\Phi/\partial z$ in boundary condition (30), that means, by substituting:

$$V_y^{\,c}(\lambda,z)=\sqrt{\frac{2}{\pi}}\int_0^\infty V_y(x,z)\cos\lambda x dx\,,\Phi^s(\lambda,z)=\sqrt{\frac{2}{\pi}}\int_0^\infty \Phi(x,z)\sin\lambda x dx\,. \quad (33)$$

We obtain the following system of ordinary differential equations for unknown functions $V_y^{\,c}(\lambda,z)$, $\Phi^s(\lambda,z)$:

$$-\lambda^2 V_y+\frac{d^2V_y^{\,c}}{dz^2}-Ha^2V_y^{\,c}+Ha\lambda\Phi^s=0\,, \quad (34)$$

$$-\lambda^2\Phi^s+\frac{d^2\Phi^s}{dz^2}+Ha\lambda V_y^{\,c}=0\,. \quad (35)$$

We apply also transforms (33) to the boundary conditions:

$$z=0:\quad V_y^{\,c}=0,\quad \frac{d\Phi^s}{dz}=\chi_0\left[A-F(1,0)\right]\sqrt{\frac{2}{\pi}}\sin\lambda\,; \quad (36)$$

$$z\to\infty: V_y^{\,c},\Phi^s\to 0, \quad (37)$$

where $F(1,0)=\dfrac{\partial\Phi}{\partial x}$ at $x=1, z=0$ is an unknown constant. The solution of the problem (34)-(37) has the form:

$$\Phi^s(\lambda,z)=\chi_0\sqrt{\frac{2}{\pi}}\left[-F(1,0)+A\right]\frac{\sin\lambda}{2\lambda^2}\left(k_1e^{k_2z}+k_2e^{k_1z}\right)\,, \quad (38)$$

$$V^c(\lambda,z)=\chi_0\sqrt{\frac{2}{\pi}}\left[-F(1,0)+A\right]\frac{\sin\lambda}{2\lambda}\left(e^{k_1z}-e^{k_2z}\right), \quad (39)$$

where $k_1=-(\sqrt{\lambda^2+\mu^2}+\mu)$, $k_2=-(\sqrt{\lambda^2+\mu^2}-\mu)$, $2\mu=Ha$.

Applying the inverse Fourier sine and cosine transforms to formulae (38), (39), we obtain the solution of problem (27)-(31), containing unknown constant $F(1,0)$:

$$\Phi(x,z)=\frac{\chi_0}{\pi}\left[-F(1,0)+A\right]\int_0^\infty\left(k_1e^{k_2z}+k_2e^{k_1z}\right)\frac{\sin\lambda}{\lambda^2}\sin\lambda x d\lambda\,, \tag{40}$$

$$V_y(x,z)=\frac{\chi_0}{\pi}\left[-F(1,0)+A\right]\int_0^\infty\left(e^{k_1z}-e^{k_2z}\right)\frac{\sin\lambda}{\lambda}\cos\lambda x d\lambda\,. \tag{41}$$

Using the components $j_x$ and $j_z$ of the induced current density and performing some transformations of the solution we obtain the formula for unknown constant $F(1,0)$:

$$F(1,0)=-\frac{\chi_0}{2\pi}A\frac{1}{1-\frac{\chi_0}{2\pi}}\,. \tag{42}$$

Asymptotic analysis of the solution for the case $Ha\to\infty$ is also performed in Section 1.3. Several regions of the flow are found for high Hartmann numbers, namely, Hartmann boundary layer, the flow core and the distant wake.

The graphs of the $z$ -component $j_z(x,z)$ of the current are shown in Fig. 4. Calculations are done with Mathematica.

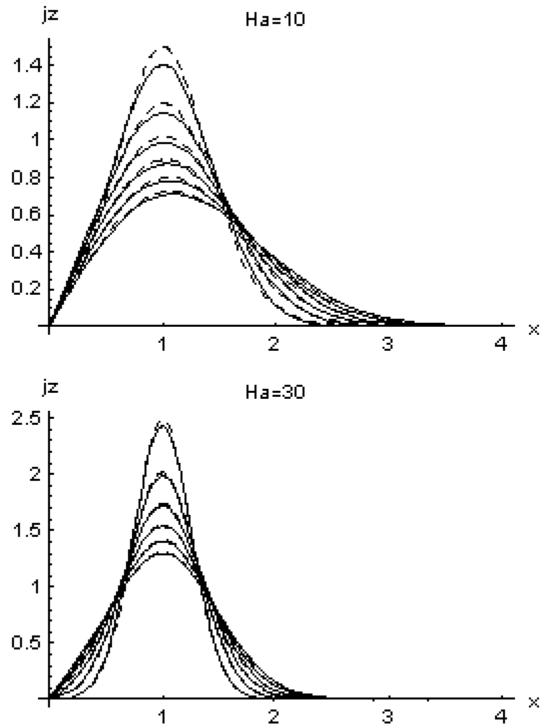


Figure 4. The graphs of the $z$ -component of current by exact formula (---) and by approximate formula ( ______ ) from $z=1$ (two upper lines) to $z=3.5$ (two lower lines) through $\Delta z=0.5$. Function $j_z(x,z)$ is odd with respect to $x$.

The streamlines of the current $\vec{j}(x,z)$ in region $0 \le x \le 1$ are shown in Fig. 5 for two values of $Ha$.

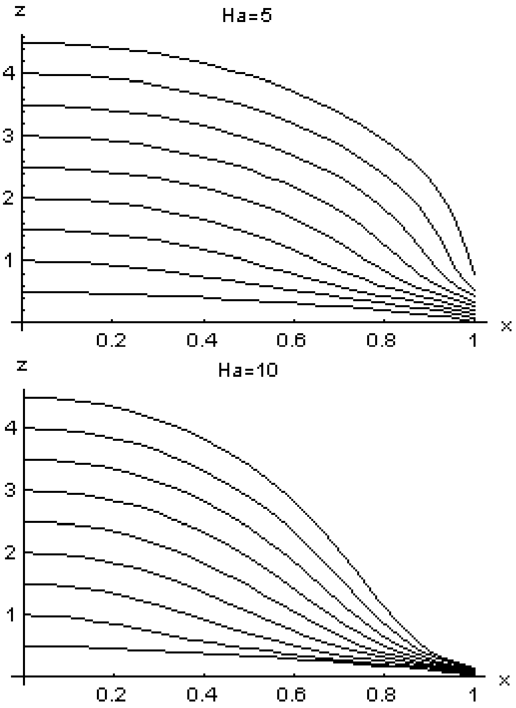


Figure 5. The streamlines of current $\vec{j}(x,z)$ in region $0 \le x \le 1$ at Ha=5 and at Ha=10.

The following conclusions can be drawn from the solution of the problem:

4. The analytical solution is obtained at the single approximate assumption that the height of the roughness is small .The solutions for the $y$ component of the velocity of the fluid and for the $x$ component of the induced current are obtained in the form of improper integrals of elementary functions. On the other hand, the $z$ component of the induced current is expressed through the Bessel functions.
5. The asymptotic solution of the problem at Hartmann number $Ha \to \infty$ is obtained in the form of elementary functions. For Hartmann numbers Ha ≥ 10 the exact and the asymptotic solutions practically coincide.
6. Several boundary layers for the velocity of the fluid and for the $x$ and $z$ components of the current at large Hartmann numbers are found.

Analytical solution described in Section 1.3 is generalized in Section 1.4 for the case of roughness of the form

$$\tilde{z} = \tilde{F}(\tilde{x}) = \begin{cases} \tilde{\chi}_1, & |\tilde{x}| < L_1 \\ \tilde{\chi}_0, & L_1 < |\tilde{x}| < L \\ 0, & |\tilde{x}| > L \end{cases}.$$

The graph of this function is shown below.

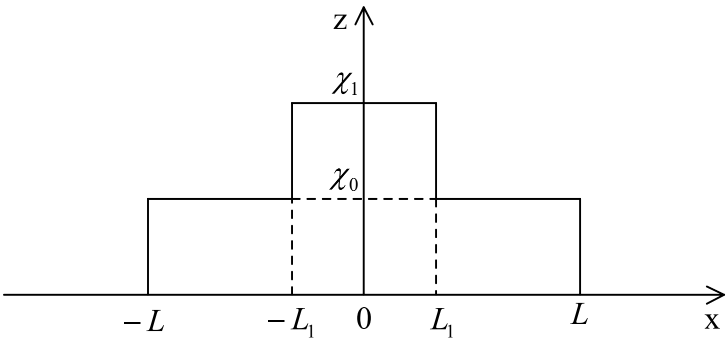


Figure 6. The geometry of the flow.

## Chapter 3. Evaluation of improper integral.

In this chapter the integral of the form

$$\int_0^\infty \frac{P_n(\lambda^2)}{Q_m(\lambda^2)} e^{-a\sqrt{\lambda^2+b^2}} \frac{\cos\lambda\cos\lambda x}{\lambda^2-\frac{\pi^2}{4}} d\lambda , \tag{43}$$

where $P_n(\lambda^2)$, $Q_m(\lambda^2)$ are polynomials of degrees $n$ and $m$, respectively, and $m \geq n,\ a > 0,\ \ b > 0$, $x > 0$ are some positive parameters is transformed into integrals of monotone functions using the convolution theorem for product of two Fourier cosine transforms.

We suppose that all zeros of polynomial $Q(\lambda^2)$ are simple and have the form: $\lambda_k{}^2 = -a_k{}^2$, $k = 1, 2, \cdots 3, ..., n$.

Let

$$F_c(\lambda) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x)\cos\lambda x dx \tag{44}$$

be the Fourier cosine transform of the function $f(x)$.

We use the theorem (see [4]):

If $F_c(\lambda)$ and $\Phi_c(\lambda)$ are the Fourier cosine transforms of functions $f(x)$ and $\varphi(x)$, respectively, then

$$\int_0^\infty F_c(\lambda)\Phi_c(\lambda)\cos\lambda x d\lambda = \frac{1}{2}\int_0^\infty \varphi(\xi)\left[f(|x-\xi|) + f(x+\xi)\right]d\xi . \tag{45}$$

Let $\dfrac{P_n(\lambda^2)}{Q_m(\lambda^2)} \dfrac{\cos\lambda}{\lambda^2-\frac{\pi^2}{4}} = \Phi_c(\lambda), e^{-a\sqrt{\lambda^2+b^2}} = F_c(\lambda)$. (46)

To obtain the functions $\varphi(x)$, $f(x)$ it is necessary to evaluate the integrals:

$$I_1 = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{P_n(\lambda^2)}{Q_m(\lambda^2)} \frac{\cos\lambda \cos\lambda x \, d\lambda}{\lambda^2 - \frac{\pi^2}{4}} = \varphi(x), \; I_2 = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-a\sqrt{\lambda^2+b^2}} \cos\lambda x \, d\lambda = f(x)$$

For evaluation of $I_2$ we use the integral known in the literature [74]:

$$\int_0^\infty \frac{e^{-a\sqrt{\lambda^2+b^2}}}{\sqrt{\lambda^2+b^2}} \cos\lambda x \, d\lambda = K_0(b\sqrt{a^2+x^2}), \tag{47}$$

where $K_0(z)$ is the modified Bessel function of order 0 of the second kind.

Differentiating formula (47) with respect to a we evaluate integral $I_2$:

$$I_2 = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-a\sqrt{\lambda^2+b^2}} \cos\lambda x \, d\lambda = \sqrt{\frac{2}{\pi}} \frac{K_1(b\sqrt{a^2+x^2})}{\sqrt{a^2+x^2}} = f(x) \tag{48}$$

where $K_1(z)$ is the modified Bessel function of order 1 of the second kind.

For evaluation of integral $I_1$ we use the residue method (see [6]):

$$I_1 = \sqrt{\frac{2}{\pi}} \frac{1}{2} \mathrm{Re}\left\{ \left( 2\pi i \sum_{k=1}^{m} \operatorname*{Res}_{a_k i} + \pi i \operatorname*{Res}_{\pi/2} \right) \frac{P_n(z^2)}{Q_m(z^2)} \left[ e^{iz(1-x)} + e^{iz(1+x)} \right] \right\}, \tag{50}$$

where $\operatorname*{Res}_{z_0} \frac{\varphi(z)}{\psi(z)} = \frac{\varphi(z_0)}{\psi'(z_0)}$,

at the condition, that $\varphi(z)$ and $\psi(z)$ are the analytical functions at $z_0$ and on some small neighborhood where $\psi(z_0) = 0$, $\psi'(z_0) \neq 0$. It follows from (49) that

$$I_1 = \sqrt{\frac{2}{\pi}} \frac{\pi}{2} \sum_{k=1}^{m} \frac{P_n(-a_k^2)}{Q_m'(-a_k^2)} \left[ e^{-a_k|1-x|} \operatorname{sign}(1-x) + e^{-a_k(1+x)} \right] +$$

$$+ \frac{1}{2} \frac{P_n\left(\frac{\pi^2}{4}\right)}{Q_m'\left(\frac{\pi^2}{4}\right)} \left[ \sin\left(\frac{\pi}{2}|1-x|\right) sign(1-x) + \sin\left(\frac{\pi}{2}|1+x|\right) \right] = \varphi(x), \tag{51}$$

where $sign(1-x)$ means the sign of $(1-x)$.

Using (43), (46) and (50) and taking into account that

$|1-x|^2 = (1-x)^2$, we transform integral (43) into integral of non-oscillatory function:

$$\int_0^\infty \frac{P_n(\lambda^2)}{Q_m(\lambda^2)} e^{-a\sqrt{\lambda^2+b^2}} \cos\lambda \cos\lambda x d\lambda =$$

$$= \frac{ab}{\pi}\int_0^\infty \varphi(\xi)\left[\frac{K_1\left(b\sqrt{(x-\xi)^2+a^2}\right)}{b\sqrt{(x-\xi)^2+a^2}} + \frac{K_1\left(b\sqrt{(x+\xi)^2+a^2}\right)}{b\sqrt{(x+\xi)^2+a^2}}\right] d\xi\,, \qquad (52)$$

where $\varphi(\xi)$ is given by formula (49).

Similarly, it can be shown that

$$\int_0^\infty \frac{e^{-z\sqrt{\lambda^2+\mu^2}}}{\frac{\pi^2}{4}-\lambda^2} \cos\lambda \cos\lambda x d\lambda =$$

$$= \frac{2\mu z}{\pi}\int_0^1 \left[\frac{K_1\left(\mu\sqrt{z^2+(x-\xi)^2}\right)}{\sqrt{z^2+(x-\xi)^2}} + \frac{K_1\left(\mu\sqrt{z^2+(x+\xi)^2}\right)}{\sqrt{z^2+(x+\xi)^2}}\right] \cos\frac{\pi}{2}\xi\, d\xi\,. \quad (53)$$

Integral (52) can be easily evaluated using package "Mathematica" for all values of the parameters $x \geq 0$ and $z \geq 0$. As it can be seen from formula (52), the advantages of these transformations are:

3. The parameter $x$ goes from an argument of oscillatory function cosine into the argument of a monotone Bessel function $K_1$;
4. The limits of integration are changed in the bounded region $0 \leq \xi \leq 1$.

The integrals (52) and (54) are used to evaluate or transform the solution of problems about MHD flows arising due to the roughness of the surface (see [2], [13]). One such example is given in Section 3.2.

## Chapter 4. Corrosion of Eurofer steel and magnetic confinement of plasma in reactors.

### Section 4.1. Deuterium-Tritium reaction and its use in Reactors.

During this century, the world's population will double from six billion people and it will rise to ten billions by 2050. More importantly, a lot more energy will be used than we use today, energy consumption will probably be two times higher by the middle of the century with an even stronger increase in electricity consumption.

Table 1 below shows the pattern of energy consumption in 2007.

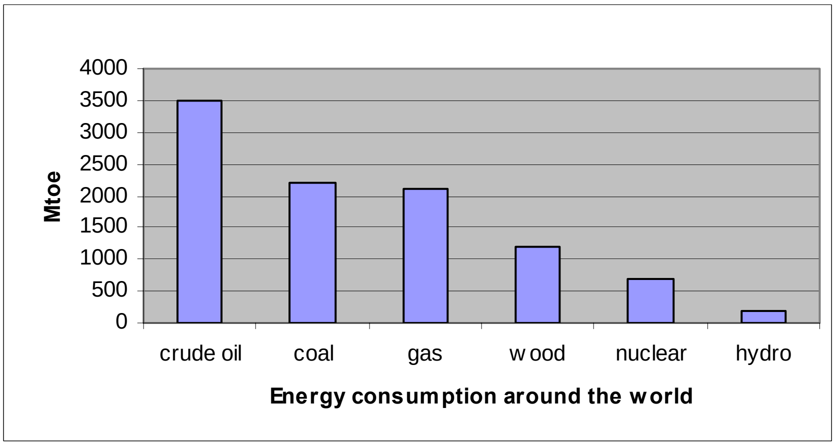


Table 1. Energy consumption by the year 2007 [Mtoe (Million Tonnes Of Oil Equivalent)]
(The exact values are respectively 3500, 2200, 2100, 1200, 700, and 200)

Scientists from all European member states and G8 countries associated with the EURATOM fusion program have been trying to reproduce the Deutrium-Trithium Plasma reaction process on Earth inside od the blanket of a reactor as:

$$ {}_{1}^{2}\mathrm{H} + {}_{1}^{3}\mathrm{H} \rightarrow {}_{2}^{4}\mathrm{He} + {}_{0}^{1}\mathrm{n} + 17.6\ \mathrm{MeV} \qquad (53) $$

(see [1], [32], [33], [57], [49]).

The fusion energy ( 17.6 MeV ) appears as kinetic energy of neutrons (14.1 MeV ) that need to be saved inside of a reactor using lead, and of alphas ( 3.5MeV ) that are evacuated as ashes from the chimney of a certain reactor [1], [36], [37], and [64].
Deutrium is generously present in seawater but Tritium is a radioactive element rarely existent naturally on Earth. However it can be bred inside the reactor using the reaction of the neutrons in a blanket containing lithium, an abundant light metal in the nature as:

Li6 + n ---- T + He4 + 4.8 MeV

Li7 + n ---- T + He4 – 2.5 MeV

Ten grams of deuterium which can be extracted from 500 liters of water and 15 gr of tritium produced from 30 g. the lithium would produce enough fuel for the lifetime electricity needs of a person in an industrialized country (see [1], [32], [39], [35], [49], [55], [56], [64]).

Progress of the D-T plasmas confinement inside of reactors is discussed starting from the **JET** project (Joint European Torus) reaching ITER (International Thermonuclear Experimental Reactor) which will provide the technical data necessary for future decisions for other future reactors such as DEMO and PROTO which will become the first proto-type power station with complete reactor and ancillary systems that would generate electricity on a commercial scale. The construction suppose to start in 2050s and its operation in 2070s (see [1], [33], [35], [49], [57], [64]).

**Section 4.2. Analysis of MHD phenomena influence on the corrosion of EUROFER steel in the Pb-17Li flow**

Consider MHD flow of a conducting fluid with roughness of the surface in the form $\tilde{z} = \tilde{\chi}_0 \cos(\pi\tilde{x}/2L)$ located in the half space $\tilde{z} > 0, -\infty < \tilde{x}, \tilde{y} < +\infty$. The external magnetic field is $B^c = B_0 e_z$. Corrosion of EUROFER Steel in the Pb-17Li flow can be considered as a consequence of roughness where the Hartmann surfaces flows are perpendicular to the flow as well.

Despite of the fact that corrosion of steel in the Pb-17Li flow is a small but important part of the reactor work, we notify the importance and newest results attained of the corrosion process done in the Physics Institute in Latvia [55], [56]. This experiment was performed on different samples with flow velocities of 2,5 cm/s and 5cm/s and magnetic current of 0, 1,5 and 1.7 T. Results gained in these investigations demonstrated essential influence of magnetic field on the corrosion processes both in the intensity of corrosion and its character (see Fig. 7).

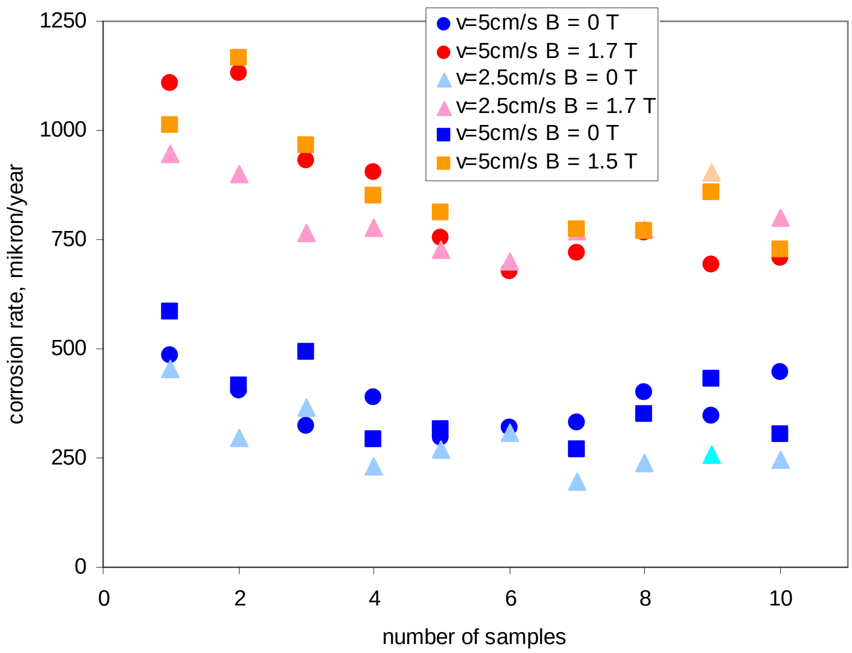


Figure 7. Comparison of corrosion rate of EUROFER samples in magnetic field and without magnetic field.

The other experiment showed that the corroding surfaces were oriented in the melt flow direction (see Fig. 8).

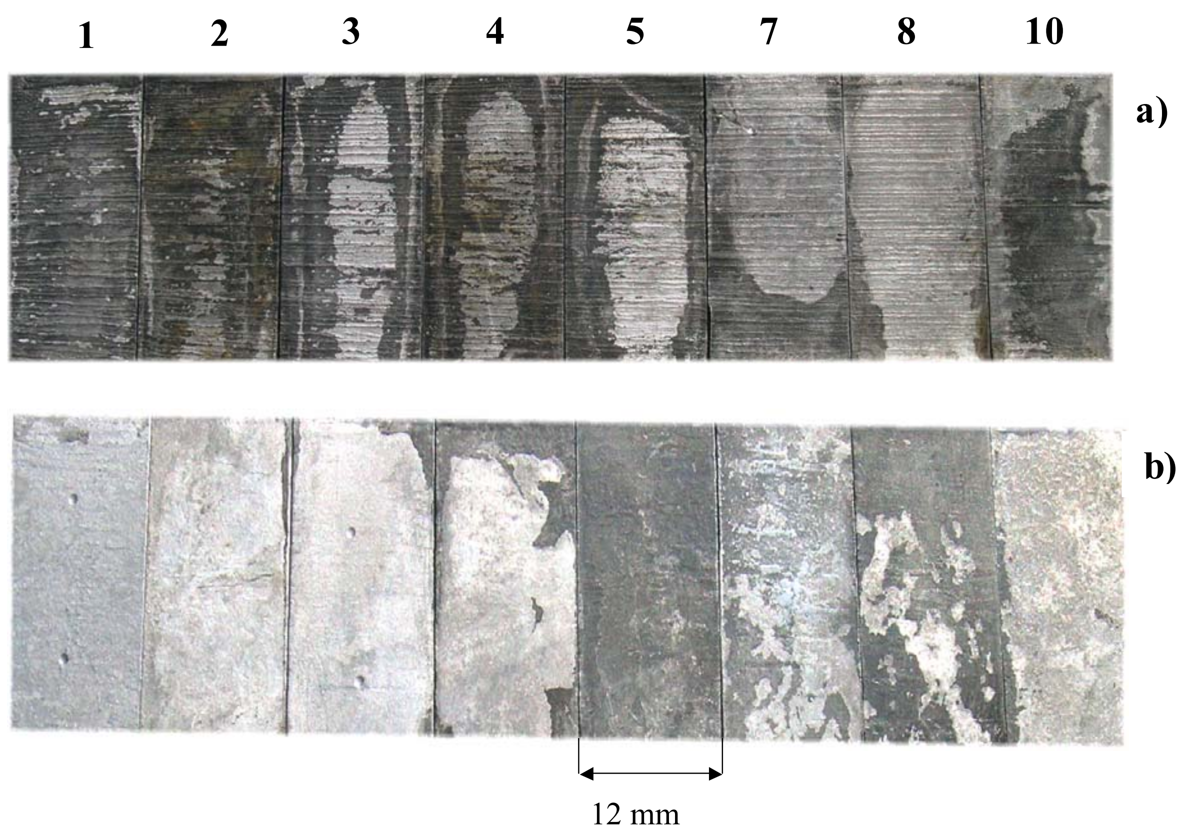


Figure 8. Surface relief of EUROFER samples subjected to corrosion in Pb17Li during 2000 hours.

There is a hope that before the end of this century scientists will be able to achieve success of ITER project. This will provide the physical and technological basis for the construction in the future electrically generating power plants like DEMO and PROTO . Then a new clean and cheap source of energy would be a part of humans' life (see [1], [9], [20], [28], [32]-[37], [40] , [49] , [51], [55] , [56] , [62] , [70], [73]).

## Chapter 5. Ginzburg-Landau equation for stability analysis of shallow water flows in weakly nonlinear regime.

Losses due to turbulent friction are often described in hydraulics by means of empirical (or semi-empirical) formulas like Chezy of Manning's formulas [22]. In particular, the Chezy formula is used to represent the bottom friction force $\vec{F}$ in the form

$$\vec{F} = \frac{\rho g A c_f}{h} \vec{v}|\vec{v}|,$$

where $\rho$ is the density of the fluid, $g$ is the acceleration due to gravity, $A$ is the cross-sectional area, $h$ is water depth, $c_f$ is the friction (or roughness) coefficient, $\vec{v}$ is the velocity vector and $\vec{F}$ is the friction force. The coefficient $c_f$ is estimated by means of several empirical formulas which can be found in the literature. One example is Colebrook formula [66] which relates $c_f$ to the Reynolds number of the flow.

Consider the base flow of the form

$$\vec{U} = (U(y), 0) \tag{54}$$

where

$$U(y) = 1 - \frac{2R}{1-R}\frac{1}{\cosh^2(\alpha y)}. \tag{55}$$

The base flow (55) is suggested in [19] after careful analysis of available experimental data for deep water flows behind circular cylinders. The profile (55) is also adopted in the present study. The parameter $R$ is the velocity ratio: $R = (U_m - U_a)/(U_m + U_a)$, where $U_m$ is the wake centerline velocity and $U_a$ is the ambient velocity, and $\alpha = \sinh^{-1}(1)$. It is shown in [44] that under the rigid-lid assumption the linear stability of wake flows in shallow water is described by the following eigenvalue problem:

$$\varphi_1''(U - c + SU) + SU_y\varphi_1' + \left(k^2 - U_{yy} - k^2U - \frac{S}{2}kU\right)\varphi_1 = 0 \tag{56}$$

$$\varphi_1(\pm\infty) = 0, \tag{57}$$

where the perturbed stream function of the flow, $\psi(x, y, t)$, is assumed to be of the form

$$\psi(x, y, t) = \varphi_1(y)\exp[ik(x - ct)] + c.c. \tag{58}$$

Here $\varphi_1(y)$ is the amplitude of the normal perturbation, $k$ is the wavenumber, $c$ is the wave speed of the perturbation, and $c.c.$ means "complex conjugate". The linear stability of the base flow (55) is determined by the eigenvalues, $c_m = c_{rm} + ic_{im}$, $m = 1,2,...$ of the eigenvalue problem (56), (57). The flow (55) is linearly stable if $c_{im} < 0$ for all $m$ and linearly unstable if $c_{im} > 0$ for at least one value of $m$.

The linear stability problem (56), (57) is solved by means of a pseudospectral collocation method based on Chebyshev polynomials. The computational procedure is briefly described below (details of the numerical method can be found in [44]). The interval $-\infty < y < +\infty$ is mapped onto the interval $-1 < r < 1$ by means of the transformation $r = \frac{2}{\pi}\arctan y$. The solution to (56), (57) is sought in the form

$$\varphi_1(r) = \sum_{k=0}^{N} a_k(1 - r^2)T_k(r), \tag{59}$$

where $T_k(r)$ is the Chebyshev polynomial of degree $k$. The collocation points $r_j$ are

$$r_j = \cos\frac{\pi j}{N},\ j = 0,1,...,N. \tag{60}$$

| $R$ | $k$ | $S_c$ | $c$ |
|---|---|---|---|
| -0.3 | 0.892 | 0.11819 | 0.69814 |
| -0.4 | 0.909 | 0.15689 | 0.65964 |
| -0.5 | 0.926 | 0.19548 | 0.62394 |
| -0.6 | 0.944 | 0.23409 | 0.59083 |
| -0.7 | 0.962 | 0.27286 | 0.55925 |
| -0.8 | 0.980 | 0.31189 | 0.52882 |

Table 2. Critical values of the stability parameter $S$.

The critical values of the stability parameter $S$ are shown in Table 2.

Following [67], we perform weakly nonlinear stability analysis in the neighborhood of the critical point. The amplitude evolution equation (the complex Ginzburg-Landau equation) for the evolution of the most unstable mode above the threshold for shallow water flows is derived in [44], all the formulas for the calculation of the coefficients of the Ginzburg-Landau equation are also given in [44]. The complex Ginzburg-Landau equation has the form

$$\frac{\partial A}{\partial \tau} = \sigma A + \delta \frac{\partial^2 A}{\partial \xi^2} - \mu |A|^2 A \tag{61}$$

Using the method described in [44] we calculate the coefficients of the CGLE (61) for different values of $R$. The results are summarized in Table 3.

| $R$ | $\sigma$ | $\delta$ | $\mu$ |
|---|---|---|---|
| -0.3 | 0.063 + 0.004i | 0.060 – 0.206i | 4.673 + 13.294i |
| -0.4 | 0.078 + 0.003i | 0.090 – 0.195i | 3.796 + 10.938i |
| -0.5 | 0.090 + 0.000i | 0.115 – 0.184i | 3.895 + 10.119i |
| -0.6 | 0.100 – 0.003i | 0.136 – 0.172i | 4.375 + 10.109i |
| -0.7 | 0.109 – 0.007i | 0.153 – 0.161i | 5.149 + 10.590i |
| -0.8 | 0.116 – 0.012i | 0.167 – 0.152i | 6.302 + 11.448i |

TABLE 3
Coefficents of the CGLE (61)

One of the major conclusions drawn from weakly nonlinear analysis applied to quasi-two-dimensional flows in [22] was the effect of strong dependence of the Landau constant $\mu_r$ on the form of the base flow profile. Calculations presented in [22] showed that the values of the Landau constant differed by a factor of 3 for two base flow velocity profiles whose linear stability characteristics differed by only 20%. As a result, it was concluded in [22] that it would be impossible to apply methods of weakly nonlinear theory in practice since the base flow profile cannot be determined very precisely in experiments. In other words, it was concluded in [22] that the problem of determination of the Landau constant from weakly nonlinear theory is ill-posed so that small variations of the base flow profile lead to large changes in the Landau constant.

The calculations presented in Table 2 and 3 in this chapter demonstrate that the coefficients of the CGLE are not so sensitive to the variation of the parameter $R$ of the base flow profile (55) as claimed in [22]. In fact, not only the Landau constant is not so sensitive to the changes in the profile (55) but all the coefficients of the CGLE do not vary too much.

## CONCLUSIONS

The present thesis is a theoretical work dealing with analysis of the structure of MHD flows and stability of shallow water flows. Solutions of some MHD problems in the presence of roughness element on the walls are obtained. The solutions are obtained in terms of improper integrals containing Bessel functions. The integrals are oscillatory at large $x$. We transform these integrals into integrals of monotone functions using the convolution theorem for product of two Fourier cosine transforms. Applications to some MHD problems are considered. We report the newest results of the three recently planned experimental sessions (each 2000 hours long) which have been finished successfully in Salaspis Latvia. The results gained in these investigations demonstrated essential influence of magnetic field on the corrosion processes both in the intensity of corrosion and its character. Besides, new results concerning the profile of corrosion are obtained [56]. The process of investigation of EUROFER corroded samples showed the existence of sufficient distinction of corrosion processes between samples located in the zone outside magnetic field ($\boldsymbol{B}$ = 0) and those located in zone with magnetic field ($\boldsymbol{B}$ = 1.7 T). Such investigations are done for the purpose of fusion control in reactors. Especially, of D-T ( Deutrium- Tritium) plasma fusion concept. One of the main things in this program is the problem of liquid metals breeder blanket behavior. Structural material of blanket should meet high requirements because of extreme operating conditions. Therefore the knowledge of the effect of metals flow velocity, temperatures and also a neutron irradiation and a magnetic field on the corrosion processes are necessary. At the moment the eutectic lead –lithium (Pb-17Li) is considered as the most suitable tritium breeder material of the reactor (see [1], [55], [56]).

We analyze stability of shallow water flows in a weakly non-linear regime by using the complex form of Ginzburg-Landau equation. In our work the bottom friction is modeled by a nonlinear Chezy formula [66]. The coefficients of the CGLE do not change much in the interval $-0.8 \le R \le -0.3$. This interval of the $R$ values corresponds to convectively unstable regime [44]. As a result, it is plausible to conclude that the complex Ginzburg-Landau equation can be used for the analysis of shallow wake flows in a weakly nonlinear regime.

The first two chapters are devoted to the analysis of MHD flows under roughness elements. Analytical solutions of the corresponding problems are obtained for the case of roughness of different forms. In particular, in the problem introduced in Section 1.3.1 we proved that the two dimensional MHD flow arises in the direction opposite to the $y$ axis, only if the roughness of the boundary is present. The solutions

for the $y$ component of the velocity of the fluid and for the $x$ component of the induced current are obtained in the form of improper integrals of elementary functions. On the other hand, the $z$ component of the induced current is expressed through the Bessel functions. The asymptotic solution of the problem at Hartmann number $Ha \to \infty$ is obtained in the form of elementary functions. For Hartmann numbers $Ha \geq 10$ the exact and the asymptotic solutions practically coincide.
Moreover, in the problem introduced in this section, it is proved that the induced magnetic field has only a $y$-component. Solutions for the system of MHD equations for the velocity fluid and for the potential of the induced current are obtained. In addition, the equations for the $x$ and $z$ components of pressure gradients are obtained. The velocity of the fluid in the core flow at large Hartmann numbers is constant. That means that it does not depend on $Ha$. With the increase of Hartmann number only the height of the core region is increased. The MHD solutions described in our work facilitate the investigation of the redistribution of the fluid in a region where the magnetic field is strong (the Hartmann number is large). These conclusions are important and can be helpful to other problems dealing with electrically conducting fluid through ducts in various area of Technology and Engineering such as MHD power generation, MHD flow-meters, MHD pumps, etc.

In Chapter 3 we consider the solutions of certain problems about MHD flow of conducting fluid in the half space that are expressed in terms of improper integrals of the product of some meromorphic function and the function $\exp(-a\sqrt{\lambda^2 + b^2})\cos\lambda\cos\lambda x$. Here $a > 0$ and $b > 0$ are some parameters, $x > 0$ is the $x$-coordinate in Cartesian coordinate system. These functions are strongly oscillating at large $x$, what make difficult the calculation of these integrals numerically. In Chapter 3 these integrals are transformed into integrals of monotone functions using the convolution theorem for product of two Fourier cosine transforms. The obtained results can be used to estimate the effect of roughness of the surface on the MHD flow in strong magnetic fields.

Chapter 4 is devoted to the practical investigation of EUROFER corrosion in the Pb17Li flow where we describe the results of the three recently planned experimental sessions which have been successfully completed. Results gained in these investigations demonstrated essential influence of magnetic field on the corrosion processes both in the intensity of corrosion and its character. Besides, new results concerning the profile of corrosion are obtained [56]. The process of investigation of EUROFER corroded samples showed that magnetic field sufficiently influence on corrosion: visual observation of test samples removed from the test section after experiments showed sufficient distinction of corrosion processes between samples located in the zone outside magnetic field ($\boldsymbol{B}$ = 0) and those located in zone with magnetic field ($\boldsymbol{B}$ = 1.7 T). Search of new energy sources draws the increasing attention to the use of reactors for this purpose. EUROATOM program scientists are designing how fusion reactors might properly operate using D-T plasma fusion concept. The plans are to built JET power plant following by ITER in the process of DEMO, and reaching PROTO at the later stage. PROTO will be the power plant that all nations around the world are waiting for as being the plant that will be purely generating a fully controlled power of energy that will be directly connected to electricity networks. Besides, it is one of the very few options potentially acceptable from the environmental safety (totally free from $CO_2$ emissions) and economic points of view. The results obtained in the thesis can be used to assess the effect of corrosion in a magnetic field.

Chapter 5 is devoted to the stability analysis of shallow water flows in a weakly non-linear regime. Calculations presented in [22] showed that the values of the Landau's constants differ by a factor of 3 for two different velocity profiles with linear stability characteristics that differ by not more than 20%. In other words, the Landau's constant in [22] was found to be quite sensitive to the shape of the base flow profile. In our work the bottom friction is modeled by a nonlinear Chezy formula [66]. The analysis of data from Table 2 and Table 3 shows that for shallow wake flows of the form (55) the changes in the linear stability characteristics resulted in even smaller changes in the coefficients of the CGLE. The coefficients of the CGLE do not change much in the interval $-0.8 \le R \le -0.3$. This interval of the $R$ values corresponds to convectively unstable regime [44]. As a result, it is plausible to conclude that the complex Ginzburg-Landau equation can be used for the analysis of shallow wake flows in a weakly nonlinear regime.

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