The European Consortium
for Mathematics in Industry
(ECMI)

Daugavpils University

University of Latvia

Vilnius Gediminas
Technical University

14th International Conference

Mathematical Modelling and Analysis

Abstracts

Daugavpils
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May 27 - 30
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14^{th} International Conference

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Daugavpils University, 2009

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APPLICATION OF THE BROWNIAN BRIDGE PROCESS IN A TERM STRUCTURE OF INTEREST RATES MODEL

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Jesper Lund in his work analyses the effect of planned EMU membership on domestic yield curves in the potential member states [1]. In order to describe dynamics of the short-term spread (between domestic and euro interest rate) under the real probability measure, Lund applies the standard Vasiček process (Ornstein-Uhlenbeck), while extends risk-neutral process with the second factor - stochastic price of risk which also follows the Vasiček process. It is not difficult to prove that in order to avoid arbitrage opportunity, short-term spread should converge to zero by time of entering the currency area. Lund's specification does not ensure fulfillment of this condition. I develop a term structure model of interest rates for a country which will join the euro area in the future. The specification of the short-term interest rate spread dynamics is defined by the Brownian bridge process, see, e.g., [2] for a more general case. This stochastic process has a feature to converge to zero at a specific time moment. Therefore, this specification ensures convergence of the short-term spread to zero by country's entrance to the currency area and thus avoids an arbitrage opportunity at this moment in time. An econometric counterpart of the theoretical model is also developed. Since observations are discretely sampled, it is needed to transform the continuous-time state-space system into a discrete-time form. The paper provides a framework of the econometric model using the extended Kalman filter.

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COMBINED SPLINES IN SMOOTHING HISTOPOLATION

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Our talk deals with space $S(T, A_1 \times A_2)$ of combined splines [1] defined by continuous linear operators $A_1 : X \to I\!R^n, \quad A_2 : X \to I\!R^m$ and $T : X \to Y$ in Hilbert spaces X and Y. Such splines give us a possibility to take into account interpolation conditions of two different types described by A_1 and A_2 correspondingly.

For given vectors $\boldsymbol{u} \in I\!R^n, \boldsymbol{v} \in I\!R^m$ and parameters $\delta, \omega, \varepsilon_i > 0,\ i = 1, \ldots, n$, we consider the following conditional minimization problems:

$$||Tx||^2 + \frac{1}{\omega}||A_1x - \boldsymbol{u}||^2 \longrightarrow \min_{A_2x = \boldsymbol{v}}, \qquad ||Tx|| \longrightarrow \min_{||A_1x - \boldsymbol{u}|| \le \delta,\ \ A_2x = \boldsymbol{v}},$$

$$||Tx|| \longrightarrow \min_{|(A_1x)_i - u_i| \le \varepsilon_i,\ \ i = 1, \ldots, n,\ \ A_2x = \boldsymbol{v}}.$$

The aim of this talk is to present some results on solutions of these problems obtained under the assumptions:

$\mathrm{Ker}T \cap \mathrm{Ker}A_1 \cap \mathrm{Ker}A_2 = \{0\}, \quad A_1(X) = I\!R^n, \quad A_2(X) = I\!R^m, \quad T(\mathrm{Ker}A_1 \cap \mathrm{Ker}A_2)$ is closed.

In particular we consider the problem of approximation of a given histogram with boundary conditions by taking

$$Tx = x^{(r)}, \quad (A_1x)_i = \int_{t_{i-1}}^{t_i} x(t)dt,\ i = 1, \ldots, n, \quad (A_2x)_1 = x(a), \quad (A_2x)_2 = x(b), \quad x \in W_2^r[a, b].$$

This investigation is closely related to our previous works on smoothing histopolation [2], [3].

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CHANGE OF NUMBER OF PERIOD ANNULI IN LIENARD TYPE EQUATIONS

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We consider Lienard type equation

$$x'' + \frac{2x}{1-x^2}x'^2 + g(x) = 0, \tag{1}$$

where $g(x) = -x(x+a)(x^2-c^2)(x-b)$, parameters a, b, c are positive and $a > c$ and $b > c$. $G(x)$ is a primitive of $g(x)$.

We are looking for so called period annuli.

THEOREM 1. *Let M_1 and M_2 be non-neighboring points of maximum of the function $G(x)$. Suppose that any other local maximum of $G(x)$ in the interval (M_1, M_2) is strictly less than $min\{G(M_1); G(M_2)\}$. Then there exist at least one nontrivial period annulus.*

We use transformation by Sabatini [1] which allows the reduction of equation (1) to a conservative one of the form

$$u'' + h(u) = 0. \tag{2}$$

We consider also the respective primitive function $H(u) = \int_0^u h(s)\, ds$.

The existence of period annulus is dependent on the system

$$H(a) > 0 \quad and \quad H(b) > 0. \tag{3}$$

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COMPUTATIONAL MODELLING OF BIOSENSOR-BASED ANALYTICAL SYSTEMS

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Biosensors are sensing devices made up of a combination of a specific biological entity, usually an enzyme, that recognizes a specific target analyte and a transducer that translates the bio-recognition event into an electrical signal [1]. The signal is usually proportional to the concentration of the analyte (substrate). The biosensors are reliable, cheap and sensitive for environmental, clinical and industrial applications. Practical biosensors form sandwich-like structures. An electrode acting as a transducer is covered by a selective membrane following a layer of the immobilized enzyme and an outer porous or perforated membrane.

To improve the efficiency of the design of an analytical system and to optimize its configuration, a model of a certain biosensor should be build. In the enzyme region, the enzyme-catalysed reaction is coupled with the mass transport by diffusion and is described by a system of non-linear reaction-diffusion equations [2; 3]. Assuming the quasi steady state approximation the dynamics of the biosensor action can be described as follows:

$$\frac{\partial S}{\partial t} = D_S \Delta S - \frac{V_{max} S}{K_M + S}, \quad \frac{\partial P}{\partial t} = D_P \Delta P + \frac{V_{max} S}{K_M + S}, \tag{1}$$

where Δ is the Laplacian, S is the substrate concentration, P is the concentration of the reaction product, V_{max} is the maximal enzymatic rate, K_M is the Michaelis constant. Outside the enzyme region only the mass transport by diffusion is considered. The porous membrane was assumed as a periodic media, and a homogenization process was applied. The governing equations together with appropriate initial, boundary and matching conditions form a boundary-value problem, which was solved numerically by applying the finite difference technique.

The biosensor geometry and the catalytic parameters significantly influence the sensitivity and stability of the biosensors [1]. The numerical simulation was applied to investigate pecularities of the biosensors response and to optimize the biosensors characteristics [2; 3].

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MATHEMATICAL MODELING OF MASS TRANSFER IN CRITICAL REGIMES OF VERTICAL TWO-PHASE FLOWS

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A system of differential equations describing main regularities of mass transfer process in critical regimes of two-phase flows is formed. Using this system, several similarity criteria are determined.

Equations of discrete-stationary solid phase distribution in such flows along the channel height are derived.

The analysis of this system allowed us to define optimal conditions of the process of bulk materials classification in vertical channels.

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CHAIN FRACTIONS IN THE MATHEMATICAL MODEL OF MULTI-STAGE SEPARATION

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Multi-stage (cascade) separation is used in various branches of industry for separating solid bulk materials into fractions by particle sizes or densities. Separation of mixtures of liquids, gases and isotopes into components is also realized by multi-stage processes.

Despite the different physical nature of these processes, all of them are characterized by common regularities of cascade separation. While developing a detailed mathematical model of such processes, we have formulated their principal regularities. In this sense, they are of generalizing character. We have succeeded to describe the cascade separation mechanism most comprehensively using chain fractions.

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ON USING SPLINES FOR THE APPROXIMATION OF A JOINT DENSITY FUNCTION

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This paper deals with the problem of the approximation of a density function underlying a given histogram on the rectangular grid $\triangle_x \times \triangle_y$, where $\triangle_x = \{x_0, x_1, ..., x_n\}$ and $\triangle_y = \{y_0, y_1, ..., y_m\}$ are two sequences of strictly increasing knots with $h_i^x = x_i - x_{i-1}$, $h_j^y = y_j - y_{j-1}$. Let

$$F_{x\times y} = \{f_{ij}, i = 1, \ldots, n, j = 1, \ldots, m\}$$

be a given histogram. The quantity f_{ij} presents the frequency of a data set on the subrectangle

$$R_{ij} = [x_{i-1}, x_i] \times [y_{j-1}, y_j].$$

We are interested in having a function $g(x, y)$ that satisfies the volume matching histopolation conditions

$$\iint_{R_{ij}} g(x, y)dxdy = h_i^x h_j^y f_{ij}, \quad i = 1, \ldots, n, j = 1, \ldots, m$$

The problem of histopolation is solvable, but not uniquely.

Different approaches to the solution of this problem by using splines are observed in this paper: the variational approach (e.g. [2]) and the approach, which concerns reproducing the main geometric characteristics of the histogram (positivity, directional monotonicity, local maxima and minima) by choosing the corresponding class of splines and creating free parameters which should be suitably chosen (e.g. [1]). One of the method for the solution of the problem of histopolation is considered in a more detailed way.

The possibility for the approximation of the joint distribution function by using splines is discussed.

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SOME NEW MODELS AND THEIR SOLUTIONS FOR INTENSIVE STEEL QUENCHING

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The conventional steel quenching is usually performed in environmentally unfriendly oil or water/polymer solutions. Contrary to traditional method the intensive quenching process uses environmentally friendly highly agitated water or low concentration of water/mineral salt solutions [1] - [3]. Traditionally for the mathematical description of the intensive quenching process, classical heat conduction equation is used. We propose in the year 2005 to use hyperbolic heat equation for more realistic description of the intensive quenching (IQ) process (especially for process initial stage). However, the utilization of this hyperbolic type partial differential equation brings serious difficulty. It is practically impossible to determine experimentally the initial heat fluxes. In our previous papers we have constructed various one and two dimensional analytical exact and approximate [4], [5] solutions for IQ processes. Here we consider few other models and construct solutions for direct and inverse problems of hyperbolic heat conduction equation. Here are both approximate (on the basis of conservative averaging method), and exact (on the basis of Green function method). We obtain the solution for initial heat flux in the form of the iterated integral equation.

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MATHEMATICAL MODELLING OF 2D MAGNETOHYDRODYNAMICS AND TEMPERATURE FIELDS, INDUCED BY ALTERNATING CURRENT FEEDING ON THE BAR TYPE CONDUCTORS IN A CYLINDER

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The heating of buildings by ecologically clean and compact local devices is interesting and actual problem. One of the modern areas of applications developed during last years is effective use of electrical energy produced by alternating current in production of heat energy. This work presents the mathematical model of one of such devices.

In papers [1; 2; 3; 4] we had modelled cylinder form electrical heat generators with six or nine circular conductors - electrodes. In this work we analyze different type of conductors. They have forms of bars and they are placed parallel to the cylinder axis in the electroconductive liquid. Let the cylindrical domain $\Omega = \{(r, \phi, z) : 0 < r < R, 0 \leq \phi \leq 2\pi, -\infty < z < \infty\}$, where R is the radius of the cylinder. The alternating current is fed to N infinite discrete conductors of forms of bars, which are placed parallel to the cylinder axis in the liquid in the domain $r < r_0 < R$.

In the weakly conductive liquid-electrolyte the current creates the radial $B_r(r, \phi)$ and the azimuthal $B_\phi(r, \phi)$ components of the magnetic field as well the axial component of the induced electric field $E_z(r, \phi)$, which, in its turn, creates the radial $F_r(r, \phi)$ and azimuthal $F_\phi(r, \phi)$ components of the Lorentz' force.

For the calculation the electromagnetic fields outside the electrodes, the averaging method over the time interval $2\pi/\omega = 1/f$ is used. The averaged values of electromagnetic force $< F_r(r, \phi) >$, $< F_\phi(r, \phi) >$ give rise to a liquid motion, which can be described by the stationary Navier-Stokes equation in the ring $r_0 < r < R$.

The 2D averaged magnetic field, source terms for the tempeature and Lorenz' forces, induced by alternating current with 3, 6 and 9 bar type electrodes are calculated in cross-section of cylinder by computer program MATLAB. With the finite difference method the distributions of magnetohydrodynamics flows and maximal temperature depending of the connections of electrods are obtained.

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MULTI-PORT PHASE LOCKING UNDER NOISY CALIBRATING OSCILLATOR

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In radio electronics synchronization of generators with the help of phase locking is widely used. In an ideal case, a signal from an standard generator without distortions and noises reaches a phase detector as a harmonious signal with the frequency ω_{st}. If ω_0 is the angular frequency of the generator tuned, then $\Omega_{in} = \omega_{st} - \omega_0$ is the initial disorder of the generators frequency. Let $K(P)$ - transfer function of filter in operator form, φ - instant phase difference of generators, $F(\varphi)$ - standardized characteristics of a phase detector, Ω_r - maximum possible disorder(retention band). Then phase locking system equation in operator form is: $P\varphi + \Omega_r K(P) + F(\varphi)K(P)\Omega_{in}$.

This equation is called main phase locking system equation in an operator form. If the passage band of filter is sufficiently wide, then $K(P) = 1$. In this case dynamics of phase locking system can be analyzed using the first order differential equation with the delay:

$$\frac{d\varphi}{dt} + a(F(\varphi(t-\tau) + \xi(t) - \gamma) = 0$$

where $xi(t)$ is noise process. The result is obtained that in an ideal phase locking system with small stationary phase perturbations of standard generator on the border of stability domain, with the time the condition sets in which is close to oscillations with the frequency φ_0.

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CONSTRUCTION OF CHAOTIC DYNAMICAL SYSTEM

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Dynamical systems occur in all branches of sciences, from the differential equations of classical mechanics in physics to the difference equations of mathematical economics and biology. The basic goal of the theory of dynamical systems is to understand the eventual or asymptotic behavior of an iterative process. A discrete dynamical system can be characterized as a function f that is composed with itself over and over again

$$x, f(x), f^2(x), \ldots, f^n(x), \ldots,$$

where $f^2(x) = f(f(x)$, $f^3(x) = f(f^2(x)) = f(f(f(x))$, etc. If we let $x_n = f^n(x)$, then we obtain the first-order difference equation $x_{n+1} = f(x_n)$.

Difference equations and discrete dynamical systems represent two sides of the same coin. Difference equations represent analytic theory of the subject but discrete dynamical systems represent its geometrical and topological aspects.

We know that mappings $h_4(x) = 4x(1-x)$ and $D(x) = 2x\,(mod\ 1)$ are chaotic in $[0;1]$ ([1], [4]). We consider the definition of R.Devaney for a chaotic mapping ([3]): a mapping $f : A \to A$ (A is a subset of a metric space) is said to be chaotic if the set of it periodic points is dense in A, it is transitive and f has a sensitive dependence on initial conditions. In [5] is shown that doubling mapping D is topological semi-conjugate with the shift map in one-sided infinite sequences space Σ_2. The shift map is chaotic too. Models with chaotic mappings are not predictable in long-term.

We have found family of chaotic mappings in space Σ_2 ([1]). We use the idea of conjugacy and so we can construct a family of mappings in the unit segment such that it is chaotic ([2]).

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ASYMPTOTIC METHODS FOR MARKOV DYNAMICAL SYSTEMS

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Averaging principle and diffusion approximation procedures are the most frequently used asymptotic methods for analysis of nonlinear dynamical systems subjected to permanent random perturbations (see, for example, [1] and references there). To employ these approaches to contemporary applied research of statistical nonlinear dynamics one should introduce a small positive parameter $\varepsilon \in (0, \varepsilon_0)$ and divide phase variables to slow $x^\varepsilon(t) \in \mathbf{X}$ with vector field $F(x^\varepsilon(t), y^\varepsilon(t), \xi^\varepsilon(t))$ and fast $y^\varepsilon(t) \in \mathbf{Y}$, which vector field is proportional to negative powers of ε, that is has a form $\varepsilon^{-1}G(y^\varepsilon(t)) + H(x^\varepsilon(t), y^\varepsilon(t), \xi^\varepsilon(t))$, where $\xi^\varepsilon(t)$ is fast oscillating random perturbations. We assume that $\xi^\varepsilon(t)$ is ergodic homogeneous Markov process with invariant measure $\mu(d\xi)$ on compact space $\mathbf{\Xi}$ defined by weak infinitesimal operator $\varepsilon^{-1}\mathbb{Q}$. According to [2] this mathematical model is called Markov dynamical systems defined as Markov process on $\mathbf{X} \times \mathbf{Y} \times \mathbf{\Xi}$ with weak infinitesimal operator $\mathbb{L}(\varepsilon)v(x,y,\xi) := (F(x,y,\xi), \nabla_x)v(x,y,\xi) + (H(x,y,\xi), \nabla_y)v(x,y,\xi) + \varepsilon^{-1}(G(y), \nabla_y)v(x,y,\xi) + \varepsilon^{-1}\mathbb{Q}v(x,y,\xi)$. The classical averaging principle suggests a good approximation $\bar{x}(t)$ of the slow motion $x^\varepsilon(t)$ on any finite time intervals can be obtained as a solution of averaged equation $\frac{d\bar{x}}{dt} = \bar{F}(\bar{x})$ where $\bar{F}(x) := \lim\limits_{t\to\infty} \frac{1}{t}\int_0^t \int_{\mathbf{\Xi}} F(x, \bar{y}(s), \xi)\mu(d\xi)ds$, and $\bar{y}(s)$ is solution of "truncated" equation $\frac{d\bar{y}}{dt} = G(\bar{y})$. If $\bar{F}(x) \equiv 0$ and above mentioned "truncated" equation for $y(t)$ has unique asymptotically stable bounded solution one can ([2], [3], [4]) proceed to "slow" time εt and to construct stochastic approximation of initial dynamical systems in a form of stochastic Ito differential equation. One can prove that above averaged and diffusion approximation equations may be successfully used not only for approximate analysis of initial system on finite time interval but also for Lyapunov stability analysis of motion $x^\varepsilon(t)$ [4]. The paper also proposes method and algorithm of stochastic stability analysis for more often application-oriented dynamical systems [1] with unbounded solutions of "truncated" equation. Our approach may be of use also to approximating stochastic modeling of commonly encountered in financial econometrics impulse type Markov dynamical systems defined by weak infinitesimal operator $L(\varepsilon)v(x,y,\xi) := (F(x,y,\xi), \nabla_x)v(x,y,\xi) + (H(x,y,\xi), \nabla_y)v(x,y,\xi) + \varepsilon^{-1}(G(y), \nabla_y)v(x,y,\xi) + \varepsilon^{-1}a(\xi)\int_{\mathbf{\Xi}}(v(x + \varepsilon f(x,y,\xi), y + \varepsilon h(x,y,\xi), \zeta) - v(x,y,\xi))p(\xi, d\zeta)$ where $p(\xi, d\zeta)$ is transition probability of embedded in the compound Poisson process $\xi^\varepsilon(t)$ Markov chain.

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ON STOCHASTIC VOLATILITY MODELING

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Many econometrical studies (see [1],[2] and references there) have documented that financial time series tend to be highly heteroskedastic. This has many implications for many areas of macroeconomics and finance, including the term structure of interest rates, option pricing and dynamic capital-asset pricing theory. In the same time econometricians have also been very active in developing models of conditional heteroskedasticity. The most widely used models of dynamic conditional variance have been the ARCH models first introduced by [1]. In most general form, a univariate ARCH model makes conditional variance at time t a function of exogenous and lagged endogenous variables, time, parameters and past residuals. According to Nelson [2] approach to stochastic modeling one should fit to sampling data with in discrete time GARCH(1,1) process $\sigma_{t+1}^2 = \omega_h + \sigma_t^2[\beta_h + h^{-1}\alpha_{hh}Z_t^2]$ for conditional variance $\sigma_t^2 := \mathbf{D}\left\{\frac{S_{t+1}-S_t}{S_t}\Big/{}_hZ_s, s \le t\right\}$ where h is small positive parameter, and $\{{}_hZ_t, t \in \mathbb{Z}\}$ are independent $\mathbf{N}(0,h)$ random variables. Under assumptions $1 - \alpha_h - \beta_h = h\theta + o(h), \omega_h = h\omega + o(h), \alpha_h = \frac{\sqrt{h}}{\sqrt{2}}\alpha + o(h)$ author of paper [2] derives continuous time approximation for conditional variance in a form of stochastic Ito differential equation

$$d\sigma_t^2 = (\omega - \theta\sigma_t^2)dt + \alpha\sigma_t^2 dw(t) \tag{1}$$

But sometimes as we will show analyzing real data, hypothesis on non-correlatedness of cumulative excess returns residuals may be rejected. Our paper discusses a possible correlation effect assuming that random process ${}_hZ_t$ as before is stationary $\mathbf{N}(0,h)$ but with correlation coefficient ρ. Applying the method and results of the paper [3] we have derived continuous time approximation for conditional variance in a form of diffusion process satisfying stochastic Ito differential equation

$$d\sigma_t^2 = \left(\omega + \left(\frac{\alpha^2\rho^2}{1-\rho^2} - \theta\right)\sigma_t^2\right)dt + \alpha\sqrt{\frac{1+\rho^2}{1-\rho^2}}\sigma_t^2 dw(t) \tag{2}$$

with coefficients dependent on correlation parameter ρ. Analyzing ergodic property of this equation we have shown that it is important to take into account possible serial correlation in conditional variance process. Our results are also illustrated and confirmed by statistic simulation.

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ASYMPTOTICAL METHODS FOR CONDITIONAL SECOND MOMENTS OF LINEAR MARKOV DIFFERENCE EQUATIONS

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The paper deals with n-dimension linear stochastic difference equation in $\mathbb{R}^n$ defined by equality

$$x_t = A(y_t)\, x_{t-1},\ t \in \mathbb{N} \tag{1}$$

where $\{y_t, t \in \mathbb{N}\}$ is a homogeneous ergodic Feller Markov chain with invariant measure $\mu(dy)$ and transition probability $p(y, dz)$ on the metric compact space $\mathbb{Y}$, and $\{A(y), y \in \mathbb{Y}\}$ is uniformly bounded continuous $n \times n$ matrix function. We will suppose that matrix $A(y)$ has a near to constant form

$$A(y, \varepsilon) := M + \sum_{k=1}^{l} \varepsilon^k A_k(y), \tag{2}$$

where ε is a small positive parameter and matrix M has spectrum in a following form: $\sigma(M) = \sigma_0(M) \cup \sigma_\gamma(M)$ divided into two parts $\sigma_0(M) \subset \{|\lambda| = 1\}$ and $\sigma_\gamma(M) \subset \{|\lambda| \le \gamma < 1\}$. This problem arises in financial econometrics at the stock dynamical analysis within infinite time interval (see, for example, [2]). The paper proposes a convenient for application asymptotic method of stochastic stability analysis for the equation (1) with near to constant coefficients (2). Applying the methods and results of [1] we produce an algorithm which allows to reduce the above problem to testing of positive definition property of a solution of the specially constructed matrix equation or on spectral properties of linear continuous operator $(\mathbb{A}(\varepsilon)q)(y, \varepsilon) := \int_Y A^T(z, \varepsilon) q(z) A(z, \varepsilon) p(y, dz)$ acting in the Banach space $\mathbf{V}$ of symmetric uniformly bounded continuous $n \times n$ matrix functions $\{q(y), y \in Y\}$ with norm $||q|| := \sup_{y \in Y, |x|=1} |(q(y)x, x)|$. Our results are based on existence [1] such positive numbers d and ε_0 that for any $\varepsilon \in (0, \varepsilon_0)$ equation (1) with matrix (2) is exponentially mean square stable if and only if the equation $(\mathbb{A}(\varepsilon)q)(y, \varepsilon) - q(y, \varepsilon) = -I$ has solution in a form of Laurent series $q(y, \varepsilon) = \sum_{k=-d}^{\infty} \varepsilon^k q_k(y)$, $d \ge 1$ with positive defined main part $\widehat{q}(y, \varepsilon) := \sum_{k=-d}^{0} \varepsilon^k q_k(y)$.

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SOLVABILITY OF CERTAIN SELF-SIMILAR BOUNDARY VALUE PROBLEM ARISING IN HYDRODYNAMICS

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The paper [1] considers an urgent problem for the slip flow model in hydrodynamics. Introducing the self–similar variables one from the governing boundary layer equations and associated boundary conditions obtains the boundary value problem for the Blazius equation

$$f''' + \frac{1}{2} f f'' = 0;$$

$$f(0) = 0, \quad f'(0) - \kappa f''(0) = 0, \quad f'(\infty) = 1,$$

where the parameter $\kappa \in [0, +\infty)$ depends on the slip length, and following boundary value problem for the second order homogenous linear differential equation

$$\Theta'' + \frac{Pr}{2}(\Theta' f - \Theta f') = 0;$$

$$\Theta'(0) = -1, \quad \Theta(\infty) = 0.$$

The qualitative analysis and numerical solution of these problems with respect to parameters are carried out.

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PARALLEL NUMERICAL ALGORITHMS FOR NONSTATIONARY DIFFUSION-REACTION EQUATIONS ON GRAPHS

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We consider a reaction-diffusion parabolic problem on branched structures. The Hodgkin–Huxley reaction–diffusion equations are formulated on each edge of the graph. The problems are coupled due some conjugation conditions at branch points. On the branched structure (E, P) we consider a system of parabolic linear problems for a set of functions $\{u^k(x,t)\}$:

$$\frac{\partial u^k}{\partial t} = \frac{\partial}{\partial x}\Big(d^k \frac{\partial u^k}{\partial x}\Big) - q^k u^k + f^k, \quad 0 < x < l_k, \;\; k = 1, \ldots, K, \tag{1}$$

$$0 < d_0 \le d^k(x,t) \le d_M, \quad q^k(x,t) \ge 0, \quad f^k = f^k(x,t).$$

Here function $u^k(x,t)$ is defined on the edge e_k and the properties of each neuron can be different. At the first type branch points $p_j \in P_1$ the axial current is conserved:

$$\sum_{e_k \in N^+(p_j)} d^k \frac{\partial u^k}{\partial x}\Big|_{x=l_k} = \sum_{e_m \in N^-(p_j)} d^m \frac{\partial u^m}{\partial x}\Big|_{x=0}, \quad \forall p_j \in P_1. \tag{2}$$

At the branch points of the second type (corresponding to the soma in neuron models) the current flowing through the point $p_s \in P_2$ is a sum of currents coming in and leaving out through the local edges:

$$c\frac{\partial u_s}{\partial t} + q_s u_s = \sum_{e_m \in N^-(p_s)} d^m \frac{\partial u^m}{\partial x}\Big|_{x=0} - \sum_{e_k \in N^+(p_s)} d^k \frac{\partial u^k}{\partial x}\Big|_{x=l_k} + f_s, \quad \forall p_s \in P_2. \tag{3}$$

In this paper we investigate efficient parallel numerical algorithms for solution of such problems We study three different types of finite-difference schemes. In order to decouple computations at each edge of the graph we consider two modifications of the implicit backward Euler scheme. In the predictor algorithm the values of the solution at branch points are computed by using the explicit approximation of the conservation equations. the predictor or predictor – corrector techniques. The stability analysis is done using the maximum principle method. In the predictor–corrector method in addition to the previous algorithm, the values of the solution at the branch points are recomputed by the implicit algorithm, when the discrete solution is obtained on each sub-domain. The stability of this algorithm is investigated only numerically.

Results of computational experiments are presented and the efficiency of the proposed parallel algorithms is investigated.

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ALTERNATING DIRECTION METHOD FOR THE TWO–DIMENSIONAL DIFFUSION EQUATION WITH NONLOCAL INTEGRAL CONDITION

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We consider the implicit alternating direction method for solving the folowing two-dimensional time-dependent diffusion equation:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + f(x,y,t), \qquad 0 \leqslant x,y \leqslant 1, \quad 0 < t < T$$

with initial condition

$$u(x,y,0) = \varphi(x,y),$$

and boundary conditions

$$\begin{aligned} &u(0,y,t) = \mu_1(y,t), \quad u(1,y,t) = \mu_2(y,t), \\ &u(x,1,t) = \mu_3(x,t), \quad u(x,0,t) = \mu_4(x)\mu(t), \end{aligned}$$

and the nonlocal boundary condition

$$\int_0^1 \int_0^1 u(x,y,t)dxdy = m(t),$$

where $u(x,y,t)$ and $\mu(t)$ are unknown functions.

We solve the system of one-dimensional difference equations by two different methods [1], [2].

The influence of the condition $\mu(0) = \mu(1) = 0$ is analysed.

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DESIGN AND ANALYSIS OF ODE MODELS WITH VARIABLE TIME DELAYS FOR TUMOUR DEVELOPMENT

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In this talk we present the main considerations related to design and basic stability and periodicity analysis of ODE and time delay ODE models for dynamics of systems consisting of solid tumours and their protein and vascular environments.

In [1] several ODE and time delay ODE systems have been defined encoding the most essential observations and assumptions about the development of systems enabling tumour development. An example of a model from this series is

$$\begin{cases} \dot{N} = f_1(E_{\tau_1})N \\ \dot{P} = f_2(E)N - \delta P \\ \dot{E} = f_3(P_{\tau_2})E - f_1(E_{\tau_1}))E. \end{cases} \tag{1}$$

In order to uncover the basic features of the models such as feedback loops the Hopf point analysis of the models has to be performed. The main conclusion of [1] is the existence of Hopf bifurcations with nonzero time delays. This conclusion may lead to practical results since Hopf bifurcation imply existence of small periodic oscillations around equilibrium states. Such periodic oscillations may be enforced by suitable therapies and/or surgeries.

The models given in [1] can be generalized allowing time delays to be dependent on state variables. We describe such models and perform analysis similar to that given in [1].

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DISCRETIZATION OF ELLIPTIC CONTROL PROBLEMS BY FINITE ELEMENTS

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We consider an optimal control problem for systems by elliptic partial differential equations, with state constraints and a *minimax* objective function. Using the finite element method theory, we discretize the problem and linearize the minimax control problem.
Let Ω be a bounded polygonal domain in $\mathbb{R}^k$ with a piecewise smooth boundary $\Gamma = \Gamma_1 \bigcup \Gamma_2 \bigcup \Gamma_3$.The state constrained boundary control problem is stated as

$$\begin{array}{lrcll} \min\limits_{u} \quad \max\limits_{\Omega} & y(x), & & & \\ s.t. & -\nabla \cdot (K \nabla y) + by & = & f & in\ \Omega, \\ & y & \geq & \psi & in\ \bar{\Omega}, \\ & y & = & u & on\ \Gamma_1, \\ & y & = & g & on\ \Gamma_2, \\ & \frac{\partial y}{\partial \nu} & = & q & on\ \Gamma_3, \end{array} \tag{1}$$

where f, g and q are given functions and u represents the boundary control on Γ_1. We discrete the problem (1) by finite element method.

THEOREM 2. *When k=2, let $\mathcal{T}_h$ be a valid triangulation of Ω. The discrete problem satisfies the discrete maximum principle for h small enough if there exists $\varepsilon > 0$ such that for all h, all the angles of the triangles of $\mathcal{T}_h$ are less than or equal to $\frac{1}{2}\pi - \varepsilon$.*

When the discrete maximum principle holds, a valid linear formulation of the discrete boundary control problem is

$$\begin{array}{ll} \min & s, \\ s.t. & \tilde{e}s - \tilde{\beta} \geq 0, \\ & A\beta + \tilde{A}\tilde{\beta} = F, \quad \beta \geq \Psi, \quad \tilde{\beta} \geq \tilde{\Psi}. \end{array} \tag{2}$$

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ON CONSTRUCTIONS OF SEQUENCES TO SOLUTIONS OF BVP

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Abstract. On approximation of solutions to the Dirichlet boundary value problems by non-monotone sequences of solutions

We consider the Dirichlet problem $x'' = f(t, x, x')$, $x(a) = A$, $x(b) = B$ under the assumption that there exist the upper and lower functions. We distinguish between two types of solutions, the first one, which can be approximated by monotone sequences of solutions (the so called Jackson - Schrader's solutions) and those solutions of the problem, which cannot be approximated by monotone sequences. We discuss the conditions under which it is possible to approximate the second type solutions of the Dirichlet problem.

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CONSTRAINED PHOTONS

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We can combine Maxwell's equations in vacuum with the evolution of an immaterial fluid (corresponding to the velocity vector field $\mathbf{V}$), by using the following set of model equations (see [2]):

$$\frac{\partial \mathbf{E}}{\partial t} \ = \ c^2 \mathrm{curl}\mathbf{B} \ - \ \rho \mathbf{V} \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} \ = \ - \ \mathrm{curl}\mathbf{E} \qquad\qquad \mathrm{div}\mathbf{B} \ = \ 0 \tag{2}$$

$$\frac{D\mathbf{V}}{Dt} \ = \ - \ \mu(\mathbf{E} + \mathbf{V} \times \mathbf{B}) \ - \ \frac{\nabla p}{\rho} \tag{3}$$

where $\mathbf{E}$ is the electric field and $\rho = \mathrm{div}\mathbf{E}$, $\mathbf{B}$ is the magnetic field, c is the speed of light, μ is a dimensional constant, and p is some kind of pressure.

We would like to show the results obtained in [1], corresponding to the numerical simulation of electromagnetic waves trapped in bounded 3-D regions of space. In the framework of fluid dynamics, these structures are perfectly similar to vortex rings, where, instead of a rotating fluid, we have an electromagnetic wave. The geometry of the regions may display a shape varying from the one of a standard annulus to that of a spherical Hill's type vortex (a toroid-shaped domain where the central hole is reduced to a segment).

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NUMERICAL MODELLING OF PEROXIDASE-BASED OPTICAL BIOSENSOR

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Biosensors are sensing devices made up of a combination of a biological entity, usually an enzyme, that recognizes a specific analyte and the transducer that translates the biorecognition event into an electrical signal [1]. Optical biosensors are based on the measurement of absorbed or emitted light resulting from a biochemical reaction. A mathematical model of a peroxidase-based optical biosensor has been recently developed [2].

In order to define the main governing parameters, the corresponding dimensionless mathematical model of the peroxidase-based optical biosensor has been derived. The modelling biosensor comprises two compartments, an enzyme layer and an outer diffusion layer. The governing equations for the enzyme layer in a dimensionless form are defined as follows:

$$\begin{gathered}\frac{\partial S}{\partial T}=\frac{\partial^2 S}{\partial X^2}-\alpha_2 CS \qquad \frac{\partial P}{\partial T}=\frac{D_P}{D_S}\frac{\partial^2 P}{\partial X^2}+\alpha_2 CS, \quad \frac{\partial H}{\partial T}=\frac{D_H}{D_S}\frac{\partial^2 H}{\partial X^2}-\alpha_1 EH, \\ \frac{\partial E}{\partial T}=-\alpha_1 EH+\alpha_2 CS, \quad \frac{\partial C}{\partial T}=\alpha_1 EH-\alpha_2 CS, \qquad 0<X<1,\ T>0,\end{gathered} \tag{1}$$

where S, P, H, E, C are the substrate, product, hydrogen peroxide, enzyme and compound I dimensionless concentrations in the enzyme layer, D_S, D_P, D_H are the diffusion coefficients, X and T stand for dimensionless space and time. The dimensionless parameters α_1 and α_2 are known as the diffusion modules or Damköhler numbers [3].

The influence of the dimensionless diffusion modules on the biosensor response and the sensitivity was investigated. A quasi-steady state model for the peroxidase-based optical biosensor was also developed and investigated for the ability to accurately predict the biosensor response. The digital simulation was carried out using the finite difference method.

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ON OSCILLATION OF N^{TH} ORDER ODE

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We consider positively homogeneous the N^{th} order differential equations of the type

$$x^{(n)} = h(t, x) \tag{1}$$

where h possesses the property that $h(t, cx) = ch(t, x)$ for $c \geq 0$. This class includes the linear equations $x^{(n)} = p(t)x$ and piece-wise linear ones $x^{(n)} = k_2x^+ - k_1x^-$. We use the definition.

Definition 3. *[I.T. Kiguradze, T.A. Chanturia, [2]]*
Let us call equation (1) like $[l, n-l]-$ *oscillatory equation in interval* I, if there exist $t_1, t_2 \in I$, $t_1 < t_2$, and nontrivial solution of (1) such that

$$\begin{aligned} x^{(i)}(t_1) &= 0, \quad i = 0, \ldots, l-1, \\ x^{(i)}(t_2) &= 0, \quad i = 0, \ldots, n-l-1. \end{aligned}$$

We describe the oscillatory behavior of positively homogeneous equations in terms of $(N-2, 2)-$oscillation in $[a, +\infty)$.

Properties of $(N-2, 2)-$solutions of positively homogeneous equations are useful for investigations of some nonlinear boundary value problems.

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EFFICIENT MULTIGRID METHOD ON HIERARCHICAL TRIANGULAR GRIDS FOR THE BIOT'S CONSOLIDATION PROBLEM

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The classical quasi-static Biot model for soil consolidation, describes the time dependent interaction between the deformation of an elastic porous material and the fluid flow inside of it. This model can be formulated as a system of partial differential equations for the unknowns displacement $\mathbf{u} = (u, v)$ and pressure p. Here, we consider the case of a homogeneous, isotropic and incompressible medium Ω so the governing equations are given by

$$-\mu\Delta\mathbf{u} - (\lambda+\mu)\text{grad}\,\text{div}\,\mathbf{u} + \text{grad}\,p = \mathbf{g}(\mathbf{x},t),$$
$$\frac{\partial}{\partial t}(\text{div}\,\mathbf{u}) - \frac{\kappa}{\eta}\Delta p = f(\mathbf{x},t), \quad \mathbf{x}\in\Omega,\ 0<t\leq T.$$

where λ and μ are the Lamé coefficients, κ is the permeability of the porous medium and η the viscosity of the fluid. A stabilized finite element scheme for the poroelasticity equations, based on the perturbation of the flow equation, was proposed in [1]. This allowed us using continuous piecewise linear approximation spaces for both displacements and pressure, obtaining solutions without oscillations independently of the chosen discretization parameters.

To approximate solutions of mathematical physics problems defined on irregular domains it is very common to apply regular refinement to an unstructured input grid. Assuming that the coarsest grid is rough enough in order to fit the geometry of the domain, a hierarchy of globally unstructured grids is generated. This kind of meshes are suitable for use with geometric multigrid and this method is implemented using stencil-based operations to remove the limitations on the size of the problem that can be solved by using this process of refinement. In this framework, we are interested in the design of an efficient geometric multigrid method on hierarchical triangular grids for a linear finite element discretization of the poroelasticity problem. To design this geometric multigrid method, a Local Fourier Analysis on triangular grids, which was introduced in [2], is necessary. This tool permits to choose the suitable components of the algorithm in order to obtain an efficient multigrid method.

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DEVELOPMENT OF EFFICIENT MULTIGRID FINITE ELEMENT METHODS ON SEMI-STRUCTURED TRIANGULAR GRIDS

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We are interested in the design of efficient geometric multigrid methods on hierarchical triangular grids for problems in two dimensions. Assuming that the coarsest grid is rough enough in order to fit the geometry of the domain, a hierarchy of globally unstructured grids is generated. This kind of meshes are suitable for use with geometric multigrid. To discretize problems with constants coefficients on these type of meshes, explicit assembly of the global stiffness matrix for the finite element method is not necessary and this can be implemented using stencils. As the stencil for each coarsest triangle is the same for all unknowns that are interior to it, one stencil suffices to represent the discrete operator reducing drastically the memory required. Fourier analysis is a well-known useful tool in multigrid for the prediction of two-grid convergence rates. With the help of the Fourier Analysis on triangular grids [1], we design efficient geometric multigrid methods for different problems on hierarchical triangular grids.

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MATHEMATICAL MODELLING OF 2D MAGNETOHYDRODYNAMICS FLOW IN THE RING BY EXTERNAL MAGNETIC FIELD

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The viscous electrically conducting incompressible liquid is located between two infinite coaxial cylinders (rings), surfaces of them with different angular velocity rotate can. The external magnetics fields (homogenous, radial or axial) are imposed. It is important to mix an electrically conducting liquid, using various magnetics fields [1; 2; 3]. We analyze the 2D viscous electrically conducting incompressible flow between two infinite coaxial cylinder by different type of the external magnetics fields and angular velocity. This process is considered with the so-called inductionless approximation.

Let the cylindrical domain $\{(r, \phi, z) : r_0 < r < R, 0 \leq \phi \leq 2\pi, -\infty < z < \infty\}$ contain viscous electrically conducting incompressible liquid, where r_0, R are the radiuses of the coaxial cylinders. The surfaces of these cylinders can correspondingly with angular velocities Ω_0, Ω_1 rotate. The external 2D magnetics fields are added in following form:
1) uniform homogenous magnetic field with the the radial $B_r(\phi) = B_0 \sin(\phi)$ and the azimuthal $B_\phi(\phi) = B_0 \cos(\phi)$ components (this field is parallel to Ox axis) [3],
2) radial magnetic field with the radial $B_r(r) = B_0/r$ component ($B_\phi = 0$),
3) axial magnetic field with the the azimuthal $B_\phi(r) = B_0/r$ component ($B_r = 0$).
Here B_0 is the scale of the induction for magnetic field. These components of the external magnetics fields creates the radial $F_r(r, \phi)$ and azimuthal $F_\phi(r, \phi)$ components of the Lorentz' force $\mathbf{F}$.
The axial component of the vector's **curl F** increases the liquid motion. The stationary 2D flow of incompressible viscous liquid in a cylinder is described by the system of the Navier - Stokes equations in the polar coordinates (r, ϕ).

The distribution of electromagnetic forces and the 2D magnetohydrodynamics flow induced by the external magnetics field in the cross-section of the rings are obtained by finite difference methods. The original method is used to calculate the circular matrix is used.

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FUZZY POS CATEGORY AND AGGREGATION OF FUZZY ORDER RELATIONS

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Since the first introduction of the concept of a fuzzy set by L. A. Zadeh and its generalization by J. A. Goguen [1] in the second half of the 20th century, fuzzy analogues of basic concepts of classical mathematics were introduced and investigated, in particularly fuzzy order relation [5]. In our work, we use the notion of a fuzzy order relation and construct a fuzzy POS category (L-POS) whose objects are fuzzy ordered sets and morphisms - "potential" order-preserving mappings (in the fuzzy sense); for the concept of fuzzy category see [4]. Actually, we involve L-fuzzy subclass of the class of morphisms as a mapping from the class of morphisms to GL-monoid L:

$$\mu : MOR(L\text{-}POS) \to L.$$

The intuitive meaning of the value $\mu(f)$ is the degree to which a morphism f is an order-preserving mapping.

We continue by constructing an aggregation process in the fuzzy POS category.
Let $n \geq 2$, $A : [0,1]^n \to [0,1]$ and let $R_1, R_2, ..., R_n$ be fuzzy relations ($R_i : X \times X \to L$).
An aggregation fuzzy relation R_A ($R_A : X \times X \to L$) is described by the formula

$$R_A(x,y) = A(R_1(x,y), ..., R_n(x,y)),\ x,y \in X.$$

We say an aggregation operator A preserves a given property of fuzzy relations if for all fuzzy relations $R_1, R_2, ..., R_n$ having this property, R_A also has this property. It is important to investigate which aggregation operators are able to preserve properties of fuzzy relations in aggregation process. There are many works devoted to this topic, see e.g. [3]. In our work we involve the degree to which aggregation operator preserves properties of fuzzy relations. In order to calculate how good the operator A preserves the properties of fuzzy relations $R_1, R_2, ..., R_n$ we use the mapping ξ: $\xi(A) = \inf\limits_i \mu_i$, where μ_i is the degree to which an aggregation operator A preserves the property of the corresponding fuzzy relation R_i. Further we investigate properties and behavior of the mapping ξ and illustrate the obtained theoretical results by the real world example.

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ON TIME MAP FORMULAE

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Let us consider a function $U(\alpha, \lambda)$ which has the following meaning: for given $\alpha > 0$ and $\lambda > 0$ $U(\alpha, \lambda)$ is the distance from $t = a$ to the first (after a) zero of a solution to the initial value problem

$$x'' + \lambda f(x) = 0, \quad x(a) = 0,\ x'(a) = \alpha, \tag{1}$$

in other words, the function $U(\alpha, \lambda)$ is the first zero function (time map) for a solution of the problem (1). We study the function U as a function of two arguments, looking for expressions of the first and second order partial derivatives and obtaining the consequences of them.

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THE IMPLEMENTATION OF SINGULAR SPECTRAL ANALYSIS IN DIGITAL SIGNAL PROCESSING (DSP)

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The whole set of digital signal processing procedures can be used in a very different and looking not related with each other practical applications - exact measurements, technical systems control, image processing, medical diagnostics, financial mathematics, economical forecasting [1]. One of such procedures is signal extraction from the mixture "signal plus noise".

There are many methods for signal extraction form mixture with noise and the common property of them is that the better we know structure of signal, the better we can separate it from the noise. For signals with unknown structure under conditions of unknown statistical properties of noise the task of signal extraction from mixture is very complicated.

Singular spectral analysis (SSA) is one of the best methods, which allows to separate signal from noise [2] under very poor knowledge about noise statistical properties. The base scheme of SSA consists of four steps. The first step is called the embedding step and converts one-dimensional time series into L - dimensional vectors $X_i = (f_{i-1}, \ldots, f_{i+L-2})^T$, $1 \leq i \leq K$. The second step is called the singular value decomposition and represents trajectory matrix as the sum or resultant matrices $\mathbf{X} = \mathbf{X}_1 + \cdots + \mathbf{X}_d$. The third step is called the grouping and distributes the eigenvalues $\lambda_1, \ldots, \lambda_L$ into m subsets $I_1, \ldots, I_m$. The fourth step is called as diagonal averaging and calculates new averaged values of resultant matrices for each $I_1, \ldots, I_m$, representing initial time series as sum of m series. Our results are illustrated by the set of MATLAB programmes for SSA implementation for different practical tasks, including industrial mathematics, space technologies and financial mathematics.

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DETERMINING TEMPERATURE CONDUCTION COEFFICIENT AS PER TEMPERATURE MEASUREMENTS AT VARIABLE BOUNDARY CONDITIONS

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Majority of methods for determining temperature conduction coefficient are based on temperature measurements at inner points of a specimen, by presuming that process of thermal conductivity is described by a mathematical model with constant boundary conditions of the first or the second type. Such presumption causes a model error as leap of delivered power that is actually provided at a starting moment does not cause any leap of either temperature or heat flow at borders of the specimen. Thus, it is topical to discuss methods for solution of inverse thermal conductivity problems which allow boundary conditions of the first or the second type variable in time.

Let us presume that a specimen wherein temperature is measured is a plate, and mathematical model of thermal conductivity is as follows:

$$\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2} \tag{1}$$

$$u(x,0) = 0, \quad u(0,t) = u(b,t) = f(t), \quad x \in [0,b]. \tag{2}$$

Let us presume that at inner point of the plate $x_1 \in (a,b)$ change of temperature is known in time $u(x_1,t)$. Problem: to find temperature conduction coefficient a^2.

As it follows from [1], solution of the problem (1), (2) is the following:

$$u(x,t) = \int_0^t f(\tau) G(x, t-\tau) d\tau, \tag{3}$$

where $G(x,t)$ - problem (1) Green's function.

Integral included in expression (3) may be put into generalized Taylor series [2], resulting in the following:

$$u(x,t) = \sum_{k=0}^{\infty} G^{(k)}(x,0) \cdot f^{(-k-1)}(t), \tag{4}$$

$$f^{-1}(t) = \int_0^t f(\tau) d\tau, \ldots, \ f^{(-k)}(t) = \int_0^T f^{(-k+1)}(\tau) d\tau.$$

Such projection into the series in applied also in the paper [3]. If measured temperature t_j in inner point of the specimen x_1, where moments of time are denotated with $j = 0, 1, 2, \ldots, N$, then we can write down

$$u(x_1, t) = \sum_{k=0}^{N} G^{(k)}(x_1, 0) \cdot f^{(-k-1)}(t_j), \quad j = 0, 1, 2, \ldots, N. \tag{5}$$

Expression (5) is a system of linear algebraic equations in connection with Green's function derivatives $G^{(k)}(x_1, 0)$. Its left side is known from measurement results, but integrals $f^{(-k-1)}(t_j)$ are easily calculable. By inserting (5) into equation (1), we obtain:

$$\sum_{k=0}^{\infty} \left(a^2 G^{(k+2)}(x_1, 0) \cdot f^{(-k-1)}(t_j) - G^{(k)}(x_1, 0) f^{(-k)}(t_j) \right) = 0,$$

where from it follows that

$$a^2 = \frac{G^{(k)}(x_1, 0) f^{(-k)(t_j)}}{G^{(k+2)}(x_1, 0) f^{(-k-1)}(t_j)}.$$

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THE FINITE-DIFFERENCE METHOD FOR THE SOLUTION OF PSEUDOPARABOLIC EQUATION WITH NONLOCAL INTEGRAL CONDITIONS

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In this work we consider the one-dimensional nonlinear pseudoparabolic equation with two nonlocal integral conditions and one initial condition [1]:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - \eta \frac{\partial^3 u}{\partial x^2 \partial t} + f(x,t,u), t \geqslant 0, 0 < x < 1, \tag{1}$$

$$\int_0^1 u(x,t)\mathrm{d}x = \mu_1(t), \int_0^1 xu(x)\mathrm{d}x = \mu_2(t), \tag{2}$$

$$u(x,0) = \varphi(x). \tag{3}$$

We investigate implicit difference scheme for the solution of this problem

$$\frac{u_i^{j+1} - u_i^j}{\tau} = \Lambda u_i^{j+1} - \eta \frac{\Lambda u_i^{j+1} - \Lambda u_i^j}{\tau} + f_i^j(u_i^j), \tag{4$_a$}$$

$$h\Big(\frac{u_0^{j+1} + u_N^{j+1}}{2} + \sum_{i=1}^{N-1} u_i^{j+1}\Big) = \mu_1^{j+1}, \qquad h\Big(\frac{u_N^{j+1}}{2} + \sum_{i=1}^{N-1} ihu_i^{j+1}\Big) = \mu_2^{j+1}, \tag{5}$$

where Λ is a differential operator $\Lambda u_i^{j+1} = \frac{u_{i-1}^{j+1} - 2u_i^{j+1} + u_{i+1}^{j+1}}{h^2}$.

For differential equation (1) we analyze two another difference analogues:

$$\frac{u_i^{j+1} - u_i^j}{\tau} = \Lambda u_i^{j+1} - \eta \frac{\Lambda u_i^{j+1} - \Lambda u_i^j}{\tau} + f_i^j(u_i^{j+1}), \tag{4$_b$}$$

$$\frac{u_i^{j+1} - u_i^j}{\tau} = \frac{\Lambda u_i^{j+1} + \Lambda u_i^j}{2} - \eta \frac{\Lambda u_i^{j+1} - \Lambda u_i^j}{\tau} + f_i^j(\frac{u_i^{j+1} - u_i^j}{2}). \tag{4$_c$}$$

The results of numerical solution are presented.

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MODELLING DYNAMICS OF AGGREGATE CONSUMPTION FOR LITHUANIAN ECONOMY

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Household consumption is the largest component of aggregate expenditure in most economies. In Lithuania it accounts for about 65% of spending. Therefore it is important for macroeconomists to be able to explain the determinants of consumer spending via a well-specified consumption function. Although literature on modelling consumption is large [2], in Lithuanian academic literature the analysis of household consumption, at macroeconomic level, is relatively scarce. Typically consumption is an integral part of a larger structural macroeconomic model [4],[5] and there is one recent publication devoted only for the modelling of consumption [3]. In the latter the authors use consumption as the error-correction type of model. Their results are quite close to the general ideas of Friedman's permanent income hypothesis. Modern theories of consumption are based on analysis of optimal consumption behaviour over time under constraint [1]. In equilibrium, a rational consumer chooses optimum levels of consumption in each period so as to maximize utility.

In this paper, we model household consumption from the perspective of the modern representative agent-based approaches. Household chooses a stochastic consumption plan to maximize the expected value of their time-additive nonlinear utility function subject to asset budget constraint. This multi-period problem can be solved by using the Bellman equation. The first order condition is the Euler equation which is typically estimated using the general method of moments. We employ numerical methods to compute equilibrium. Empirical analysis is conducted using quarterly Lithuanian data covering period from year 1995 to 2008.

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PARALLEL NUMERICAL ALGORITHMS FOR OPTIMIZATION OF ELECTRICAL CABLES

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In modern cars electrical and electronic equipment is of great importance. For engineers the main aim is to determine optimal conductor cross-sections in electro cable bundles in order to minimize the total weight of cables. A quantitative description of the thermo-electrical characteristics in the electrical cables can be obtained by using mathematical models based on parabolic partial differential equations. Let $T(X,t)$ describe a distribution of the temperature in electrical cables [1]:

$$\rho(X)c(X,T)\frac{\partial T}{\partial t} = \sum_{i=1}^{2}\frac{\partial}{\partial x_i}\Big(k(X)\frac{\partial T}{\partial x_i}\Big) + f(X,T), \quad (X,t) \in D \times (0,t_F], \tag{1}$$

Initial and boundary conditions:

$$T(X,0) = T_a, \quad X \in \bar{D} = D \cup \partial D, \tag{2}$$

$$k(X,T)\frac{\partial T}{\partial \eta} + \alpha_k(T)\big(T(X,t) - T_a\big) + \varepsilon\sigma\big(T^4 - T_a^4\big) = 0, \quad X \in \partial D. \tag{3}$$

We have proposed efficient parallel numerical algorithms for simulation of temperature distribution in electrical cables and have solved an inverse problem for fitting the diffusion coefficient of the air-isolation material mixture to the experimental data [2; 3]. The goal is to minimize the total mass of the metal in all wires under the constraints that the temperature of the wires remains bounded by some given maximal value. Since the formulated optimization problem is NP-hard, we restrict to an heuristic algorithm which is based on a greedy type search method. Parallel multilevel algorithms are developed using the master-slave and data decomposition paradigms [4]. Results of computational experiments are presented and the efficiency of the proposed parallel algorithms is investigated.

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UNIQUENESS RESULTS FOR SOME PARABOLIC INTEGRO-DIFFERENTIAL INVERSE PROBLEMS

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We present uniqueness results for two identification problems for parabolic integro-differential equations (PIDE) containing convolutions over time with kernels h. These results essentially use extremum principles.

The first problem is to determine an unknown x-dependent (i.e. space-dependent) factor of a source term by means of final over-determination of the solution of PIDE. We prove a positivity principle for PIDE with scalar kernels and use this principle to establish the uniqueness for the inverse problem. As a corollary, the uniqueness for inverse problems to determine unknown x-dependent coefficients of PIDE follows, too.

Secondly, we consider an inverse transmission problem for PIDE in an open domain Ω where the subdomains of continuity are Ω_1 and $\Omega_2 = \Omega \setminus \overline{\Omega}_1$ such that $\partial\Omega_2 = \partial\Omega_1 \cup \partial\Omega$ and $\mathrm{dist}\{\partial\Omega_1, \partial\Omega\} > 0$. The restrictions of h in the subdomains Ω_1 and Ω_2 are assumed to be scalar and denoted by h_1 and h_2, respectively. The inverse problem consists in determination of h_1 from measured flux in $\partial\Omega$ over the time. This problem is severely ill-posed, because it reduces to an equation with an infinitely smoothing operator. In case the problem is formulated in the infinite time interval $(0, \infty)$, we apply the Laplace transform and use the extremum principle and Giraud theorem for the resulting elliptic problem to prove the uniqueness for the inverse problem. The uniqueness in the finite time interval $(0, T)$ is an open problem in the moment of submission of this abstract, as it requires an extremum principle for PIDE with non-scalar h.

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THE EIGENVALUE PROBLEM FOR ONE-DIMENSIONAL DIFFERENTIAL OPERATOR WITH VARIABLE COEFFICIENT SUBJECT TO INTEGRAL CONDITIONS

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We consider the eigenvalue problem for one-dimensional differential operator with variable coefficient subject to nonlocal integral conditions

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big(p(x)\frac{\mathrm{d}u}{\mathrm{d}x}\Big) + \lambda u = 0, 0 < x < 1, \tag{1}$$

$$u(0) = \gamma_1 \int_0^1 u(x)\mathrm{d}x, \tag{2}$$

$$u(1) = \gamma_2 \int_0^1 u(x)\mathrm{d}x. \tag{3}$$

At the beginning we investigate theoretically how eigenvalues depend on the type of function $p(x)$ (symmetric, increasing or decreasing). After that, this problem is solved numerically. Also we analyze how the eigenvalues depend on the parameters γ_1, γ_2.

We prove some properties of the spectrum for this differential problem. We consider the cases in which there appears zero, negative and positive eigenvalues. The area of multiple or complex eigenvalues is analyzed. The results of numerical experiments are presented.

The case of constant coefficient in differential equation was analyzed earlier in the articles [1], [2].

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EFFECTIVE METHODS FOR THE SOLUTIONS OF DIFFUSION PROBLEMS IN MULTILAYERED 3D DOMAIN

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The averaging method were applied for the mathematical simulation of the mass transfer process in multilayered underground systems[1; 2]. In this talk we consider the metal concentration in the 2 layered peat block. A specific feature of these problems is that it is necessary to solve the 3-D boundary-value problems for elliptic type partial differential equations of second order with piece-wise diffusion coefficients in two layered domain.Therefore we develop here an effective finite-difference method for solving of a problem of the above type with periodical boundary condition in x direction. This procedure allows to reduce the 3-D problem to a 2-D problems.

The process of diffusion is consider in 3-D parallelepiped $\Omega = \{(x, y, z) : 0 \leq x \leq l, 0 \leq y \leq L, 0 \leq z \leq Z\}$. The stationary 3-D problem of the linear diffusion theory for multilayered piece-wise homogenous materials of N layers $\Omega_i = \{(x, y, z) : x \in (0, l), y \in (0, L), z \in (z_{i-1}, z_i)\}, i = \overline{1, N}, z_0 = 0, z_N = Z$ is described by following partial differential equations (PDE)

$$D_{ix}\partial^2 c_i/\partial x^2 + D_{iy}\partial^2 c_i/\partial y^2 + D_{iz}\partial^2 c_i/\partial z^2 + f_i(x, y, z) = 0.$$

Here D_{ix}, D_{iy}, D_{iz}, are constant diffusions coefficients, $c_i = c_i(x, y, z$ – the concentrations functions in every layer, $f_i(x, y, z)$ - the fixed sours function.
The 3D diffusion problem in layered domain are approximate with the 2D boundary value problem for a system of PDEs . This system is solved by the finite difference method.

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ON FULLY DISCRETE COLLOCATION METHODS FOR SOLVING WEAKLY SINGULAR INTEGRO-DIFFERENTIAL EQUATIONS

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A popular class of methods for solving initial or boundary value problems for weakly singular integro-differential equations of the form

$$u^{(n)}(t) = \sum_{i=0}^{n-1} a_i(t)u^{(i)}(t) + \sum_{i=0}^{n} \int_0^b K_i(t,s)u^{(i)}(s)ds + f(t)\,,\ 0 \le t \le b,$$

$$\sum_{j=0}^{n-1} \left[\alpha_{ij}u^{(j)}(0) + \beta_{ij}u^{(j)}(b)\right] = d_i\,, \quad i = 1,\dots,n\,,$$

is piecewise polynomial collocation method. In order to implement those methods one has to compute exactly certain integrals that determine the linear system to be solved. Unfortunately those integrals usually can not be computed exactly and even when analytic formulas exist, their straightforward application may cause unacceptable roundoff errors resulting in apparent instability of those methods in the case of highly nonuniform grids. Therefore it is very useful to consider fully discrete analogs of the collocation methods, where the system integrals are computed by using quadrature formulas.

We discretize the integrals of the form $\int_0^b K_i(t,s)u_m^{(i)}(s)\,ds$, where K_i are the kernels of the integral operators and u_m is the approximate solution corresponding to the collocation method, as follows. Namely, we introduce a nonuniform grid of the form

$$\Pi_1 = \{-b, -b\left(\tfrac{M_i-1}{M_i}\right)^{\rho_i}, \dots, -b\left(\tfrac{1}{M_i}\right)^{\rho_i}, 0, b\left(\tfrac{1}{M_i}\right)^{\rho_i}, \dots, b\left(\tfrac{M_i-1}{M_i}\right)^{\rho_i}, b\},$$

where $M_i > 0$, $\rho_i \ge 1$, and define a division of $[0,b]$ into subintervals by the points of the form $[0,b] \cap \{t - s\ :\ s \in \Pi_1\} \cup \Pi_0$, where Π_0 is the grid of the collocation method. In each of the subintervals we use the Gaussian quadrature formula with n_i, $2n_i \ge m + n - i$ knots, where $m-1$ is the order of polynomials in the collocation method. We show that the additional error (in the L^∞ norm) caused by this discretization of i-th integral operator is bounded by the quantity

$$c\begin{cases} M_i^{-\rho_i(1-\nu_i)}, & 1 \le \rho_i < \frac{2n_i-m-n+i+1}{1-\nu_i}, \\ M_i^{-(2n_i-m-n+i+1)}(\log(M_i)+1), & \rho_i = \frac{2n_i-m-n+i+1}{1-\nu_i}, \\ M_i^{-(2n_i-m-n+i+1)}, & \rho_i > \frac{2n_i-m-n+i+1}{1-\nu_i}, \end{cases}$$

where ν_i is a parameter characterizing the singularity of the kernel K_i on the diagonal $t = s$, and give choices for ρ_i and M_i in the case of various integro-differential equations so that the order of convergence of the original collocation methods for those equations is preserved.

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ELECTRICAL CONDUCTIVITY DETERMINATION OF METAL PLATES WITH PLANAR CIRCULAR COILS

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Planar circular coils are widely used in eddy-current nondestructive inspection of metal samples. There are several advantages of planar coils in comparison with conventional air-cored eddy current coils: (1) straightforward manufacture using printed-circuit-board technology; (2) high sensitivity to cracks; (3) possibility of inspection of complex geometries. In the present paper a theoretical model for determination of electrical conductivity of metal plates with planar circular coil is developed. Suppose that a planar circular coil carrying an alternating current is situated at a height $z = h$ above an isotropic electrically conducting plate of constant thickness d. The electrical conductivity of the plate is constant (but unknown). The relative magnetic permeability of the plate is equal to 1. In this case the system of Maxwell's equations can be written in terms of the vector potential A which has (due to axial symmetry) only one non-zero component. The corresponding system of linear partial differential equations for the components of the vector potential in free space and in the plate can be solved analytically by the method of Hankel transform.

Traditional approach in eddy current nondestructive testing is based on the evaluation of the change in impedance of the coil due to the presence of a conducting medium. The change in impedance of a planar circular coil due to the presence of the conducting plate is obtained in terms of improper integral containing Bessel and Struve functions. The final expression for the change in impedance is a nonlinear function of σ (the electrical conductivity of the plate). The value of σ can be estimated by minimizing the L_2 norm between experimentally obtained values of the change in impedance of the coil due to the presence of the conducting plate at different frequencies and theoretical values expressed in terms of improper integral. In addition, corrections for non-ideal coil behavior can also be applied in order to improve the accuracy of the obtained estimate. Software package "Mathematica" is used to solve the corresponding nonlinear optimization problem in order to estimate the value of σ. Reasonable agreement between the actual and predicted values of the electrical conductivity of the plate is found.

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A MATHEMATICAL FORMALIZATION OF MASLOW'S THEORY OF MOTIVATION

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A mathematical formalization of the well known theory of motivation proposed by Abraham Maslow can be given on a basis of the following equation:

$$M_i = C_i e^{\frac{D-A_{i-1}}{B_i-D}}, \tag{1}$$

M_i - partial motivation (generated by the i-th group of needs);
C_i - a constant;
D - total potential income;
A_i - threshold of satisfaction of the needs of the i-th level;
B_i - threshold of saturation of the needs of the i-th level. This value is equal to the total potential income that ensures the complete satisfaction of the needs of the i-th level so that they stop participating in this employee's motivation.

$$M = \Phi(M_1, M_2, \ldots, M_n),$$

M - total motivation;
n - total number of levels of needs. Different versions of the Maslow's model include either 5 or 7 levels. For economical reasons,

$$M_i = \begin{cases} C_i e^{\frac{D-A_{i-1}}{B_i-D}}, & A_{i-1} \le D \le B_{i-1}, \\ 0, & D < A_{i-1},\ D > B_i. \end{cases} \tag{2}$$

Using the modified Heavyside function Heav(x):

$$\mathrm{Heav}(x) = \begin{cases} 0, & x \le 0, \\ 1, & x > 0. \end{cases} \tag{3}$$

So the formula (1) will look as follows:

$$M_i = C_i e^{\frac{D-A_{i-1}}{B_i-D}} \mathrm{Heav}(D - A_{i-1})\mathrm{Heav}(B_i - D). \tag{4}$$

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NUMERICAL STUDY OF THE ROSENSWEIG INSTABILITY IN A MAGNETIC FLUID SUBJECT TO DIFFUSION OF MAGNETIC PARTICLES

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Taking into account the process of diffusion of ferromagnetic particles under the action of a nonuniform magnetic field is a distinctive feature of a contemporary mathematical modeling of hydrostatics problems for a magnetic fluid. The steady-state distribution of the volumetric particle concentration C in the fluid volume V is described by the equation $\nabla \cdot (\nabla C - \xi C L(\xi H)\nabla H) = 0$ with the Robin-type boundary condition $\partial C/\partial n - \xi L(\xi H)(\partial H/\partial n)C = 0$ and the condition of particle mass conservation $\int_V C dV = C_0 V$ where L is the Langevin function; H, the magnetic-field intensity; $\xi = \mu_0 m/kT$; μ_0, the magnetic constant; m, the magnetic moment of a particle; k, the Boltzmann constant; T, the fluid temperature; C_0, a constant corresponding to a uniform distribution of particles. The exact solution of the problem is given in [1] and is of the form $C = \varphi C_0 V \big/ \int_V \phi dV$, $\varphi = \sinh(\xi H)/(\xi H)$. The present study is devoted to the classical problem on stability of a horizontal semi-infinite layer of a magnetic fluid under the influence of gravity and a uniform magnetic field normal to the plane free surface [2]. It is well-known that development of small perturbations leads to the formation of a periodical peak-shaped structure on the surface when the magnetic-field intensity exceeds a critical value. The numerical modeling of the overcritical surface shapes is based on an axisymmetric statement. The coupled problem consists of three interconnected subproblems – on equilibrium shapes of a free surface, on a structure of magnetic field, on diffusion of magnetic particles. The generalized Young-Laplace parametric equations are governing equations in the first subproblem and the Maxwell equations formulated in terms of the potential of magnetic field, in the second. For the first the finite-difference scheme [3] and for the second, the finite-element method are used. The particle concentration C is computed by the formula above. Note that heretofore the problem has been solved in the uniform concentration approximation [4]. Numerical results show a significant influence of the particle diffusion on the overcritical shapes. Specifically, amplitude of peaks arising on the surface is appreciably higher and corresponds to experimental data better than in the case of the uniform concentration [4; 5].

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γ-AGOPS AND SOME ASPECTS OF GENERALIZED AGGREGATION PROBLEM

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We explore questions related to the aggregation problem. For the basic concepts of the aggregation theory a reader can refer e.g. to [1].
First we recall the definition of an aggregation operator (hereinafter - agop), and then we introduce a special class of agops, called γ- agops where $\gamma \in (0, 1]$:

DEFINITION 4. $A : \cup_{n\in N}[0, 1]^n \to [0, 1]$ is a γ-agop on the unit interval if the following conditions hold:
(A1) $A(0, ..., 0) = 0$, (A2) $A(1, ..., 1) = 1$
(A_γ) if $(\forall i = \overline{1, n}, \gamma \in (0, 1])$ $(\varphi_\gamma(x_i) \le \varphi_\gamma(y_i))$ then $A(x_1, ...x_n) \le A(y_1, ..., y_n)$, where
$\varphi_\gamma : [0, 1] \to \{0\} \cup [\gamma, 1]$, $\varphi_\gamma(x) = \begin{cases} 0, & \text{if } x < \gamma, \\ x, & \text{if } x \ge \gamma \end{cases}$

(A1), (A2) are known as boundary conditions of an aggregation operator and (A_γ) is a modification of the monotonicity property. Further we give examples of γ-agops and study properties of γ-agops. The second part of our talk is devoted to the generalization of the problem of aggregation: for such generalization we use the notion of pointwise extension introduced in [2]:

DEFINITION 5. Let $F(X)$, $\prec$ and A be correspondingly the set of fuzzy subsets of X, an order relation on $F(X)$ and an ordinary aggreagation operator on the unit interval.
$P_1, ..., P_n \in F(X)$, $\tilde{A} : \cup_{n\in N}F(X) \to F(X)$, then $\tilde{A}$ is a pointwise extension of A if the following holds:

$$\forall x \in X, \mu_{\tilde{A}(P_1,...,P_n)}(x) = A(\mu_{P_1}(x), ..., \mu_{P_n}(x).$$

We study pointwise extension of γ-agops w.r.t. different order relations. Some new results are obtained.

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TOWARDS HIERARCHICAL APPROACH FOR LARGE NONLINEAR MULTI-OBJECTIVE PROBLEMS

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Many problems in economics, business and engineering are of the multi-objective type and large scale. One of the approaches to solve multi-objective optimization problems is to convert them into a problem with single objective function. There are several ways of reducing a multicriterial problem into a monocriterial one. Frequently, multi-objective optimization problems are solved by the scalarization method (weighting method). The aim is then to find the Pareto point for the numerical values of target functions. However, as a rule, the best points of target functions are different and a certain compromise point must be found in the parameters space. The idea of weighting method is to associate each objective with a weighting factor and thereafter optimize the weighted sum of objectives.

The generation of either optimal or Pareto optimal points for large and complex systems can be easier if the problem can be decomposed and solved as a set of smaller coordinated subproblems which may be treated indipendently. Problems of high dimension, e.g those arising in electrical, mechanical and aeronautical engineering can sometimes only be solved by the exploitation of their special structure. In order to cope with large scale problems and to develop many optimum plans a multi-level (hiearchical) approach may be useful. The idea of hiearhical decision making is to reduce the overall complex problem into smaller and simpler (of lower dimension) subproblems which can be distributed over a large number of processors or computers. One way to do so is the use of decomposition-coordination schemes, designating processors (computers) as the master and slaves.

In this talk we present some results concerning the study of two most widely used decomposition-coordination approaches based on either the generation of feasible or non-feasible points respectively. In the latter only the values reached at the end of the procedure are assured to be feasible. For finding proper values of coordination parameters some rapidly convergent iterative methods are developed based on the classical cubically convergent method if the feasible approach is used [1]. For the non-feasible approach a few gradient-type methods are examined. Convergence properties and computational aspects of the methods under discussion are studied. Problems of their global convergence and polyalgorithmic strategy of their implementation are discussed as well.

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INTERPOLATION OF THE MARKOV CHAIN AND ESTIMATION

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The possibility of identifying nonlinear time series using nonparametric estimates of the conditional mean and conditional variance is studied. Most nonlinear models satisfy the assumptions needed to apply nonparametric asymptotic theory. Sampling variations of the conditional quantities are studied by simulation and explained by asymptotic arguments for the first-order nonlinear autoregressive processes. The paper deals with the identification and prediction problems of the autoregressive models of nonlinear time series. We will remind that assumption about normal distribution of time series allows to calculate the conditional expected value of phase variable as linear functional of its past values $\{x_t, s < t\}$. We should deal with the estimation of unknown function in nonlinear difference equation of the first order with usual kind of information about the distribution law. In many applied problems of time series regression analysis one can write

$$x_{n+1} = f(x_n) + \sigma_n \xi_{n+1}, \tag{1}$$

where ξ_n is (i.i.d.)-random error, with zero average and variance one. For searching for the function $f(x_n)$ we consider the model in which unknown function depends on the elements of Markov chain. We can write that the conditional expected value of random variable looks like

$$\mathbf{E}\{x_{n+1}|\mathcal{F}^n\} = \mathbf{E}\{x_{n+1}|x_n\} = \sum_y p(x_n, y)y = f(x_n), \tag{2}$$

that determines non-linearity of functional dependence x_{n+1} from x_n . The next step is to investigate the Markov chain built on the equation $u_{n+1} = h(u_n) + g(u_n)\xi_{n+1}$, which is closely connected with equation (1). So we need to express the functions $h(u_n)$ and $g^2(u_n)$ through the transition probabilities. For this purpose the main task is to find the transition probabilities of Markov chain on the basis of observed values of the time series.

If we denote $R(l)$ as the set of matrix of Markov chain's observations $L = ||l_{kj}||, (k, j = 1, ..., m)$ having property $\sum_{k,j=1}^{m} l_{kj} = l$ with the initial state U_k of the Markov chain, then the unbiased estimates of transition probabilities $P_{kj}^{(l)}$ from state U_k to state U_j for l steps are

$$\hat{P}_{kj}^{(l)} = \frac{\sum\limits_{L \in R(l)} P_{kj}^{(l)}(L) \cdot P_{kj_n}^{(n-l)}(N-L)}{P_{kj_n}^{(n)}}, N \in R(n), \tag{3}$$

where j_n - final state U_{jn} index.

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ADAPTIVE CONTROL OF ELECTRICAL SOLAR SAIL

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Research results of optimal control and identification algorithms of the dynamic systems in extreme conditions and limited prior information are discussed. The model of electrical solar sail is chosen as an object for research. The electrical solar sail consists of a net of the thinnest electric send-offs with a diameter of 10 microns, whose general length is more than 30 000 meters and complex of electronic equipment, providing the permanent functioning of this system in space. Features of structure and function scheme of the object control system are considered.

The dynamic pressure of the solar wind varies but is on average about 2 nPa at Earth distance from the Sun. This is about 5000 times weaker than the solar radiation pressure. Due to the very large effective area and very low weight per unit length of a thin metal wire, the electric sail is still efficient however [1, 2]. A 20 km long electric sail wire weighs only a few hundred grams and fits in a small reel, but when opened in space and connected to the spacecraft's electron gun, it can produce a one square kilometer effective solar wind sail area which is capable of extracting 1-2 mN force from the solar wind. For example, by equipping a small, 200 kg, spacecraft with 100 such wires, one may produce acceleration of about 1 mm/sec^2 . After acting for one year, this acceleration would produce a significant final speed of 30 km/sec. Small payloads could be moved quite fast in space using the electric sail, a Pluto fly could occur in less than five years, for example.

The dynamical characteristics of the electrical solar sail confirm that the closed-loop system of the one section wire is unstable. To design the optimal control law which guarantees the stability of the control system we use the algorithms of parametric identification to compute coefficients of matrices A and B. These coefficients are used to compute matrix coefficients of the optimal control system solving Riccati equation [3].

The dynamical characteristics (impulse response, Bode Diagram) of the electrical solar sail confirm that the closed-loop system of the electrical solar sail is stable when the algorithms of parametric identification are used in the control system. Therefore, the controlled system is stable [4].

Modelling results of time-optimal adaptive system with a linear-square regulator in a feedback loop and parametric identification algorithm confirm that the synthesized control system of electric solar sail is steady. The comparison of efficiency of the adaptive control system of electric solar sail with the similar systems using Pontryagin's maximum principle confirms its advantage.

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SEVERAL THEOREMS ON λ-SUMMABLE SERIES

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G. H. Hardy [1] proved: necessary and sufficient condition that the series

$$\sum_{k=0}^{\infty} a_k \tag{1}$$

should be summable $(C,1)$ to sum A is that

$$\xi_n + (n+1)b_{n+1} \to A, \tag{2}$$

where

$$\xi_n = \sum_{k=0}^{n} a_k \tag{3}$$

and

$$b_n = \sum_{k=n}^{\infty} \frac{a_k}{k+1}. \tag{4}$$

We consider here some problems of rapidity at Hardy's result.

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COMPARISON OF SIMULATION TOOLS FOR BACTERIA REGROWTH MODELING IN WATER DISTRIBUTION SYSTEMS

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Bacteria regrowth in water distribution networks is an issue that has been addressed by many authors. Significant efforts were made to develop a comprehensive model for simulating growth of bacteria in drinking water distribution networks. Although bacteria development is affected by many factors and it is virtually impossible to take them all into account, there were attempts to determine the most significant factors influencing bacteria growth. As a result, there are several models available that in one hand include significant factors and in other hand are not too bulky enabling quick calculations without high requirements of computing power. However additional efforts are needed to compare models and simulation tools and check reproducibility of results obtained with different models and various packages of simulation software. The goal of this research is to compare results obtained with the help of Fortran IMSL library and Epanet-MSX software. A mathematical model describing growth of bacteria in water distribution pipes is proposed in [1]. In the present paper the model is incorporated into Epanet MSX and Fortran code that includes IMSL library. Solutions have been obtained for a straight pipe and a simple network for various conditions and parameters. Epanet MSX does not take dispersion into account, so the dispersion term is excluded there. The results obtained with Fortran routine agree well with the results of Epanet-MSX software for a certain range of parameters. However, in some cases, significant difference between the results is observed. A conclusion can be drawn that the dispersion term may be insignificant in some cases, however, in some range of parameters it is to be taken into account.

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CENTRAL PART INTERPOLATION AND PRODUCT INTEGRATION METHOD FOR WEAKLY SINGULAR FREDHOLM INTEGRAL EQUATIONS

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In the first part of the talk the regularity of a solution to a linear Fredholm integral equation of the second kind is discussed. The kernel of the integral equation may have weak diagonal and boundary singularities. Assuming certain differentiability properties of the kernel and free term, we discuss the growth rates of the derivatives of the exact solution near the boundary of the domain of integration. In the second part we perform a change of variables which improves the boundary behaviour of the kernel and exact solution. After that, using a central part interpolation by polynomials on the uniform grid, we solve the transformed equation by the product integration method. Global convergence estimates are presented and a collection of numerical results is given.

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AGGREGATION OPERATORS AND T-NORM BASED OPERATIONS WITH L-FUZZY REAL NUMBERS

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Our work deals with a fuzzy analogue of a real number. In the literature on fuzzy mathematics one can find several different schemes for defining fuzzy numbers. We consider the notion originating from B.Hutton paper [1] and later developed by several authors.

Let $L = (L, \wedge, \vee)$ be a completely distributive lattice with lower and upper bounds $0_L, 1_L \in L$. An L-fuzzy real number is a function $z\colon \mathbb{R} \to L$ such that

(i) z is non-increasing;

(ii) $\bigwedge_x z(x) = 0_L, \bigvee_x z(x) = 1_L$;

(iii) z is left semi-continuous, i.e. $\bigwedge_{t<x} z(t) = z(x)$.

The set of all fuzzy real numbers is called the fuzzy real line and is denoted by $\mathbb{R}(L)$. The operations of L-fuzzy addition and L-fuzzy multiplication as they are defined in [2] are jointly continuous extensions of addition and multiplication from the real line $\mathbb{R}$ to the L-fuzzy real line $\mathbb{R}(L)$.

The aim of this talk is to present alternative definitions for arithmetic operations with L-fuzzy numbers which are based on a triangular norm (recall that a triangular norm, or a t-norm for short, is an associative, commutative binary operation on a lattice L which is non-decreasing in each argument and has the neutral element 1_L [3]). For this aim we use the t-norm extension $\tilde{A}$ of an aggregation operator A which is defined by the following formula [4]:

$$\tilde{A}(z_1, \ldots, z_n)(x) = \bigvee_{x=A(x_1,\ldots,x_n)} T(z_1(x_1), \ldots, z_n(x_n)),$$

where $z_1, \ldots, z_n \in \mathbb{R}(L)$, $x_1, \ldots, x_n \in \mathbb{R}$ and T is a t-norm. Basic algebraic properties of these arithmetic operations are discussed. Examples illustrating the role of a t-norm in the definition of operations are given. In particular we consider the cases of minimum, product and Lukasiewicz t-norms.

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DYNAMICAL MODELS OF ECONOMIC COMPETITION: A GEOMETRICAL APPROACH

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The laws of movement of dynamical systems are best derived as extremals of suitable functionals:

$$c(t) \longmapsto \int_0^1 \mathcal{L}(\dot{c}(t))\, dt. \tag{1}$$

satisfying the constraints on the configuration and variations spaces. Computing motion in this form is a classical problem in Geometric Mechanics defined with lagrangians $\mathcal{L}$ or hamiltonians $\mathcal{H}$. Models of the economic growth of a nation and neoclassical model of the economy have been treated in [1], [3] in such Geometric Approach. Different lagrangian functions $\mathcal{L}$ and hamiltonians $\mathcal{H}$ with different sets of constraints result in different laws of economy. Dynamic models of consumer behaviour, are also discussed in the literature [2] in this setting.

Dynamic models of competition and behaviour of players in the market, such as proposed in [4], should allow different players to develop with less risk their best suited strategies of adaptation to the situation on the market. As investing in the development, production, marketing and sales can be very costly, weaker players in the market can not survive and monopolies appear. But from customers' perspective, the situation of a monopoly, or almost monopoly, is bad and should be avoided.

We propose and discuss in a geometric approach, a model of competition on a market driven by a limited number of producers and distributors. We show that parameters of a proposed model have well established meaning and can be measured. Statistical data of chosen markets prove useful in validating our method. In geometric setting we are able to exhibit conservation laws, along the temporal evolution of the system, to better determine the initial dynamic system.

Computer simulations show the usefulness of the proposed method.

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PARTITIONING OF RIGHT HEART VENTRICLE MEDIAL AXIS

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The right heart ventricle has a complex and irregular shape. This obscures analysis of ventricle surface movements. We strive to quantitavely evaluate morphological changes of right ventricle surface. These changes appear during heart beat cycle, between different age groups, due to heart diseases, and other factors. Method for such evaluation would open new possibilities for insights into heart function, disease course extrapolation and other.

As a base for evaluation algorithm we chose medial axis transform of segmented right ventricle images. There are several unique advantages of using medial axis or skeleton to model geometric objects. First, it provides localisation of features such as anatomical landmarks (which are extremely valuable in bio-medical applications). Second, it separates thickness information (e.g., radius of medial axis or skeleton) from orientational and topological information, i.e., shape features can be subdivided into radial, orientational and location information in order to facilitate statistical analysis. Third, shape differences between objects can be quantified in a more intuitive and accurate way. Fourth, it is more expeditious to capture coarse-scale changes from the acquired models, making it more stable and robust to handle noisy datasets [1].

For a thorough analysis of ventricle surface and its medial axis we need to simplify medial axis. We strive to automatically partition it into two dimensional medial scaffolds which would allow simplified analysis and parametrisation of ventricle surface.

Our proposed algorithm first computes medial axis from sampled ventricle surface. For this we use algorithm proposed by Dey [2]. Computed medial axis is a subcomplex of Voronoi diagram. Second step is medial axis partitioning.

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INVESTIGATION OF SELECTION STRATEGIES IN BRANCH AND BOUND ALGORITHM WITH SIMPLICIAL PARTITIONS AND COMBINATION OF LIPSCHITZ BOUNDS

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One class of global optimization methods is Lipschitz optimization. A multivariate function $f : D \to R, D \subset R^n$ is said to be Lipschitz if it satisfies the condition

$$|f(x) - f(y)| \leq L \|x - y\|, \quad \forall x, y \in D,$$

where $L > 0$ is a constant called Lipschitz constant, the domain D is compact and $\|\cdot\|$ denotes the norm.

Branch and bound algorithms with rectangular or simplicial partitions and computationally cheap but rather crude lower bounds are used in most methods for multivariate unconstrained Lipschitz optimization of problems with dimension greater than two.

In the algorithm under consideration we use simplicial partitions and a combination of two types of Lipschitz bounds. The first type is improved Lipschitz bound with the first norm, where the graph of the upper bounding function is intersection of n-dimensional pyramids and its maximum is found solving a system of linear equations [3]. The other type is a combination of simple bounds with different norms [2].

The algorithm is implemented using a branch and bound template [1]. Specific rules of the algorithm are described while a common structure is implemented in the template.

Performance of the algorithm with different selection strategies (best first, depth first, breadth first, statistical) has been investigated experimentally solving multidimensional test problems for global optimization.

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ASYMPTOTIC ANALYSIS OF LINEAR IMPULSE DYNAMICAL SYSTEM WITH MARKOV JUMPS

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Let $\{y(t), t \geq 0\}$ be the right continuous homogeneous Markov process on some probability space $(\Omega, \mathcal{F}, \mathbf{P})$ with the infinitesimal operator $\mathcal{Q}$ defined by the formula

$$\mathcal{Q}v(y) = a(y) \int_Y (v(\xi) - v(y)) P(y, d\xi)$$

Y is a compact, $P(y, A)$ is a transition probability of a Markov chain with Feller's property. We assume that the spectrum of the operator $\mathcal{Q}$ has the form $\sigma(\mathcal{Q}) = \alpha\sigma_{-\rho} \cup \{0\}, \sigma_{-\rho} \subset \{z \in C : \mathbf{R}e \leq -\rho < 0\}$ and zero has the multiplicity one. The process $y(t)$ is ergodic and has the unique invariant probability measure $\mu \in C^*(Y)$. Let $\{\tau_j, j \in \mathbf{N}\}$ be switching moments of the above Markov process. Time periods between jumps are exponential distributed with intensity $a(y)$. We consider the system of

- the differential equation for the linear variable

$$\frac{dx(t)}{dt} = \varepsilon f_1(t, y(t), z(t))x(t) + \varepsilon^2 f_2(t, y(t), z(t))x(t), t \in (\tau_{j-1}, \tau_j), j \in \mathbf{N}$$

- the differential equation for the nonlinear variable

$$\frac{dz(t)}{dt} = \varepsilon h_1(t, y(t), z(t)), t \in [\tau_{j-1}, \tau_j), j \in \mathbf{N}$$

- and the jump equation for the linear variable

$$x(t) = x(t-) + \varepsilon g(t-, x(t-), y(t-), y(t), z(t)), t \in \{\tau_j, j \in \mathbf{N}\}$$

We have investigated the stability of this system with respect to the variable $x(t)$. After the transition to the fast time it was proved,that $\{x_\varepsilon(t), y_\varepsilon(t), z_\varepsilon(t)\}$ is the Markov process and derived its infinitesimal generator. Analyzing the infinitesimal operator of the above Markov process we construct a stochastic approximation in a form of the system of stochastic differential equations

$$\begin{aligned} dX &= B_{11}(Z)Xdt + B_{12}(Z)Xdw_{11}(t) + B_{13}(Z)Xdw_{12}(t) \\ dZ &= B_{21}(Z)dt + B_{22}(Z)dw_{21}(t) \end{aligned}$$

One can prove that processes $\{x_\varepsilon(t), z_\varepsilon(t)\}$ converge weakly as to the diffusion processes $\{X(t), Z(t)\}$. Under ergodic properties of $Z(t)$ one can calculate Lyapunov index for exponential stability analysis of $X(t)$. As the example the model of stochastic oscillator $\ddot{x} + \delta\dot{x} + (\omega^2 + h\cos(\nu t + y(t)) = 0$ was considered.

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THE INVESTIGATION OF THE COMPLEX SPECTRUM FOR THE ONE STATIONARY PROBLEM WITH TWO-POINT NONLOCAL BOUNDARY CONDITION

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Let us analyze the complex spectrum of the Sturm–Liouville problem with one classical boundary condition

$$-u'' = \lambda u, \qquad t \in (0,1), \tag{1}$$

$$u(0) = 0, \tag{2}$$

and other nonlocal second type boundary condition

$$u'(1) = \gamma u'(\xi), \tag{3}$$

$$u'(1) = \gamma u(\xi) \tag{4}$$

with the parameters $\gamma \in \mathbb{R}$ and $\xi \in [0,1]$. The real part of the spectrum is analyzed in [1; 2; 3].

In this study we investigate the behaviour of the complex eigenvalues when $\gamma \in \mathbb{R}$. We prove some properties of critical points in the complex plane. Critical points are important for investigation of multiple eigenvalues. Generalized eigenfunctions can exist for these points. We will present the modeling results.

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ON NUMERICAL REALIZATION OF APOSTERIORI REGULARIZATION PARAMETER CHOICE RULES IN (ITERATED) TIKHONOV METHOD

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We consider a linear problem $Au = f$, $f \in \mathcal{R}(A)$, where $A \in \mathcal{L}(H, F)$ is a continuous operator between infinite-dimensional Hilbert spaces H and F. We do not suppose that range $\mathcal{R}(A)$ is closed. For approximation of the solution u_* of this ill-posed problem we use the Tikhonov method $u_\alpha = (\alpha I + A^*A)^{-1}A^*f_\delta$ or the iterated Tikhonov method $u_\alpha = u_{\alpha,m}$, where $u_{i,\alpha} = (\alpha I + A^*A)^{-1}(\alpha u_{i-1,\alpha} + A^*f_\delta)$ $(i = 1, 2, \ldots, m)$. Here $\alpha > 0$, I is the identity operator and f_δ is a known approximation to exact data f with noise level δ: $\|f_\delta - f\| \le \delta$. To choose the regularization parameter $\alpha = \alpha(\delta)$ we consider several quasioptimal parameter choice rules (see [1]) in the form $\varphi(\alpha) = b\delta$, $b = \text{const}$, where $\varphi(\alpha) = \langle Au_{\alpha,m} - f_\delta, Au_{\alpha,m+1} - f_\delta\rangle^{1/2}$ in case of the modified discrepancy principle, $\varphi(\alpha) = \alpha^{-k+1/2}\langle A^*(Au_{\alpha,m+k} - f_\delta), Au_{\alpha,m+k} - f_\delta\rangle$, $k \in \mathbb{N}$ in case of Rule R1, $\varphi(\alpha) = \|Au_{\alpha,m+1} - f_\delta\|^{-1}\langle Au_{\alpha,m} - f_\delta, Au_{\alpha,m+1} - f_\delta\rangle$ in case of the monotone error rule and $\varphi(\alpha) = \dfrac{\kappa_\alpha\|A^*(Au_{\alpha,m+1} - f_\delta)\|^2}{\sqrt{\alpha}\langle AA^*(Au_{\alpha,m+1} - f_\delta), Au_{\alpha,m+2} - f_\delta\rangle^{1/2}}$, $\kappa_\alpha = (1 + \alpha\|A\|^{-2})^{1/2}$ in case of Rule R2 [2]. These rules require computing of additionally iterated approximations. We propose for the realization of these parameter choice rules alternative numerical schemes, using instead of additional iterations linear combinations of approximations with different parameters (see [3]). We report also numerical experiments with test problems from P. C. Hansen's "Regularization toolbox".

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CONVEX EXTENSION OF FUNCTIONS AND APPLICATIONS IN MATHEMATICAL PROGRAMMING

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The convex extension f_c of function $f : X \to R(X \subset R^n)$ is the majorant convex function $f_c : convX \Rightarrow R$ where $f_c(x) = f(x)$ if $x \in X$. The function $f : X \to R$, $X \subset R^n$, is called discrete-convex if for all $x_i \in X(i = 1, \ldots, n+1); \lambda_i \geq 0(i = 1, \ldots, n+1)$ and $\sum_{i=1}^{n+1} \lambda_i = 1$; $\sum_{i=1}^{n+1} \lambda_i x_i \in X$ holds $f(\sum_{i=1}^{n+1} \lambda_i x_i) \leq \sum_{i=1}^{n+1} \lambda_i f(x_i)$. The use of all n+1 elements convex combinations follows from the well-known theorem of Caratheodory. Important special case is $X = Z^n$, where $Z^n = \underbrace{Z \times \ldots \times Z}_{n}$ and Z={. . . , -1,0,1, . . . } The graph of a discrete-convex function is a part of the graph of a convex function.

Theorem 1
The function $f : X \to R$, $(X \subset R^n)$ can be extended to the convex function on $convX$ if f is a discrete -convex on X. The convex extension f_c of f is

$$f_c(x) = \min_{x_i, \lambda_i} \Big\{ \sum_{i=1}^{n+1} \lambda_i f(x_i) \mid x = \sum_{i=1}^{n+1} \lambda_i x_i; \lambda_i \geq 0(i = 1, \ldots, n+1), x_i \in X(i = 1, \ldots, n+1) \Big\} \quad (1)$$

Theorem 2
The convex extension f_c of f is

$$f_c(x) = \begin{cases} \max\limits_{a,b} \{\langle a, x\rangle + b \mid \langle a, y\rangle + b \leq f(y), y \in X\}, x \notin X \\ f(x), x \in X \end{cases} \quad (2)$$

The convex extension is so called point-wise maximum over all linear functions not exceeding the given function. The convex function of a discrete-convex function is a piecewise linear function. Each discrete-convex function has a unique convex extension. The class of discrete-convex functions is the largest one to be extended to the convex functions. A class of iteration methods of local searching for solving discrete mathematical programming problems is developed. On each step of the iteration the calculation of the value of objective function is required only on some vertices of some kind of unit cube.

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3D CENTER LINE EXTRACTION ALGORITHM FOR CURVILINEAR STRUCTURES

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The blood vessel skeleton can be used as a solution for variety of problems. Those are: blood vessel description, abnormalities detection, branching point identification or data reduction. Skeletonization is an iterative process frequently used to reduce the pictorial content of an object.

The proposed algorithm is expansion of C. Steger work on center line extraction to 3 dimensions [1]. The main idea of algorithm is to approximate the blood vessel region locally by two dimensional second order polynomial [2]. This polynomial is placed in a plane orthogonal to the direction of blood vessel. This direction is determined by constructing Hessian matrix for each image point, and choosing the smallest eigenvalue of it. Points, where the polynomial reaches maximal value are determined. This way we obtain the sub-pixel precise skeleton of blood vessel.

The presented algorithm was applied on MRI angiography images with head blood vessels and with prepared test images, results will be presented.

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FRACTIONAL LANGEVIN MODELS FOR HUMAN MOTION TRACKING IN RECURSIVE BAYESIAN ESTIMATION ALGORITHMS

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The source motion for the human tracking task for a single coordinate component x can be modelled as Langevin process [1]:

$$\frac{d^2x}{dt^2} + \beta_x \frac{dx}{dt} = F_x(t), \tag{1}$$

where β_x is a rate constant and $F_x(t)$ is so-called thermal excitation process. This corresponds to the discrete process with discretization step ΔT in state-space form:

$$\begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} = \begin{bmatrix} 1 & \Delta T \\ 0 & a_x \end{bmatrix} \begin{bmatrix} x_{k-1} \\ \dot{x}_{k-1} \end{bmatrix} + \begin{bmatrix} 0 \\ b_x \end{bmatrix} F_{x_k}. \tag{2}$$

Here $a_x = \exp(-\beta_x \Delta T)$, $b_x = \bar{v_x}\sqrt{1-a_x^2}$, $\bar{v_x}$ - is the steady-state RMS velocity and F_{x_k} being drawn from $\mathcal{N}(0,1)$, where a standard model of source dynamics [4] is obtained for $\beta = 0$. Here (1) can be seen as a special case of generalized Langevin equation with a special choice on retarded effect of frictional force [2]. In this work we propose to extend the source dynamics model [4] with the fractional versions of Langevin equation [3]. We report on construction of approximations for Bayesian Recursive Estimators, such as Kalman Filters, for developed fractional-order models and compare their performance to some conventional adaptive and non-adaptive filtering schemes based on integral-order models for human motion tracking. Additionally, the performance with respect to different approximation schemes of fractional-order operators is analyzed.

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GREEN'S FUNCTIONS FOR PROBLEMS WITH NONLOCAL BOUNDARY CONDITIONS

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We analyze Green's functions for the stationary differential problem

$$Lu := -(p(x)u')' + q(x)u = f(x) \tag{1}$$

where $p(x) \geq p_0 > 0$, $p \in C^1[0,1]$, $q \in C[0,1]$, with nonlocal boundary conditions

$$\langle l_0, u\rangle = \gamma_0 \langle k_0, u\rangle, \tag{2}$$

$$\langle l_1, u\rangle = \gamma_1 \langle k_1, u\rangle, \tag{3}$$

where $\langle l_i, u\rangle := \langle l_i(x), u(x)\rangle$ is the classical part and $\langle k_i, u\rangle := \langle k_i(x), u(x)\rangle$, $i = 0, 1$, is the nonlocal part of these conditions.

We find Green's functions for problems with various types of conditions, using the general formula that we have obtained for problem (1)–(3). For example, the classical part of conditions can be

$$\langle l_i, u\rangle = \alpha_i u'(i) + \beta_i u(i), \quad \text{where } |\alpha_i| + |\beta_i| > 0,\ i = 0, 1,$$

and the nonlocal part can be

$$\langle k_i, u\rangle = \sum_{j=1}^{N} (\delta_{ij} u'(\xi_{ij}) + \gamma_{ij} u(\xi_{ij})) \quad \text{or} \quad \langle k_i, u\rangle = \gamma_i \int_0^1 \rho_i(t) u(t)\, dt,$$

where $\rho_i(t)$, $i = 0, 1$, are weight functions.

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AEROELASTIC CALCULATION FOR PRELIMINARY MANOEUVRE OF ULTRA LIGHT PLANE

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The expansion of computer technologies allows using numerical simulation in the early stages of aircraft design more and more often. The role of both wind tunnels and initial test flights used to verify and validate the solutions seems to be diminishing. Big systems for three-dimensional simulations of Fluid-Structure Interactions (FSI) constitute highly specialized and costly software. Most of the codes are based on many simplifications.

In this paper fluid-structure interaction, taking into account the preliminary manoeuvre of ultra light plane, is concerned. This phenomenon has important influence in many aeronautical applications. The method and developed system is demonstrated on ultra light I23 plane. For the first flow the comparison with experiment made in Institute of Aviation Warsaw is presented. Finally, aeroelastic simulation of full I23 aircraft configuration presents the capability of used numerical codes to analyze large-scale complex geometries for preliminary manoeuvre. All computations were carried out in parallel environment for CFD mesh of order of millions tetrahedral elements.

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ON INTEGRATION OVER FUZZY SET WITH RESPECT TO FUZZY VALUED MEASURE

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There are many works where measures of fuzzy sets and integrals over fuzzy sets were studied. A detailed survey of the most important concepts and results in this direction can be found in [2],[3]. As different from this approach our interest is to develop a theory of measure and integral where not only sets are fuzzy, but also measure and integral take fuzzy real values in the sense of [4].

The aim of the present talk is to define and investigate fuzzy valued integral $\int\limits_E f \, d\mu$ of a function $f : X \to \mathbb{R}$ over a fuzzy set $E \in [0,1]^X$ with respect to a fuzzy valued measure μ. In order to achieve this purpose we introduce the concept of a fuzzy valued measure (first defined in [1] and developed in our talk at the conference FSTA - "Fuzzy Sets: Theory and Applications" in 2008 in Slovakia). We assume that f is a measurable function with respect to a finite measure ν defined on a σ-algebra $\Phi \subset 2^X$ of crisp sets and then we extend measure ν to a fuzzy valued measure μ defined on a T-tribe $\Sigma \subset [0,1]^X$ of fuzzy sets in the case when operations for fuzzy sets and fuzzy real numbers are based on a t-norm T [5]. By analogy with classical case we define an integral stepwise, first considering the case of characteristic functions, then extending it for simple non-negative measurable functions and finally for non-negative measurable functions. The basic properties of fuzzy valued integral are proved.

By using integral we define a fuzzy norm of functions on a fuzzy set E and consider functional space $\mathcal{L}(E)$. That gives us an opportunity to investigate on a fuzzy set E the error of approximation of $f \in \mathcal{L}(E)$.

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THE COMPARATIVE RESEARCHES OF HUMAN FACES GEOMETRY AND SKIN ROUGHNESS WITH MODAL ANALYSIS

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To increase the safety of the security systems (e.g. on airports) the new biometrics techniques are developed. One of the important and rapidly advancing areas of security identification and verification is face recognition [1, 2]. All new developed systems must be protected on any manipulations [3] and attempts of fake.
This article presents application of the PCA (Principal Component Analysis) method for comparative of geometrical similarity of 3D human faces. The discussed method is using the geometrical measurement (three-dimensional coordinates of points) of twin's faces as a data input. The points clouds are obtained from the structural light 3D scanner (3D biometric measurements) and used for testing level of sensitivity and accuracy.
The authors apply 3D version of PCA method for comparative the geometry of twin's faces. This numerical analysis is giving information about similarity and differences of analyzed faces. PCA decompose set of 3D objects into mean face and individual features (empirical modes), which describing deviations from mean value. Obtained mean shape describe the similarity of faces, eigenmodes present geometrical differences between faces. Eigenvalues can be used for numerical (mathematical) comparison of study faces.
In presented paper three sets of data of different type of twins (identical - monozygotic, fraternal-dizygotic) and thirteen typical faces are used and compared. The mean face and the features (eigenfaces) are presented and discussed.
Authors propose using the set of eigenfaces and corresponding coefficient values (computed from PCA) for security verification. As an example of authorization key the set of coefficient values for the faces are presented. Each key describes individual shape of face and can be decoded and compared with the original data of user to obtain access to restricted area or data.
Furthermore the high precision 3D data (face geometry) can be used for the mathematical analysis of the skin surface structure. That kind of data is important for topometric tests and investigations of new products from cosmetic industry [4].

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DAMPING OF CUTTING TOOL'S VIBRATIONS IN TRACE TURNING

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Uncontrolled vibrations occurring during metal working process considerably diminish the surface purity of detail and may cause breaking the cutter. The necessity to inquire into the origin of those vibrations initiated a number of investigations studying the influence of different factors upon the occurrence as well as their nature. Experimental analysis and simulation proved that autovibrating process in metal cutting could be satisfactory described only with regard to the mechanism of "track regeneration", i.e., the influence of unevenness formed during the previous turn of detail upon the cutter movement. Increasing stability domain is a matter of great importance for metal working on turner's bench in trace turning. Investigation of the model of cutting process provides a possibility to substantiate the phenomenon that fast oscillating stochastic perturbations of parameters of dynamical system increase stability domain. This result was formerly observed in practice. We provide the mathematical proof of this fact by means of asymptotical methods of stability investigation of stochastic dynamic model based on averaging principle.

Suppose the turning process on a turner's bench. The detail under processing moves with linear velocity v. Let us denote rigidity of cutter fastening c, energy dispersion proportional to the velocity of cutter vibrations with coefficient α, cutter mass m, the force P_n of normal pressure of shavings upon the front side of the cutter, the force P_f of friction, d_0 the desired depth of cutting, τ the time of one turn of spindle. Suppose $x(t)$ to be the deviation of the cutter from the undisturbed cutting position at the time moment t. The actual cutting depth declines from desired value as $d(t) = d_0+x(t)-x(t-\tau)$. According to experimental data force P_n is proportional to the thickness of cut off layer $P_n = qd(t)$. Considering the velocity of shavings v_0 with regard to cutter and the cutter velocity itself $\dot{x}$ we obtain $P_f = -f(v_0+\dot{x})P_n$ where f is a friction coefficient. Hence we have an equation of cutter movement $m\ddot{x} = P_f + P_\alpha + P_c$ or $m\ddot{x}(t) = -f(v_0+\dot{x}(t))q(d_0+x(t)-x(t-\tau))-\alpha\dot{x}(t)-cx(t)$. Denoting $z(t) = x(t) - \bar{x}$ the deviation from statical equilibrium position $\bar{x} = c^{-1}f(v_0)qd_0$ we have functional differential equation for cutter declination from equilibrium

$$\ddot{z}(t) + \gamma_1\dot{z}(t) + a_1z(t) + b_1(z(t) - z(t-\tau)) = l_1F(\dot{z}(t))(d_0 + z(t) - z(t-\tau)) \tag{1}$$

where $\gamma_1 = \frac{\alpha}{m}$, $a_1 = \frac{c}{m}$, $b_1 = \frac{f(v_0)q}{m}$, $l_1 = -\frac{q}{m}$, $F(\dot{z}) = f(v_0 + \dot{z}) - f(v_0)$. We propose the following algorithm of stability investigation of equation (1). Initially by means of D-partitioning of parameters' a_1 and b_1 plane we analyze stability domain of linear equation. Then with the rest of parameters chosen from the stability domain of linear equation we find restrictions on function F providing global asymptotical stability of trivial solution of equation (1). Currently let us suppose the time of one turn of spindle be a time function $\tau(t)$. It may be periodic or stochastic function. For example: $\tau(t) = \Delta(1 + \varepsilon\cos t)$ or $\tau(t) = \Delta(1 + \varepsilon\xi(t))$, where $\xi(t)$ is an ergodic Markov process with two states 0 and 1. It is proven that under certain conditions on parameters of the model this variable rotation speed may act as a damper of cutter vibrations and increase the stability domain.

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MULTIPLICITY IN PARAMETER-DEPENDENT PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

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We bring together various results of the author and collaborators concerning multiple solutions of two-point boundary value problems for ordinary differential equations.

First, equations of the type

$$x'' + \lambda f(x) = 0 \tag{1}$$

are considered together with the boundary conditions

$$x(0) = 0, \quad x(1) = 0. \tag{2}$$

Phase plane analysis and results on behavior of the time-map function for (1) are combined in order to explain the existence of multiple solutions. Emphasis is maid on positive solutions of the problem (1), (2). Comparison with the existing results is made.

Second, two-parameter equations of the form

$$x'' + \lambda f(x^+) - \mu g(x^-) = 0 \tag{3}$$

are considered, where $x^+ = \max\{x, 0\}$, $x^- = \max\{-x, 0\}$, together with the boundary conditions (2). These problems are studied in two directions. First, solution curves (λ, μ, x) are considered for sign-changing solutions $x(t)$. Then a links with Fučìk type problems are concerned and relations to Fučìk type spectra is established.

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ON THE EIGENVALUE PROBLEMS FOR STATIONARY DIFFERENTIAL AND DIFFERENCE OPERATORS WITH COUPLED BOUNDARY CONDITIONS

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We consider the eigenvalue problem for one-dimensional differential operator with given nonlocal coupled boundary conditions,

$$\frac{\mathrm{d}^2u}{\mathrm{d}x^2} + \lambda u = 0, \quad 0 < x < 1, \tag{1}$$

$$u(0) = \gamma_0 u(1), \tag{2}$$

$$\frac{\mathrm{d}u}{\mathrm{d}x}\Big|_{x=0} = \gamma_1 \frac{\mathrm{d}u}{\mathrm{d}x}\Big|_{x=1}, \tag{3}$$

and its finite-difference counterpart,

$$\frac{U_{j-1} - 2U_j + U_{j+1}}{h^2} + \lambda U_j = 0, \quad j = 1, 2, \ldots, N-1, \quad hN = 1, \tag{4}$$

$$U_0 = \gamma_0 U_N, \tag{5}$$

$$\frac{U_1 - U_0}{h} = \gamma_1 \frac{U_N - U_{N-1}}{h}, \tag{6}$$

where $\gamma_0, \gamma_1 \in \mathbb{R}$, $\gamma_0 + \gamma_1 \neq 0$. Such problem with $\gamma_0 = 0$ and $\gamma_1 = \beta$, $|\beta| > 1$, in finite-difference aspect was investigated by A. V. Gulin et al. (see [1] and references therein). We also briefly consider the similar two-dimensional differential and finite-difference problems.

The main aim of our work is to investigate the dependence of the qualitative structure of the spectrum of problems (1)–(3) and (4)–(6) on the parameters γ_0, γ_1, i.e., to formulate conditions of existence of zero, positive, negative or complex eigenvalues. We use technique, which is used, for example, in papers [2; 3] to investigate similar problems with other types of nonlocal conditions.

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NUMERICAL ANALYSIS OF MICROSTRUCTURED OPTICAL FIBERS BY THE R-FUNCTIONS METHOD

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Microstructured optical fibers (MOFs) also called a photonic crystal fibers or holey fibers are single material optical fibers with a periodic array of air holes running along their entire length [1]. MOFs have recently generated much interest thanks to the new ways provided to control and guide light. There are many research efforts have been devoted to understanding the propagation characteristics of MOFs based on different numerical methods.

This paper devoted to numerical analysis of wave propagation in holey fibers by the R-functions method (RFM) [2]. The basic feature of the RFM is construction of normalized boundary domain equations for geometric objects of complicated forms [3].

Mathematical model of the investigated processes is boundary value problem for Helmholtz equation. According to the RFM the solution of the problem constructed in analytical form by the sheaf of functions called general structure of solutions (GSS), satisfying exactly the prescribed boundary conditions. Numerical realization of elaborated method and analysis of obtained results were conducted.

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ON SOME PROBLEM WITH NONLOCAL INTEGRAL CONDITION

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We consider the Fučík equation

$$-x'' = \mu x^+ - \lambda x^-, \quad x^\pm = \max\{\pm x, 0\} \tag{1}$$

together with nonlocal boundary conditions

$$x(0) = 0, \;\; x(1) = a \int_0^1 x(s)ds, \;\; a \in \mathbb{R}. \tag{2}$$

We study the structure of the spectrum for various values of a.
By the spectrum we mean the set

$$\{(\mu, \lambda) \in \mathbb{R}^2 : \text{ the problem (1), (2) has a nontrivial solution}\}.$$

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COMPARISON OF SPEEDS OF CONVERGENCE IN CERTAIN FAMILIES OF FUNCTIONAL SUMMABILITY METHODS

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Speeds of convergence in certain family of summability methods for functions are compared in the talk. The results introduced here extend the results proved in [1] for "matrix case" to "integral case" and are partly published in [2].

1. Let us denote by X the set of the functions $x = x(u)$ defined for $u \geq 0$, bounded and measurable by Lebesgue on every finite interval $[0, u_0]$. Suppose that A is a transformation of functions $x = x(u)$ (or, in particular, of sequences $x = (x_n)$) into functions $Ax = y = y(u) \in X$. If the limit $\lim_{u\to\infty} y(u) = s$ exists then we say that $x = x(u)$ is convergent to s with respect to the summability method A, and write $x(u) \to s(A)$.

One of the basic notions in our talk is the notion of speed of convergence. Let $\lambda = \lambda(u)$ be a positive function from X such that $\lambda(u) \to \infty$ as $u \to \infty$. We say that a function $x = x(u)$ is convergent to s with speed λ if the finite limit $\lim_{u\to\infty} \lambda(u)\,[x(u)-s]$ exists. In order to characterize the speed of convergence of x also the estimation $\lambda(u)\,[x(u) - s] = O(1)$ as $u \to \infty$, is used. We say that x is convergent with speed λ with respect to the summability method A if the function $Ax = y = y(u) \in X$ is convergent with speed λ.

2. We discuss a Riesz-type family $\{A_\alpha\}$ of summability methods A_α, where $\alpha > \alpha_0$ and α_0 is some fixed number, and which transform functions $x = x(u)$ into functions $A_\alpha x = y_\alpha(u)$. This family is defined with the help of relation $A_\beta = C_{\gamma,\beta} \circ A_\gamma \quad (\beta > \gamma > \alpha_0)$, where $C_{\gamma,\beta}$ is certain integral transformation (see e.g. [2]). For example, the Riesz methods (R, α) and certain generalized Nörlund methods $(N, p_\alpha(u), q(u))$ form Riesz-type families. A Riesz-type family has the monotony property: $x(u) \to s\,(A_\gamma) \Longrightarrow x(u) \to s\,(A_\beta)$ for any $\beta > \gamma > \alpha_0$.

It is important to be able to compare the speed of convergence of $x = x(u)$ with respect to different methods A_γ and A_β if $\beta > \gamma$ and choose a method A_α close to optimal for converging x. For a given speed $\lambda = \lambda(u)$ and a fixed number $\gamma > \alpha_0$ the speeds $\lambda_\beta = \lambda_\beta(u)$ and $\lambda_\delta = \lambda_\delta(u)$ $(\beta > \delta > \gamma)$ can be found (see [2]) such that for all $\beta > \delta > \gamma$ the following implications are true:

$$\lambda(u)\,[y_\gamma(u) - s] \to t \Longrightarrow \lambda_\beta(u)\,[y_\beta(u) - s] \to t,$$

$$\lambda(u)\,[y_\gamma(u) - s] = O(1) \Longrightarrow \lambda_\beta(u)\,[y_\beta(u) - s] = O(1),$$

$$\lambda(u)\,[y_\gamma(u) - s] = O(1),\ \lambda_\beta(u)\,[y_\beta(u) - s] \to t \Longrightarrow \lambda_\delta(u)\,[y_\delta(u) - s] \to t.$$

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MATHEMATICAL MODELLING OF HEAT-MASS-TRANSFER PROCESSES FOR A HIGH INTENSITY GRANULAR DRYING [2]

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When drying a wet particle in hot flow air (at a temperature higher than the boiling one), different stages of the drying process are observed. At the initial stage, when the particle is moist in full, a significant amount of moisture is removed by evaporation at the particle surface under temperatures close to the wet bulb temperature. Here, the temperature and moisture distributions in the particle are subjected to the heat conductivity and diffusion equations, respectively. At the later stages, when the moisture content is reduced, the moisture diffusion to the particle surface becomes more restricted that causes the increase of the particle temperature. At the advanced stage, moisture is removed at the boiling regime when a granule part adjoining to surface is already dry (at a temperature above boiling) but another part of the granule is still wet (at a temperature below boiling). Here, a Stefan type problem arises, when it is required to find the boiling front position and also temperature and moisture distributions in granules.

A mathematical model of a process to high intensity drying granules is constructed that describes the temperature and moisture distributions along the particle radius in time for different stages of the process. The qualitative adequacy of the mathematical model constructed to the experimental data observed, and also a finite difference numerical method to solve the above differential problem are discussed.

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ON RESTRICTION OF THE RICHARDSON TECHNIQUE TO IMPROVE ACCURACY OF GRID SOLUTIONS FOR SINGULARLY PERTURBED REACTION-DIFFUSION EQUATIONS [3]

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In the case of singularly perturbed problems for parabolic convection-diffusion equations, order of the ε-uniform convergence rate for solutions of classical difference schemes on special grids (Bakhvalov, Shishkin), as a rule, is *not higher than one* in spatial and temporal variables. Earlier, for such a problem, using a Richardson technique, a difference scheme with *improved accuracy order* in *spatial and temporal variables* was constructed, namely, with the order close to 2 in x and 2 in t.

However, the Richardson technique turns out to be restrictedly applicable for the construction of difference schemes with *high accuracy order*. Here, on an example of a Dirichlet problem for a singularly perturbed parabolic convection-diffusion equation it is shown that for the Richardson method the rate of the ε-uniform *convergence in x* with order *higher than two* is unachievable.

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CONSERVATIVE DIFFERENCE SCHEMES FOR SINGULARLY PERTURBED PARABOLIC EQUATIONS [4]

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A Dirichlet problem is considered on a vertical strip for a singularly perturbed parabolic reaction-diffusion equation. The spatial derivatives in the differential equation, having *divergent form*, are multiplied by the perturbation parameter ε^2, where ε takes arbitrary values in the open-closed interval $(0, 1]$. The parabolic boundary layer appears in a neighbourhood of the strip boundary as $\varepsilon \to 0$.

Using the *integro-interpolational method*, a *conservative* finite difference *scheme* is constructed on piecewise-uniform meshes condensing in a neighbourhood of the boundary layer. The solution of the difference scheme and its normalized derivatives in x_1 and x_2 converge ε-uniformly at the rate $\mathcal{O}\big(N^{-2} \ln N + N_0^{-1}\big)$. Here $N = \min(N_1, N_2)$, $N_1 + 1$ and N_0 are the number of nodes in x_1 and t, respectively, $N_2 + 1$ is the maximal number of nodes in x_2 per unit length.

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FUZZY APPROXIMATION SYSTEMS AND RELATED STRUCTURES

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In 1968, that is only 3 years after L.Zadeh has published his famous work *"Fuzzy Sets"* [9] thus laying down the principles of what can be called *Mathematics of fuzzy sets*, his student C.L.Chang [1] introduced the concept of a fuzzy topological space thus marking the beginning of Fuzzy Topology, the conterpart of General Topology in the context of fuzzy sets. Later this concept was developed in various directions, see e.g. [2], [6], [8], [7]. Now Fuzzy Topology is one of the most well developed fields of Mathematics of fuzzy sets, and there are dozens of fundamental works on this subject.

In 1983 Z. Pawlak [5] has introduced the concept of a rough set which can be viewed as a certain alternative of the concept of a fuzzy set for the study of mathematical problems of applied nature. Pawlak's work was followed by many other researchers where rough sets and mathematical structures on the basis of rough sets were introduced and studied.

Although it may seem that the concepts of a fuzzy set, of a (fuzzy) topological space and of a rough set are of an essentially different nature and "have nothing in common", this is not the case. Probably, the first one to start studying the intermediate relations between topologies, fuzzy sets and rough sets was J. Kortelainen [3], see also subsequent his and co-authors papers [4], etc.

The aim of this work is to present an alternative view on the relations between fuzzy sets, fuzzy topological spaces and rough sets. In order to realize this aim we introduce the concept of an approximative system (first defined in our talk at the conference FSTA - "Fuzzy Sets: Theory and Applications" in 2008 in Liptovsky Jan, Slovakia) and thus come to the category **AS** of approximative systems. The properties of this category are studied and the connections between the category **AS** and its subcategories related to fuzzy topology, fuzzy sets and rough sets will be discussed.

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NUMERICAL METHOD FOR ANALYSIS OF FLEXIBLE CANTILEVER SUBJECTED TO DISTRIBUTED FOLLOWER LOAD

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The static analysis of the flexible rectilinear non-uniform cantilever beam under distributed follower load is considered. The angle of inclination of the load with respect to the deformed axis of the beam remains unchanged during deformation. The mathematical formulation of such a problem yields a boundary value problem for nonlinear integro-differential equation. Usually, the similar non-linear boundary value problem is reduced to a normal system of non-linear differential equations of a fourth order, which is then solved using the shooting method. According to the shooting method, the nonlinear boundary value problem is converted to a set of initial-value problems and the unknown initial value is then determined iteratively. The convergence of this iterative procedure depends upon how close the initial guess values are to the solution sought for.

At the present paper the considered boundary value problem is reduced to an initial value problem by change of variables. As a result, the problem can be solved without iterations by the Runge-Kutta integration scheme. Besides, the solution of initial value problem is unique, i.e. deformed shape is unique for any distributions of flexural stiffness and follower load. It is shown that there exist no critical loads in the Euler sense (divergence). These conclusions generalize the same results for non-uniform cantilever beams under concentrated follower forces [1];[2].

Some equilibrium configurations of the cantilever beam under distributed follower load are presented. The direct numerical method considered is simple, provides high accuracy of calculations, and needs less computational time than the shooting method. The direct method can be extended to similar problems of curved cantilever beams.

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COMPUTATIONAL MODELLING OF AMPEROMETRIC BIOSENSORS IN THE CASE OF SUBSTRATE AND PRODUCT INHIBITION

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An amperometric biosensor is a device used for measuring concentration of some specific chemical or biochemical substance in a solution [1]. Biosensors use specific biochemical reactions catalyzed by enzymes immobilized on electrodes. Once a product of a reaction reaches an electrode it oxidizes or reduces producing the anodic current which is measured.

The response of an amperometric biosensor at mixed enzyme kinetics and diffusion limitations has been modelled digitally in the case of substrate and product inhibition. The model is based on non-stationary diffusion equations containing a non-linear term related to non-Michaelis-Menten kinetics of the enzyme reaction [2].

The simulation of the biosensor response has been carried out using the finite difference technique [3]. The mathematical model and the numerical solution were validated using analytical solutions existing for the very specific cases of the model parameters [1; 2].

The dimensionless model of the biosensor with substrate and product inhibition has been constructed in order to decrease the number of biosensor properties. Governing equations in biosensor's enzyme layer in dimensionless coordinates are expressed as follows:

$$\frac{\partial S}{\partial T} = \frac{\partial^2 S}{\partial X^2} - \Phi^2 \frac{S}{(1 + P/K_p) + S\,(1 + S/K_s)}, \tag{1}$$

$$\frac{\partial P}{\partial T} = \frac{D_{p_e}}{D_{s_e}} \frac{\partial^2 P}{\partial X^2} + \Phi^2 \frac{S}{(1 + P/K_p) + S\,(1 + S/K_s)}, \tag{2}$$

where T stands for the time, X is the space, S is the substrate concentration, P is the product concentration, D_{s_e} and D_{p_e} are diffusion coefficients, K_p and K_s are product and substrate inhibition rates, respectively, Φ^2 is the diffusion modulus.

The dimensionless model has been used to investigate the biosensor behaviour in a set of numerical experiments. The sensitivity of biosensor at different values of substrate and product inhibition rate at different substrate concentrations has been analyzed.

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ON THE THEORY OF THE 3RD ORDER ORDINARY DIFFERENTIAL EQUATIONS

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Results on the 3rd order ordinary differential equations are presented with applications.

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CHARACTERISTIC FUNCTIONS FOR STURM–LIOUVILLE PROBLEMS WITH NONLOCAL BOUNDARY CONDITION

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Let us consider a Sturm–Liouville problem with the NBC:

$$-(p(t)u')' + q(t)u = \lambda u, \quad t \in (0,1), \tag{1}$$

$$< l_0, u(t) >= 0, \tag{2}$$

$$< l_1, u(t) >= \gamma < k, u(t) > \tag{3}$$

where $p(t) \geqslant p_0 > 0$, $p \in C^1[0,1]$, $q \in C[0,1]$, l_0, l_1 and k are linear functionals. For example, the functional k can describe multi-point or integral NBCs:

$$< k, u(t) >= \sum_{j=1}^{n} \big(\varkappa_j u(\xi_j) + \kappa_j u'(x_j)\big), \quad < k, u(t) >= \int_0^1 \varkappa(t)u(t)dt,$$

and the functional l_i, $i = 0, 1$ can describe local (classical) boundary conditions

$$< l_0, u(t) >= \alpha_0 u(0) + \beta_0 u'(0), \quad < l_1, u(t) >= \alpha_1 u(1) + \beta_1 u'(1),$$

where the parameters $|\alpha_i| + |\beta_i| > 0$, $i = 0, 1$.

Let $\varphi_0(t;\lambda)$ and $\varphi_1(t;\lambda)$ be two independent solutions of equation (1) and

$$D_s^t(\lambda) := \begin{vmatrix} \varphi_0(t;\lambda) & \varphi_1(t;\lambda) \\ \varphi_0(s;\lambda) & \varphi_1(s;\lambda) \end{vmatrix}, \quad < k_1 \cdot k_2, D_s^t(\lambda) > \quad := \begin{vmatrix} < k_1, \varphi_0(t;\lambda) > & < k_1, \varphi_1(t;\lambda) > \\ < k_2, \varphi_0(s;\lambda) > & < k_2, \varphi_1(s;\lambda) > \end{vmatrix}.$$

All the solutions of equation (1) are of the form $u{=}C_0\varphi_0(t;\lambda){+}C_0\varphi_1(t;\lambda)$. There exists a nontrivial solution of problem (1)–(3) if and only if $\Psi(\lambda)\gamma{=}\Phi(\lambda)$, where $\Psi :=< l_0 \cdot k, D_s^t(\lambda) >$, $\Phi :=< l_0 \cdot l_1, D_s^t(\lambda) >$. Both functions $\Psi(\lambda)$ and $\Phi(\lambda)$ are an entire functions for $\lambda \in \mathbb{C}$.

The complex characteristic function

$$\gamma_c := \Phi(\lambda)/\Psi(\lambda), \quad \gamma_c \colon \mathbb{C}_\lambda \to \overline{\mathbb{C}} \tag{4}$$

is a meromorphic function and describes complex eigenvalues.

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INFLUENCE OF THE EXTERNAL FACTORS ON THE REPRODUCIBILITY OF THE BIOSENSOR RESPONSE

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Behaviour of electrochemical glucose biosensor, based on glucose oxidase was examined in diffusion and kinetic modes of action. Mathematical model based on enzymatic conversion of substrate and diffusion of substrate was created and influence of fluctuations of membrane thickness, diffusion coefficients and pH were modelled.

Biosensors as an analytical instrument have found wide application in medicine, environment, and food-quality control. However, application of the biosensors is limited by low reliability of the biosensor action. This is due to a number of diffident parameters. Stability of the biosensor action depends on both: stability of the sensing element of the biosensor - usually enzyme or enzymatic complex, and stability of the matrix. Complicated construction of the biosensor, usually consisting of several semi-permeable layers very often is sensitive to the fluctuations of the pressure in the bulk, especially in flow-through conditions. Fluctuations of the pH, or concentration of salts can change diffusion parameters of the biosensor membranes, thereby, changing the response of the biosensor. The main goal of this paper is mathematical modelling and evaluation of the influence of three main parameters of the biosensor - pH, membrane thickness and diffusion fluctuations.

Suppose that substrate (S) conversion to product (P) was catalysed by the enzyme (E).

$$S \xrightarrow{E} P.$$

Consider biosensor as a flat amperometric device with a layer of enzyme and outer membrane. It follows that the model has two regions. In the first region (outer membrane) only mass transport limited by diffusion takes place. In the second region (enzyme layer) enzymatic conversion of glucose to gluconic acid, oxygen to hydrogen peroxide and mass transport are limited by diffusion. A mathematical model of biosensor is based on system of diffusion equations with a nonlinear term corresponding to Michaelis-Menten kinetic of the enzymatic reaction together with boundary and initial conditions and compatibility conditions on the boundary between two regions with different diffusion coefficients.

In this study we demonstrated mathematical model, allowing predict reproducibility of the biosensor and evaluate influence of membrane thickness, diffusion coefficient and pH on metrological parameters of biosensor.

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SOME ASPECTS OF NUMERICAL INVESTIGATION OF VOLUME FREE ELECTRON LASER NONLINEAR STAGE

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The main principle of vacuum electronic devices such as travelling wave tubes (TWT), backward wave tubes (BWT), free electron lasers (FEL) is based on radiation of bunches of charged particles moving over the surface or through the slow-wave system (resonator). Volume Free Electron Lasers (VFEL) are new electronic devices that are more effective on many parameters than other ones.

VFEL operates as follows. An electron beam with electron velocity u passes through a photonic crystal (resonator) of the length L. Under diffraction conditions two strong electromagnetic waves can be excited in the resonator. If simultaneously electrons are under synchronism condition, they emit electromagnetic radiation in directions depending on diffraction conditions. None of other electronic devices uses principles of dynamical diffraction of electromagnetic radiation in resonator. The system of nonlinear equations used for VFEL lasing dynamics modelling was obtained from Maxwell equations in the slowly-varying envelope approximation. Electron beam dynamics is described using method of averaging over initial phases of electrons. In the common case this system is the following:

$$\frac{\partial E}{\partial t} + a_1 \frac{\partial E}{\partial z} + b_{11} E + b_{12} E_\tau = I, \frac{\partial E_\tau}{\partial t} + a_2 \frac{\partial E_\tau}{\partial z} + b_{21} E + b_{22} E_\tau = 0,$$

$$I = \Phi \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} \left(\exp(-i\Theta(t,z,p) + \exp(-i\Theta(t,z,-p))\right) dp,$$

$$\frac{d^2\Theta(t,z,p)}{dz^2} = \Psi \left(k - \frac{d\Theta(t,z,p)}{dz}\right)^3 \mathrm{Re}\left(E(t - z/u, z) \exp(i\Theta(t,z,p))\right),$$

$$E|_{z=0} = E_0, \quad E_\tau|_{z=L} = E_1, \quad E|_{t=0} = 0, \quad E_\tau|_{t=0} = 0, \quad \Theta(t,0,p) = p, \quad \frac{d\Theta(t,0,p)}{dz} = k - \omega/u,$$

where $t > 0$, $z \in [0, L]$, $p \in [-2\pi, 2\pi]$. Amplitudes of electromagnetic fields are denoted as $E(t, z)$, $E_\tau(t, z)$. Function $\Theta(t, z, p)$ describes phase of electron beam relative to electromagnetic wave. k is a projection of wave vector on axis z. ω is a field frequency. Equation describing phase of electron beam is more complicated than usually used in simulation of different electronic devices because we take into consideration as initial phase of an electron not only the moment of time t_0 of an electron entrance in resonator at $z = 0$ (as usual) but also transverse spatial coordinate in this moment.

In our previous investigations it is shown that VFEL is a dynamical system with multiple bifurcation points and chaotic dynamics. In this work a conservation law for integro-differential system presented here is obtained. VFEL efficiency analysis is proposed. Some steady-state regimes are investigated analytically and numerically.

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PARAMETER CONTROL OF OPTICAL SOLITON IN ONE-DIMENSIONAL PHOTONIC CRYSTAL

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Interaction of laser pulse with nonlinear photonic crystal is one of the modern problems. There are many papers dealing with (i) constructing of all-optical switcher based on this interaction, (ii) light localization on defects of photonic crystal, (iii) application of photonic crystal in various laser systems. In our preceding papers [1]–[3] the optical soliton formation was found out both for self-action of laser pulse in photonic crystal with cubic nonlinear response and for dual-waves interaction in photonic crystal with combined nonlinearity. It should be emphasized that soliton appear under certain conditions on input light intensity of laser pulse, parameters of layered structure and wavelength of an optical radiation. A detail investigation of dependence of soliton formation on parameters, mentioned above, and revealing an opportunity for controlling the soliton parameters and soliton displacement in photonic crystal are the main goal of this report.

To realize the formulated goal we find soliton solution on the base of eigenfunction problem for nonlinear Schrödinger equation with periodic coefficients. To solve the eigenfunction problem we use a computer simulation. Mainly, appearance of solutions with one peak intensity is of interest for us because in previous computer simulation results deal with a such solution.

Our investigation allows to make some conclusions. First, the width of soliton essentially depends on peak intensity.This result ,obviously,is the similar to one for homogeneous medium with cubic nonlinear response. Nevertheless, for photonic crystal with alternating nonlinear and linear response there is some distinguish in comparison with homogeneous nonlinear medium. Such a way, decreasing or increasing maximum intensity of input laser pulse it is possible to create solitons spreading over several layers or localizing into one layer. Second, the area of soliton localization is sensitive to the defects of photon crystal. For example, the slightest non-symmetry in localization of the layers of the crystal can result in the shift of the soliton to the left or right boundary of layer. Third, the jump of the soliton out of the layer occurs, when the maximum intensity of light exceeds the its crucial value, which depends on such parameters as layer length or diffraction.

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SEQUENTIAL AND PARALLEL DECOMPOSITION SCHEMES FOR A SINGULARLY PERTURBED REACTION–DIFFUSION EQUATION [5]

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The Dirichlet problem for a singularly perturbed reaction–diffusion equation is considered; the highest order derivatives are multiplied by the perturbation parameter ε^2, ε takes arbitrary values in the half-open interval (0,1]. With an example of an ordinary differential reaction–diffusion equation, we construct and study continual and difference (on piecewise uniform meshes) schemes based on a domain decomposition method in the case of sequential and parallel computations. We give the conditions that ensure the parameter-uniform convergence for solutions of the decomposition schemes with increasing the number of iterations. A comparative analysis of the efficiency of the decomposition schemes for sequential and parallel computations is made. It is shown (as opposed to the convection–diffusion time–dependent case examined in [1]) that the increase in the number of solvers in parallel continual schemes, as well in the corresponding difference schemes on piecewise uniform meshes, leads to the acceleration in solving the parallel method in comparison with the sequential method, without loss in the accuracy of the solution to the decomposed scheme. The results of the study are generalized to the case of a boundary value problem on a rectangle for the singularly perturbed elliptic reaction–diffusion equation considered in [2].

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ON SOME FUZZY CATEGORIES OF MANY-VALUED TOPOLOGICAL SPACES

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The concept of a fuzzy category was introduced by A.Šostak in [1] and later was studied in a series of papers see, e.g. [2].

First we recall the concept of an (L-)fuzzy category in the form appropriate for our merits. Let $L = (L, \leq, \wedge, \vee, *)$ be a *cl*-monoid with top element 1 and bottom element 0, in particular a complete Heyting algebra (when $\wedge = *$). *An* (L)*-fuzzy category* is a pair $(\mathcal{C}, \mu)$ where $\mathcal{C}$ is an ordinary category with the class of objects $\mathcal{O}(\mathcal{C})$, the class of morphisms $\mathcal{M}(\mathcal{C})$, and $\mu : \mathcal{M}(\mathcal{C}) \to L$ an L-subclass of the class of morphism such that: $\mu(g \circ f) \geq \mu(g) * \mu(f)$ whenever composition $g \circ f$ is defined in the category $\mathcal{C}$ and for each $X \in \mathcal{O}(\mathcal{C})$ $\mu(e_X) = 1$ where e_X is the identity morphism.

On the other hand in [3] we studied some categories of many-valued sets and many-valued topological spaces, in particular, categories **SET**(L), **TOP**(L) and **FTOP**(L). The aim of this talk is to introduce new fuzzy categories $\mathcal{F}$-**SET**(L), $\mathcal{F}$-**TOP**(L) and $\mathcal{F}$-**FTOP**(L) by applying the method of fuzzification [2] to the categories **SET**(L), **TOP**(L) and **FTOP**(L), to discuss some properties of these fuzzy categories and relations between them.

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CORDIAL VOLTERRA INTEGRAL EQUATIONS

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Consider the class of Volterra operators $(Vu)(t) = \int_0^t K(t,s)u(s)ds,\ 0 \le t \le T$, having the following property (A): V is a bounded operator in $C[0,T]$ and $u_r(t) = t^r,\ 0 \le r < \infty$, are eigenfunctions of V. Such an operator in noncompact in $C[0,T]$. It occurs that V has property (A) if and only if its kernel has the form $K(t,s) = t^{-1}\varphi(s/t)$, $0 \le s \le t \le T$, where $\varphi \in L^1(0,1)$. Thus we actually study the class of operators

$$(V_\varphi u)(t) = \int_0^t t^{-1}\varphi(s/t)u(s)ds = \int_0^1 \varphi(x)u(tx)dx,\ 0 \le t \le T.$$

We call $\varphi \in L^1(0,1)$ the *core* of V_φ and V_φ itself an operator with an core, or simply *cordial operator*. We introduce in $L^1(0,1)$ a multiplication operation $\varphi \star \psi$ so that $V_\varphi V_\psi = V_{\varphi\star\psi}$.. So $L^1(0,1)$ and the class of cordial operators become commutative Banach algebras which are isometrically isomorphic. In this way we establish formulae for the spectrum $\sigma_m(V_\varphi)$ of V_φ as an operator in $C^m[0,T]$:

$$\sigma_0(V_\varphi) = \{0\} \cup \{\hat\varphi(\lambda) :\ \lambda \in \mathbb{C},\ \mathrm{Re}\lambda \ge 0\},\ m = 0,$$

$$\sigma_m(V_\varphi) = \{0\} \cup \{\hat\varphi(k) :\ k = 0, ..., m-1\} \cup \{\hat\varphi(\lambda) :\ \mathrm{Re}\lambda \ge m\},\ m = 1, 2, ...,$$

where $\hat\varphi(\lambda) = \int_0^1 \varphi(s)s^\lambda ds$. Note that $\sup\{|\ \hat\varphi(\lambda)\ |:\ \mathrm{Re}\lambda \ge m\} \to 0$ as $m \to \infty$. In particular, we localise the spectra of Diogo's, Lighthill's and some other noncompact Volterra integral operators occurring in the practice.

We also treat the Volterra integral operators of a more general form

$$(V_{\varphi,a}u)(t) = \int_0^t t^{-1}\varphi(s/t)a(t,s)u(s)ds,\ \ 0 \le t \le T,$$

where $\varphi \in L^1(0,1),\ a \in C^m(0 \le s \le t \le T),\ m \ge 0$. It occurs that $\sigma_m(V_{\varphi,a}) = a(0,0)\sigma_m(V_\varphi)$. In particular, if $a(0,0) = 0$ then $\sigma_m(V_{\varphi,a}) = \{0\}$ and $V_{\varphi,a}$ as an operator in $C^m[0,T]$ occurs to be compact.

We prove the convergence and establish error estimates of the polynomial collocation methods for the Volterra integral equation $\mu u = V_{\varphi,a}u + f$ assuming that $\mu \ne 0$, $\mu \ne a(0,0)\hat\varphi(k)$, $k = 0, 1, ...$, and that $f \in C^m[0,T]$ where m is sufficiently large so that $\mu \notin a(0,0)\sigma_m(V_\varphi)$; in particular, $m = 0$ suits if $\mu \notin a(0,0)\sigma_0(V_\varphi)$, i.e., $\mu \ne a(0,0)\hat\varphi(\lambda)$ for $\mathrm{Re}\lambda \ge 0$.

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TWO-SAMPLE PROBLEMS IN STATISTICAL DATA MODELLING

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Owen ([2], [3]) introduced the empirical likelihood method for statistical inference. This method as shown by many researchers is appealing especially for constructing confidence intervals or bands (see LaScala 1990, Owen 2001). Recently the empirical likelihood method has been proposed for the two-sample problems in a general framework (see [4]). Among them most important are probability-probability (P-P) plots, quantile-quantile (Q-Q) plots, receiver operating characteristic (ROC) curves, mean and quantile differences of two samples, structural relationship models. P-P and Q-Q plots serve as a basic tool for a statistician to check graphically the distribution of data. Simultaneous confidence bands added to these plots make it possible to do goodness of fit testing by eye (see [4]). ROC curves are of great importance and have been used in signal theory, psychology, radiology, medicine etc.

In this paper we analyse pointwise interval coverage accuracy by Monte Carlo simulations for P-P, Q-Q and ROC curves. We compare our results with the empirical coverage accuracy from the asymptotical behavior of the appropriate empirical processes.

In [1] simultaneous confidence bands for ROC curves have been derived using the smooted bootstrap method. It is straightforward to prove that the bootstrap method works also for P-P and Q-Q plots. Finally, we construct simultaneous bands for P-P and Q-Q plots of real data example using both the empirical likelihood and smoothed bootstrap method.

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LINEAR STABILITY ANALYSIS OF MIXING LAYERS IN TWO-PHASE SHALLOW FLOWS

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Linear stability analysis of mixing layers in two-phase shallow flows is performed in the present paper. Hyperbolic tangent base flow velocity profile is used to model shallow mixing layer of two merging streams. The fluid contains uniformly distributed solid particles. The nonlinear system of shallow water equations under the rigid-lid assumption is linearized in the neighborhood of the base flow and is reduced to one linear partial differential equation. The use of the method of normal modes transforms the obtained partial differential equation to eigenvalue problem for a linear ordinary differential equation. The collocation method based on Chebyshev polynomials is used to solve the corresponding linearized eigenvalue problem. The stability calculations are performed for different values of the particle loading parameter and the velocity ratio. The results show that the particle loading parameter has a stabilizing effect on the flow while the increase in the velocity ratio destabilizes the flow.

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STABILITY OF STOCHASTIC SELF-ADJUSTING DYNAMICAL SYSTEMS WITHOUT AFTEREFFECT AND WITH ETALON MODEL

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Self-adjusting systems are well known (see [1]). In these systems the parameters changing of the regulator arises from the information about parameters of external random disturbances, dynamical characteristics of the studied object or the stochastic dynamical system, and functioning, which is defined in the real time [2].

Let us suppose that the real object is described by the following linear stochastic differential equation of the n order with non-random initial conditions

$$\frac{d^n x(t,\omega)}{dt^n} + \sum_{i=1}^{n} [a_i + \Delta a_i + \delta a_i(t)] \frac{d^{n-i} x(t,\omega)}{dt^{n-i}} = \sum_{i=1}^{n} b_i \frac{d^{n-i} x(t,\omega)}{dt^{n-i}} \cdot \frac{dw_i(t,\omega)}{dt}, \tag{1}$$

$$x(t_0,\omega) = x_{00}; \frac{dx(t_0,\omega)}{dt} = x_{10}, ..., \frac{d^{n-1} x(t_0,\omega)}{dt^{n-1}} = x_{n-1,0}, \tag{2}$$

where $a_i, b_i \in R^n$, $i = \overline{1,n}$; $w_i(t,\omega) \in R^1$ are pairwise-independent Wiener random processes; $\frac{dw_i(t,\omega)}{dt}$ are white noises; $\Delta a_i \in R^1$ are unknown as now real numbers; $\delta a_i(t)$, $i = \overline{1,n}$ are produced by the self-adjusting contour. The etalon model is described by the following equation

$$\frac{d^n y(t,\omega)}{dt^n} + \sum_{i=1}^{n} a_i \frac{d^{n-i} y(t,\omega)}{dt^{n-i}} = \sum_{i=1}^{n} b_i \frac{d^{n-i} y(t,\omega)}{dt^{n-i}} \cdot \frac{dw_i(t,\omega)}{dt}, \tag{3}$$

$$y(t_0,\omega) = y_{00}; \frac{dy(t_0,\omega)}{dt} = y_{10}, ..., \frac{d^{n-1} y(t_0,\omega)}{dt^{n-1}} = y_{n-1,0}. \tag{4}$$

The basic result of this report is construction of the self-adjusting contour, i.e. obtaining of such algorithm of changing of $\delta a_i(t)$, where $\lim\limits_{t \to +\infty} E\left\{x^2(t,\omega) - y^2(t,\omega)\right\} = 0$. We suppose that the system (3), (4) is asymptotic stable in the mean square. The synthesis of the self-adjusting contour is realized with the help of the second Lyapunov's method. The exponential p-stability in the whole is investigated in the stochastic self-adjusting systems with etalon model.

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ON THE SOLVABILITY OF SOME NONLINEAR BOUNDARY VALUE PROBLEM

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Consider the second order two-point boundary value problem (BVP) of the form

$$x'' + p(t)\,x' = f(t,\,x), \tag{1}$$

$$x(0) = 0, \quad x(1) = 0, \tag{2}$$

where $t \in I := [0,\,1]$, $p \in C(I,\,\mathbb{R})$, $f \in C(I \times \mathbb{R}, \mathbb{R})$. We investigate the solvability of the BVP (1), (2) using a quasilinearization process described in [1], [2], [3]. Namely, we reduce the given nonlinear equation (1) to a quasi-linear one

$$x'' + p(t)\,x' + r(t)\,x = F(t,\,x), \tag{3}$$

where F is continuous, bounded and Lipschitzian with respect to x and the extracted linear part $\big(L_2x\big)(t) := x'' + p(t)\,x' + r(t)\,x$ is a non-resonant with respect to the boundary conditions (2).

We use a fact that modified quasi-linear problem (3), (2) has a solution, the oscillatory type of which corresponds to the type of non-resonance to the extracted linear part $\big(L_2x\big)(t)$.

Suppose that such quasilinearization is possible in some domain Ω. If a solution $\xi(t)$ of the problem (3), (2) is located in this domain of equivalence Ω than $\xi(t)$ solves the original BVP (1), (2) also.

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PROPAGATOR DIFFERENCE SCHEME FOR SOLVING OF THE DISSIPATIVE MURRAY EQUATION

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In this work implicit difference scheme, based on the propagator numerical method [1] for solving of the dissipative Murray equation is described. Previously propagator numerical method was elaborated and used for solving of advection-diffusion-reaction (ADR) equations, where it for wide range of parameters gives absolute stability.

The study problem can be described as follows:

$$\frac{\partial u}{\partial t} = \frac{D}{R_e}\frac{\partial^2 u}{\partial x^2} + f\left(\frac{\partial u}{\partial x}, u\right), \quad f = -u\frac{\partial u}{\partial x} - ru, \quad u = u(t,x), \tag{1}$$

$$0 < t \leq T, \quad x \in R^1,$$
$$u(0,x) = u_0(x),$$

where $D \geq d > 0$, and R_e is the Reynolds number. Such implicit propagator difference scheme is considered:

$$\Lambda(U_i^{l+1,m+1}) = \frac{1}{h_i^*}B_i U_{i+1}^{l+1,m+1} + \frac{1}{h_i^*}A_i U_{i-1}^{l+1,m+1} - Q_i U_i^{l+1,m+1} = exp\left(\frac{f_i^{l+1,m}}{U_i^l}\tau\right)\frac{U_i^l}{\tau}, \tag{2}$$

where

$$f_i^{l+1,m} = -U_i^{l+1,m}\left(\frac{U_{i+1}^{l+1,m} - U_{i-1}^{l+1,m}}{2h_i^*}\right) - rU_i^{l+1,m} \tag{3}$$

and

$$A_i = \frac{D}{R_e}\frac{1}{h_{i-1}}, \quad B_i = \frac{D}{R_e}\frac{1}{h_{i+1}}, \quad Q_i = \frac{1}{h_i^*}\left(A_{i+1} + B_{i-1}\right) + \frac{1}{\tau}. \tag{4}$$

Conditions for convergence and stability for the proposed difference scheme are discussed.

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ON INTERPLAY BETWEEN GLOBAL OPTIMIZATION AND VIZUALIZATION

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Objective functions in practical applications frequently do not posses theoretical properties guaranteeing availability of an efficient optimization method. In black box optimization case it is difficult also to extract features of objective functions justifying selection of a relevant optimization method. To facilitate such a selection the visualization of information collected during the search can be helpful. On the other hand implementation of visualization methods involves solution of global optimization problems. In the present paper the interrelation between global optimization and visualization of multidimensional data is considered.

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PARALLEL BRANCH AND BOUND ALGORITHM FOR MULTIDIMENSIONAL SCALING WITH CITY-BLOCK DISTANCES

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Multidimensional scaling is a technique for exploratory analysis of multidimensional data widely usable in different applications [1]. Pairwise dissimilarities among n objects are given by the matrix (δ_{ij}), $i, j = 1, \ldots, n$. A set of points in an embedding metric space is considered as an image of the set of objects. Normally, an m-dimensional vector space is used, and $\mathbf{x}_i \in \mathbf{R}^m$, $i = 1, \ldots, n$, should be found whose inter-point distances fit the given dissimilarities. Images of the considered objects can be found minimizing a fit criterion, e.g. the most frequently used least squares *STRESS* function:

$$S(\mathbf{x}) = \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \left(d\left(\mathbf{x}_i, \mathbf{x}_j\right) - \delta_{ij} \right)^2 ,$$

where $\mathbf{x} = (\mathbf{x}_1, \ldots, \mathbf{x}_n)$, $\mathbf{x}_i = (x_{i1}, x_{i2}, \ldots, x_{im})$; $d(\mathbf{x}_i, \mathbf{x}_j)$ denotes the distance between the points $\mathbf{x}_i$ and $\mathbf{x}_j$; it is supposed that the weights are positive: $w_{ij} > 0, i, j = 1, \ldots, n$. The most frequently used distances are Euclidean, but multidimensional scaling with other Minkowski distances in the embedding space can be even more informative. In the present paper the problem with the *STRESS* criterion and city-block distances in the embedding space are considered.

STRESS normally has many local minima. The case of city-block distances is different from the other cases of Minkowski distances that *STRESS* with city-block distances can be non differentiable even at a minimum point [2]. However it is piecewise quadratic, and such a structure can be exploited for tailoring of an ad hoc global optimization algorithm. A two level minimization method for the two-dimensional embedding space was proposed in [2] where a problem of combinatorial optimization is tackled by evolutionary search at the upper level, and a problem of quadratic programming is tackled at the lower level. A branch and bound algorithm for the upper level combinatorial problem is proposed in [3].

In this lecture parallel version of the branch and bound algorithm for multidimensional scaling is presented. The parallel algorithm is investigated experimentally solving geometrical and empirical data sets. The results of experiments are discussed and directions for future research are identified.

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ON PARALLEL NUMERICAL ALGORITHMS FOR THE TRAVELING WAVE MODEL

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We deal with the (2+1)-D dynamical PDE model similar to that one derived in [1; 2]. The effects in vertical y-direction are averaged, the dynamics of carriers and field propagation in longitudinal z-direction are given by TW model [1], the nonhomogeneous laser structure and field/carrier spreading or compression in lateral x-direction is represented by x-dependent device parameters and diffraction/diffusion factors. We are solving the model equations by means of finite-difference (FD) time-domain method.

The main goal of our work is to make numerical solution of the model as fast as possible, so that we could perform two- or even more parameter studies in reasonable time. By discretizing the lateral coordinate we substitute our initial (2+1)-D model by J coupled (1+1)-D TW models [1]. For typical tapered lasers J should be of order $10^2 - 10^3$. Thus, the CPU time needed to resolve (2+1)-D model is by 2 or 3 orders larger then CPU time needed to resolve a simple (1+1)-D TW model.

In the amplifier region the computation domain $(-X, X)$ should be sufficiently large if want to solve numerically a whole-space evolution of laser beams. But due to computational restrictions (CPU time and memory resources of the computer), one has to restrict the computational domain and to solve the problem only in the region of interest (or a slightly larger domain). Then the main challenge is to introduce special artificial boundary conditions which enable us to simulate accurately the asymptotical behavour of the solution and not induce numerical reflections at the boundaries. These boundary conditions must give a well posed problem and discrete approximations of the new boundary value problem should be constructed, which are stable under non-restrictive conditions on space and time steps of the discrete grids.

We use the Domain Decomposition (DD) as a general paradigm to develop our parallel algorithms. The implementation of the algorithms is done by ParSol tool. It is targeted for implementation of numerical algorithms in C++ and semi-automatic parallelization of these algorithms on distributed memory parallel computers including clusters of PCs.

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COUPLING OF ZONES WITH DIFFERENT RESOLUTION CAPABILITIES IN STRUCTURAL FINITE ELEMENT MODELS OF UNIFORM STRUCTURES

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The approach discussed in this work enables to present the finite element model of a large uniform structure by an assembly of zones of different resolution capability. A dense girder structure has been analyzed in our recent work [1], where the equivalent membrane model exhibiting similar features under static and dynamic loading has been obtained. The criterion of the behavior similarity of the two models was assumed to be the coincidence of corresponding nodal displacements. Displacements of all nodes of the models within the domain of size $L \times L$ have been taken into account when formulating the optimization task, during which the parameter identification of the membrane was performed. The penalty-type target function expressed as a sum of squares of differences between the displacements of corresponding nodes of each model was used as $T(\vec{p}_j^{\,i}, \vec{q}_j^{\,i}) = \sum_{j=1}^{m} \frac{\sum_{i=1}^{n} (\vec{p}_j^{\,i} - \vec{q}_j^{\,i})^2}{\sum_{j=1}^{n} (\vec{p}_j^{\,i})^2 + \sum_{j=1}^{n} (\vec{q}_j^{\,i})^2}$, where $\vec{p}_j^{\,i}$ is the vector of i-node displacements of the j-model of membrane, $\vec{q}_j^{\,i}$ is the vector of i-node displacements of the j-model of girder, $n = (N+1)^2$ - total number of the nodes of each model, N - number of cells along the side of the model, m - number of models.

In this work the further development of the approach is presented. The continuous membrane model, which can be used in multi-scale models and presents adequately the behavior of the girder structure under static, as well as, dynamic loads has been synthesized. The investigated example structure contains the girder structure presented as a small patch in the central zone of the equivalent continuous membrane, which should imitate a vast continuous girder structure. While actions imposed upon a structure are located in finer resolution patches, the rougher resolution zones serve as a surrounding for finer ones in order to present the overall behavior of the structure. The girder zone is coupled to the equivalent membrane by using contact elements CONTA171 and TARGE169 in ANSYS environment.

The least squares method and the artificial neural network approach have been employed for the identification of the geometric and physical parameters of the equivalent membrane. The efficiency of both approaches has been compared. In static and dynamic analysis under the same mechanical loads the equivalent membrane model provided satisfactory approximation of the behavior of the girder structure even when the membrane element side length was twice greater than the dimension of the girder cell.

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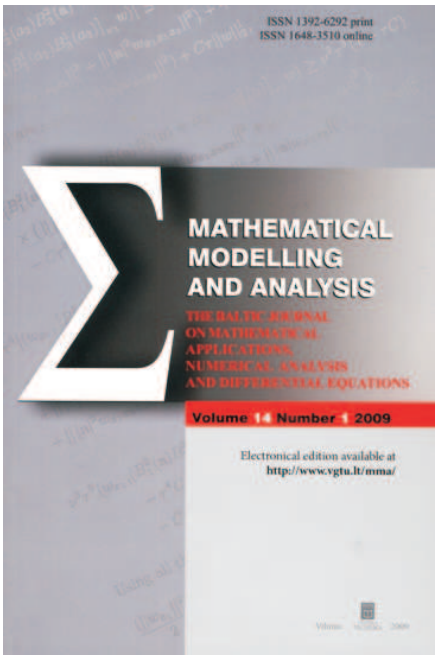

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