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A SIMULATION MODEL OF CHOOSING AN AIR FLIGHT BY A PASSENGER

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The purpose of this research is as follows: creation of a passenger's behaviour model in choice of flight. For practical calculations the simulation approach has been used. Simulation has been carried out with MathCAD software.

Keywords: *choice model, choice of flight, simulation approach*

1. Introduction

Choice models describe rather a general situation when it is necessary to make only one decision from the several available. In the choice models the subject is the person who makes a decision, and alternatives are his possible decisions.

There are a lot of different mathematical theories describing various choice models, for example, rather a detailed choice models applied to transport processes can be found at M. Ben-Akiva and S. Lerman [1].

Modern choice models are built on a multi-alternative basis and have obtained a wide distribution at the creation of the flight choice models by a passenger.

The creation of the potential passenger's behaviour analytical model includes two problems: firstly, it is necessary to create the model itself; secondly, it is necessary to estimate its parameters.

It is necessary to note, that the creation of the analytical expression for multi-alternative choice models is complex enough. Moreover, very often such models not give satisfactory results. Therefore for practical application simulation models most often are used. The example of creation of simulation multi-alternative model of a flight choice by a passenger can be found at J. Paramonov [2].

2. A Potential Passenger's Behaviour at a Flight Choice Description

When choosing an air flight the passenger may take into account a number of different factors. For example: the cost of the ticket, the departure time, the type of the flying plane, the length of flight, the type of the flight (direct flight or flight with change), service quality, etc. From all the variety of the possible factors for creation of the considered choice model the most significant and dataware provided factors have been chosen. Let us start the description of the model.

2.1. Flights' Description

A pair of cities (direction) is fixed, between these cities m available flights are executed every day. All flights are described by three parameters collected in vectors:

- the number of seats $\mathbf{r}_1 = \left(r_1^{(1)}, r_1^{(2)}, ..., r_1^{(j)}, ..., r_1^{(m)}\right)$;
- the cost of tickets $\mathbf{r}_2 = \left(r_2^{(1)}, r_2^{(2)}, ..., r_2^{(j)}, ..., r_2^{(m)}\right)$;
- the time of departures $\mathbf{r}_3 = \left(r_3^{(1)}, r_3^{(2)}, ..., r_3^{(j)}, ..., r_3^{(m)}\right)$.

2.2. Passengers' Flow Description

There is a flow of the passengers wishing to depart by one of m flights. It is supposed, that each passenger's wishes to get a ticket for a flight on a certain day. However, if there are no tickets suitable for any flight that day, then he transfers the ticket purchase next day. If on the next day he is refused too, the ticket purchase will be transferred for a day after tomorrow, and so on. Thus, the flight can be postponed for some days, but not more than for h days, after that the passenger finally cancels the trip. Let name the value h as a deadline of postponement the flight by the passenger.

On the first (prospective) day of departure the passenger tries to choose the flight from a certain time interval, being guided by a desirable departure time and some values of deviation from this time. If there is a flight, which departure time belongs to the specified range, the passenger chooses this flight. If there is a number of flights available the passenger chooses the cheapest one.

If there is not any suitable flight in the desirable departure time for the passenger, he transfers the ticket purchase on the next day. On this day and all the following days of the possible departure, the passenger chooses the flight with the minimal cost of the ticket.

It is assumed that each passenger is described by four parameters:

- X is the desirable time of departure;
- Δ_+ is the admissible value of positive deviation of the actual time of flight from the desirable one (the positive deviation is understood as a later departure of a plane in relation to desirable)
- Δ_- is the admissible value of negative deviation of the actual time of flight from the desirable one (the negative deviation is understood as an earlier departure of a plane in relation to desirable)
- h is the maximal number of days for which the passenger can postpone his flight.

Each of the listed characteristics is a random variable and has its own distribution.

The random variable X defines the desirable time of departure within a day. It is described by the corresponding density of distribution. It takes whole values from 1 up to 24 with certain probabilities which correspond to the change of intensity of demands within a day.

It is obviously, that for each passenger the values Δ_+, Δ_- and h have the random meanings as well. Let $D = \Delta_+ + \Delta_-$ is an interval of desirable by the passenger time of departure. All characteristics of the passenger can be seen on Figure 1.

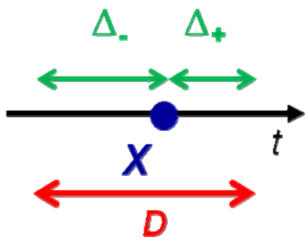

Figure 1. *Passengers' characteristics*

Let us consider the process of a flight choice beginning from the prospective day of departure. For everyone i – th passenger the modelling of a flight choice process can be described in the following way.

From all *m* flights those flights are selected, the moments of departure of which $r_3^{(j)}$ are situated within the corresponding time interval $r_3^{(j)} \in \left(X^{(i)} - \Delta_-^{(i)},\, X^{(i)} + \Delta_+^{(i)}\right)$, i.e. $r_3^{(j)} \in D^{(i)}$. If there are some flights available then we select one with the minimal cost of the ticket $r_2^{(j)}$:

$$j^{(i)} = \min_j \left\{ r_2^{(j)} : r_3^{(j)} \in \left(X^{(i)} - \Delta_-^{(i)},\, X^{(i)} + \Delta_+^{(i)}\right)\right\}. \tag{1}$$

If there is not any suitable flight, the passenger tries to depart on the following day.

If the passenger cannot depart within t days, he transfers the next attempt on $t+1$ day ($t = 1, 2, ..., h-1$) with the trip refusal probability p_t. Here $\mathbf{p} = \left(p_1, p_2, ..., p_h\right)$ is a vector of the trip refusal probability, and an increase of the value h leads to the value's $\{p_i\}$ increase. This probability reaches 1 on the last day of the possible departure.

For each passenger who agrees to depart on the t – th day after the prospective day of a departure ($t = 1, 2, ..., h-1$) the defining characteristic at the flight choice is the minimal cost of the ticket:

$$j = \min_j \left\{ r_2^{(j)} \right\}. \tag{2}$$

It was accepted that the number of potential passengers for each day, originally addressing for the ticket purchase, is a random variable N having a normal distribution with parameters μ_N and σ_N. Besides that, the values $\{N_k\}$ for different days are independent identically distributed (i.i.d.) random variables.

Modelling of the choice process has been carried out for H days, where H is the modelling horizon. As the output data by the end of the modelling horizon H the following parameters are obtained:

1) the percent of the payload capacity of flights;

$$Com_j = \frac{R_1^{(j)}}{r_1^{(j)}} \cdot 100\%\ , j = 1, 2, \ldots, m, \qquad (3)$$

where:

$r_1^{(j)}$ is the number of seats on the j-th flight;

$R_1^{(j)}$ is the average number occupied seats on the j-th flight.

2) the share of refusals $Rej/E(N)$, where Rej is the average number of refusals for a day, $E(N)$ is the average number of potential passengers of one day;
3) the share of the passengers who departed during the desirable time of the day K relative to the average number of the potential passengers for one day ($K/E(N)$);
4) the percent of the passengers who did not depart on a desirable day;
5) the average waiting time of departure by passengers (in days).

3. Simulation Model Description

A special algorithm for solving the task in view has been developed. Its novelty and originality consists in consideration of the simulation process not from the side of a newly arrived passenger, but from the side of the already existent query of claims of not departed passengers $\mathbf{Q} = (Q_0, Q_1, ..., Q_t, ..., Q_h)$, where t is the ordinal number of one of the days on which the passenger can postpone the flight (t = 0, 1, …, h), Q_t is a number of passengers of this day. Further for the model description instead of the term "the claim of the passenger" the term transaction accepted in modelling is used.

The described process of modelling is complex enough, therefore for practical calculations the simulation approach has been used. Simulation was carried out with MathCAD software.

Conceptually the simulation model is based on the consideration of the transactions turn, which corresponds to the passengers' flow, wishing to get tickets for this or that flight. Depending on the fact whether the passengers corresponding to transactions are the passengers of the prospective day of the departure ($t = 0$), or they are already the passengers of the t-th day of the postponement of flight, or they are the passengers of the final day of postponement of flight ($t = h$), the transactions describing them also differ.

Simulation model description for one run from the modelling horizon H follows below.

4. Modelling Algorithm for One Run

Begin

Cycle by the modelling horizon (by j, $j = 1, 2, ..., H$)

1. Cycle by days of expectation in reverse order (by t, $t = h, h-1, ..., 1$)
   1.1. Cycle by transactions (by k, $k = 1, 2, ..., Q_t$)
      1.1.1. For each $t-$ the day transaction the choice of flight is defined by cost of the ticket $\mathbf{r}_2$ (2). If the flight is not chosen the transaction remains in the queue of the current day transactions Q_t.
      1.1.2. For the remained $t-$the day transactions the number of the refusing the further expectation transactions is drawn. This number Rej_t has binominal distribution with parameters $n = Q_t$ and $p = p_t$: $Rej_t := rbinom(1, p, n)$.
2. Generate N transactions, which appeared on the current day ($t = 0$), and the values $X^{(i)}$, $\Delta_-^{(i)}$, $\Delta_+^{(i)}$ ($i = 1, 2, ..., N$) as well.
3. Cycle by the number of the current day transactions N (by i, $i = 1, 2, ..., N$)
   3.1. For each transaction the choice of flight is defined by two factors: the interval of the desirable time of departure D, and the cost of the ticket $\mathbf{r}_2$ (1) as well.
   3.2. If flight is not chosen the transaction passes into the queue of the current day transactions Q_0.
4. Rewrite the queue of transactions, increasing a waiting time for one day: $Q_{t+1} = Q_t$ ($t = 0, 1, ..., h-1$).

End

5. Numerical Example

In the considered below numerical example the initial data are the following:

1. Flights' characteristics:
   - the number of flights $m = 5$;
   - the vector of seats number $\mathbf{r}_1 = (80 \quad 80 \quad 100 \quad 80 \quad 100)^T$;
   - the vector of the tickets cost $\mathbf{r}_2 = (50 \quad 75 \quad 100 \quad 125 \quad 150)^T$;
   - the vector of the plane departure time $\mathbf{r}_3 = (21 \quad 12 \quad 9 \quad 17 \quad 19)^T$.
2. Passengers' characteristics:
   - the admissible values of the positive and negative deviations of the actual time of flight from the desirable by the passenger time of departure (Δ_- и Δ_+) are identical for all the passengers, besides $\Delta_- = \Delta_+$;
   - the passenger's lookup horizon $h = 3$ is the same for all passengers;
   - the vector of probabilities of refusal of departures $\mathbf{p} = (0 \quad 0 \quad 0 \quad 1)^T$.
3. Besides, it has been accepted that:
   - the modelling interval $H = 100$.
   - the initial queue of transactions (of the expecting passengers) is empty $\mathbf{Q} = (0 \quad 0 \quad 0 \quad 0)^T$;

At the given vector $\mathbf{p}$ for all the passengers the waiting time is 3 days. Thus passengers choose flights according to the minimal cost of the ticket. Exception is made by newly arrived passengers (at $t = 0$), they choose the flights in view the factors: minimal cost of the ticket, and desirable time of departure.

The number of the newly arrived passengers N of one day has a normal distribution with parameters $\mu_N = 500$ and $\sigma_N = 20$. In this case, the total sum of seats on flights for one day $\sum_{j=1}^{m} r_1^{(j)} = 440$ is less than the average number of newly arrived passengers μ_N. Thus, lack of seats for the newly arrived passengers is observed.

During the modelling horizon H there is a gradual entering the process a stationary mode. It is possible to consider, that beginning approximately from the 40th day the system is comes into the stationary mode. The following results of the modelling were obtained for a time interval from 40th to 70th day of the modelling horizon.

In this example the dependence of payload capacity of flights Com_j (in %) on the length of the interval of desirable by the passenger time of departure D ($Com_j = f(D)$ has been investigated. The results of obtained simulation are presented in Table 1 and on Figure 2.

Besides, in the given example the dependence of number of passenger's refusal Rej, on the value D is shown ($Rej = f(D)$) (see Figure 2).

Table 1. Dependence of payload capacity of flights Com_j (in %) on the length of the interval of desirable time of departure D

***D* (hours)**	**1**	**1.5**	**2**	**2.5**	**3**	**3.5**	**4**	**4.5**	**5**	**5.5**	**6**
Com_1 (%)	100	100	98	97	96	97	95	100	100	100	100
Com_2 (%)	50	69	86	99	100	98	100	100	100	100	100
Com_3 (%)	34	51	70	78	87	80	90	88	93	96	97
Com_4 (%)	42	69	89	99	100	99	100	100	100	100	100
Com_5 (%)	33	53	71	58	48	60	45	52	54	63	66

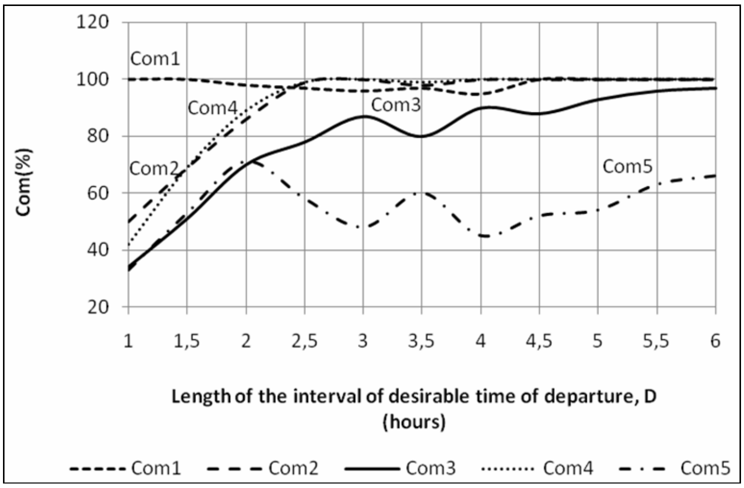

Figure 2. *Simulation results of* $Com_j = f(D)$

Analysis and comment of the obtained results is given below.

We would like to remind, that the capacity of planes has only two meanings: 80 and 100 seats. Numbers of flights are sorted in ascending order by costs of tickets. The following regularities have been revealed:

1. The cheapest flight (the first one) is filled practically completely (95 % – 100 %), without dependence on the length of an interval D.
2. The increase of value D to the meaning $D = 2$ leads to the growth of payload capacity of all flights.
3. The further increase of value D ($D > 2$) shows the steady growth of payload capacity of flights only for flights with the number of seats equal to 80.
4. Tendencies of the payload capacity of flights with number of seats 100 differ depending on cost of flight. So, the third flight with the growth of value D is filled completely, but the filling the fifth flight (the most expensive) stabilized at a level of 60 %.
5. As seen from Figure 3 the value of refusals Rej practically does not vary from the value D.

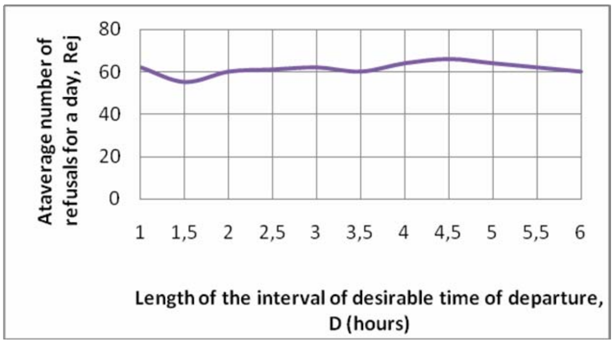

Figure 3. *Simulation results* $Rej = f(D)$

Let's comment separately the 3-rd flight. The low percentage of the payload capacity of this flight on the initial stage of modelling is connected with the inconvenient moment of the flight departure for the passenger (9 AM) and with rather high cost of the ticket. For the increase of this flight payload capacity the following variants has been considered:

- Reduction of the cost of the ticket;
- Changing of the plane departure time;
- Replacement of the plane by the plane of smaller capacity.

Let's analyse one possible variant. At the $D = 4$ the first and the second flight are filled completely because they are cheaper than the third flight. Maximal values of the third flight payload capacity are observed whether when the departure time moves at 10 AM or when the departure time is between 15 and 18 PM (see Figure 4). In the specified time intervals the third flight "takes away" passengers of more expensive fourth and fifth flights.

The analysis of results of the shown numerical examples allows drawing a conclusion that simulation results adequately describe the considered process.

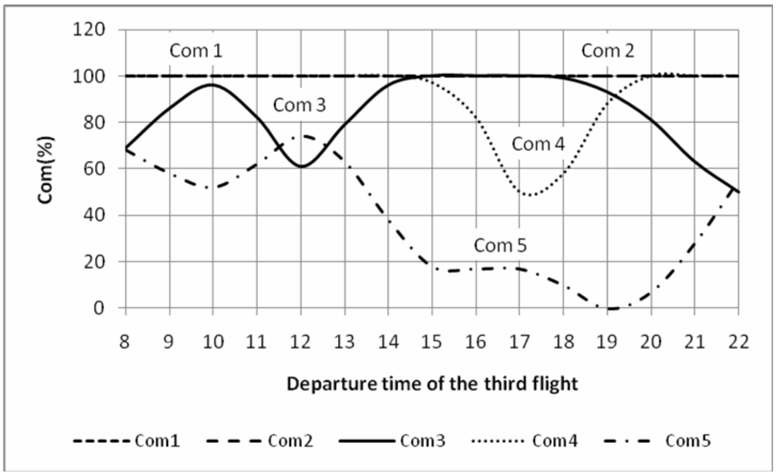

Figure 4. *Dependence of payload capacity of flights on the third flight departure time*

6. Conclusions

The passenger's behaviour model at a flight choice for one Origin-Destination pair of cities has been created out in this research. This model takes into account three characteristics of flight: the number of seats, the cost of air-ticket and the time of departure; and four characteristics of passenger as well: the desirable time of departure, the positive and negative passenger-flight deviations, and the passenger's lookup horizon. The originality of the model consists in the choice process consideration from the side of the expecting passengers queue. For practical calculation the simulation approach has been used. Numerical examples confirm the adequacy of the considered model.

References

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