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OPTIMIZATION MODELS OF DISTRIBUTION OF INVESTMENTS AT THE ENTERPRISES BY KINDS OF ACTIVITY

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Abstract. The purpose of the paper is investigation of different strategies of the development of enterprises activities. Applicability of algorithm of evaluation of different investment projects taking into account the limit of resources by its kind and quantity is shown in the paper. From the mathematical point of view modelling of processes of diversification of investment process based on use of algorithms of distribution of resources. It causes interest to existing distributive models. Research of existing methods of distribution of resources allows to reveal base strategy of development of the enterprise and also to define diversification policy of the enterprise by kinds of activity. The algorithm of definition of a degree of importance of investment projects using median's distributions is offered in the paper.

Keywords: strategy formulation, basic directions of deepened planning, optimization model.

1. Introduction

One-stage distribution of resources of the enterprise, which characterized with the absence of obvious stages of distribution, is connected with necessity to reserve the certain part of resources for realization of investment projects. In our case, using of this method of the distribution of resources is unacceptable because the result of activity of any enterprise cannot be guided only by the end of the certain period. Managers of the enterprise always have plans of strategic development of the given enterprise. In the planning process of development of the enterprise distribution of homogeneous and non-uniform resources of the enterprise should be considered. Transition from homogeneous to non-uniform resources essentially complicates problems of distribution of resources. Only homogeneous investment resources are considered in paper. Method of distribution of resources of the enterprises (investments) offered by authors can be applied also more widely for example, in the case of borrowed finances included in the model. The opportunity of ranging of objects of research in a multivariate case with use concordance ordering is considered by H. Joe [1].

The object of investigation is strategy of distributions of investments at the enterprise. The goal of investigation is the ranging different investment strategies of distributions of investments at the enterprise by kinds of its activity using median's distribution.

2. The distribution of homogenous resources

Cash flow behaviour as a main factor of investment process is considered in the paper using mehod of single time period. Puxty and Dodds [2] notify that the biggest flaw regarding single-period methods is that they do not consider the time-value of the money. This fact can not be overseen, especially not in construction projects, which as a rule are of very long duration.

As it was mentioned early, only homogenous investment resources are considered in the paper. However, with the inclusion into the model of borrowed means, which have different characteristics, using methods of this group is possible. As a basis of the offered mechanism, also in the basis of work and almost all management mechanisms, is the procedure of collective choice. Every participant of the expert group has his own point of view on what kind of policy an enterprise follow for reaching the best financial results should, and these points of view do not coincide. Collective decision takes into account the opinions of different participants (experts) about enterprise's investment development strategy. The strategy of investment at the enterprise can be received as the result of the realization of the procedure using median's distribution method. Choosing median's strategy may be different from any of the variants offered by experts, or may be coincides with the majority of experts. Setting of the task of the distribution of means is hidden in the following. Experts offer different programs of investment (resources) distributions where they reflect their point of view on the priority of directions taking into account the most important criteria. So one expert supposes that investment project gives maximum income. Another expert prefer lower but guaranteed rate of return taking into account the decrease in the risk of bankruptcy probability. It means that each expert has own system of preferences. Let us suppose that all experts' opinions have similar significance for manager (decision maker) and he (manager) wants to display all opinions in the final decision. Manager's decision will be the most "similar" from the opinions (decisions) of experts. It is possible a situation when an expert is offering not only single variant but is singling out a whole lot of investment programs that satisfy his criteria. Let us suppose that there are m experts and n action possibilities. Each expert is formulating his conditions, according to which, the enterprise will work more effectively (for example, the limitations on the spending of resources, equipment, human resources and salary). Each expert has their own target (for example, to increase the total income of the enterprise or to decrease the risk of investment, to increase the average salary, to decrease the pay-back time and the energy consumption of manufacturing) and with the fulfilment of formulated limitations the enterprise will achieve better results in a certain field.

In papers [3, 4, 5, 6] authors considered multivariate factors modelling for the investigation of stability of economic systems (in logistic, insurance, financial management). The condition of stability as the main criteria for ranging of investigated objects is considered in the mentioned papers.

In present papers authors describe the process of ranging of diferent investments projects at the enterprise ignoring the influence of factors of uncertainty. The authors suppose to investigate the influence of factors of uncertainty to the process of ranging of the investments in future.

3. Using of median's distribution at formation of strategy of investment of the enterprise

The collective decision takes into account in some form of data on a policy of the enterprise which would be chosen by separate participants. As a result of application of procedure the program which is not conterminous with one variant, offered by experts can be received, or the part of variants conterminous to parts, the specified separate participants can be allocated.

We shall put, is present **m** experts and **n** variants of activity. Each expert formulates his own opinion which (by the opinion of the expert) is the better for the enterprise investment strategy. The decision making take into account some restrictions, i.e.on the financial resources, on the equipment, on the manpower resources and restrictions on a wage fund. The description of an investment portfolio in conditions of the elementary market in view of risk elements is presented in [7, 8, 9].

Each expert allocates in metric space $\mathbf{R}^n$ of variables $\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$ (where $\mathbf{x}_i$ – a share of the means put in **i** direction) set $\mathbf{A}_j$ ($\mathbf{j} = \mathbf{1}, \ldots, \mathbf{m}$) its satisfying variants of distribution of means. The simplest case is the task each expert of set, its satisfying vectors, with the help of linear restrictions. Restrictions can have, for example, various economic senses:

$$\sum x_i = M, \qquad x_i \geq 0, \tag{1}$$

where $\mathbf{x}_i$ – the sum of means which is planned to enclose in **i** direction.

The general investments should not exceed the available sum intended to investment:

$$\sum h_{pi} * x_i \geq M_p, \quad p = 1, \ldots, r, \tag{2}$$

where restriction on the charge of **i** kind of resources; $\mathbf{h}_{pi}$ – the charge of **p** kind of resource on performance of unit of works of **i** directions (in cost expression). In tne case when $\mathbf{d}_q$ – useful fund of an operating time of a **q** kind of the equipment (for example, measured in changes):

$$\sum d_{qi} * x_i \leq (\geq)\ T_q, \tag{3}$$

where $\mathbf{d}_{qi}$ – norm of expenses of **q** tipe of equipment for performance of unit **i** kind of works means restriction for the period of work (the requirement of loading of the equipment for the term of not smaller $\mathbf{T}_q$).

$$\sum t_{si} * x_i \leq (\geq)\ L_s, \tag{4}$$

where $\mathbf{t}_{si}$ – labour input **s** tipe of professional workers for performance of unit of **i** kind of works;
$\mathbf{L}_s$ – labour input in mid-annual calculation of **s** professional group. Then if sets $\mathbf{A}_j$ are not crossed that, by analogy to the definition of Kemeny median, most precisely reflecting opinion of each expert we shall count the variant, where $\mathbf{X}_{res}$ – the sum of distances from which up to each of sets $\mathbf{A}_j$ will be minimized. The distance from the point **X** up to set **Ω** is defined under the formula:

$$d(X, \Omega) = \min \rho(X, Y), \tag{5}$$

where ρ(X, Y) – distance between points **X** and **Y**, determined under the formula:

$$\rho(X, Y) = \sum_{i=1}^{n} |x_i - y_i|. \tag{6}$$

If sets $\mathbf{A}_j$ are crossed, for the best distribution we shall search with the help of function of the common profit of the enterprise on set **Ω** determined under the formula:

$$\Omega = \bigcap_{j=1}^{m} A_j. \tag{7}$$

The decision of a problem can be received with the help of the following algorithm:

We check every sets A_j , $j = 1, \ldots, m$ not to be empty.

1. If among A_j one empty set is revealed even, about it the message is done.

2. If set $\Omega = \bigcap_{j=1}^{m} A_j \neq \emptyset$, the problem is solved using the formula:

$$\mathbf{Z}(\mathbf{X}) \to \mathbf{max},\ X \in \bigcap_{j=1}^{m} A_j \ . \quad (8)$$

Function **Z**(**X**) defines utility from distribution of means **X**, for example, total profit.

3. In the case if $\Omega = \emptyset$ we determine variant of distribution $X_{\emptyset}$. The total distance from distribution $X_{\emptyset}$ to each of sets A_j is minimal. This problem is solved using the formula:

$$\sum_{j=1}^{m} d\left(X, A_j\right) \to \min_{X \geq 0} . \quad (9)$$

Taking into account definition of distance between a point and set (5) and between two points (6), formula (9) can be written as:

$$\sum_{j=1}^{m} d\left(X, A_j\right) = \sum_{j=1}^{m} \sum_{i=1}^{n} \left| x_i - y_i^j \right| \to \min$$
$$X > 0, \quad (10)$$
$$Y^j \in A_j,\ j = 1,2,\ldots,m.$$

The decision of a problem (10) can be received as a result of performance of the following sequence of actions: For each coordinate Y_{ij} of vector Y^j and for coordinates of vector **X** it is defined the top and bottom borders a_{ij} and b_{ij}, such that:

$$a_{ij} \leq Y_{ij} \leq b_{ij} . \quad (11)$$

Authors use Monte-Carlo method for the modelling the sequence of the N pseudorandom uniformly distributed points P_k (k = 1, 2, …, .N) in a parallelepiped (11) is received. From the modelled points we select those points N_1 $(N_1 < N)$, which belong to the allowable area A_0 determined by restrictions in (10). Selected points from A_0 we substitute in criterion function in (12):

$$\sum_{j=1}^{m} d\left(X, A_j\right) = \sum_{j=1}^{m} \sum_{i=1}^{n} \left| x_i - y_i^j \right| \to \min \quad (12)$$

As a result we receive values $Z(P_1)$, $Z(P_2)$, ..., $Z(P_{N1})$, where $P=(X_1,..., X_n, Y_{11},..., Y_{1n}, ..., Y_{m1},..., Y_{mn})$.

Among $Z(P_i)$ we find the least $Z(P_i^{min})$ and we can write that $Z(P_i^{min}) \approx \min Z(P)$.

The second offered scheme of formation of resulting distribution reflects a situation at which each expert holds precisely certain opinion on a necessary level of support of each direction. Manager should to distribute resources between some final numbers of directions. We number all programs of activity, let **i** – a serial number of a direction (project) (**i** = **1**, …, **n**). Then the set of criteria on which efficiency of each project of activity will be estimated is formed. The opinion of each expert corresponds to ranging on one of criteria. The classifying of the market participants is considered by M. Joshi [10]. Gathering the initial data on each of considered programs of investment is made. We shall put that **m** parameters are estimated in algorithm.

Everyone **j** expert gives the vector of preferences:

$$Pr_j = (O_{j1}, O_{j2}, ..., O_{jn}),\ j = 1, \ldots, m, \quad (13)$$

where O_{ji} – a serial number of the project occupying in ranging by estimated criterion **j** place **i**. Ranging process is in decreasing order. In each ranging the first place takes the most attractive, from the point of view of considered criterion, for the enterprise a direction of activity.

To each vector Pr_j we shall put in conformity a vector $D_j = (D_{j1}, D_{j2}, ..., D_{jn})$ where D_{ji} – number of projects, which according to **j** individual criterion are more preferable than the direction having a serial number **i**.

Example. There are four projects of investment with the parameters – NPV, Risk and Investment (m=3). Initial data for four projects of investment are presented in (Table 1).

Table 1. Initial data for four projects of investment

Parameter	**1**	**2**	**3**	**4**
1 parameter (NPV)	75	150	100	200
2 parameter (Risk)	3	2	4	8
3 parameter (Investment)	300	320	360	340

j = 1 – evaluation (ranking) of the projects by first parameter – „NPV”:

	4	**2**	**3**	**1**
Values of parameter „NPV” are placed in decreasing ordered	200	150	100	75
Rank (D_1)	**0**	**1**	**2**	**3**

Parameter NPV with value 75 (I project) has rank 3, NPV with value 150 (II project) has rank 1 and so on. For parameter NPV (after project reordering in first row) we have:

Project	**1**	**2**	**3**	**4**
NPV	15	30	20	40
Rank (D_1)	**3**	**1**	**2**	**0**

j = 2 – evaluation (ranking) of the projects by second parameter – „Risk”:

	2	**1**	**3**	**4**
Values of parameter „Risk” are placed in encreasing ordered	2	3	4	8
Rank (D_2)	**0**	**1**	**2**	**3**

For parameter Risk we have rank vector $\mathbf{D_2}$:

	1	**2**	**3**	**4**
Values of parameter „Risk” are placed in encreasing ordered	3	2	4	8
Rank (D_2)	**1**	**0**	**2**	**3**

It means that $\mathbf{D_1}$ = (3, 1, 2, 0). $\mathbf{D_2}$ = (1, 0, 2, 3), $\mathbf{D_3}$ = (3, 2, 0, 1).

The following step is search of group ranging in which individual preferences will be in the best way submitted. In this case Kemeny median will be considered as:

$$D^* = \min_D \sum_{j=1}^{m} d\left(D, D_j\right), \qquad (14)$$

where **d**(**D**, $\mathbf{D_j}$) – distance between two rangings determined under the formula:

$$d\left(D, D_j\right) = \sum_{i=1}^{n} \left|D_i - D_{i,j}\right| . \qquad (15)$$

The procedure of evaluation of Kemeny median is:

1) to build a matrix of losses **R** = {$\mathbf{r_{kl}}$}.

– Are considered vectors, in which direction with number **i** (**i** ∈ {**1**, **2**, ..., **n**}) is located consistently from **1** up to **n** places;

D = ($\mathbf{D_1}$, $\mathbf{D_2}$,..., $\mathbf{D_k}$, ..., $\mathbf{D_n}$)– ranging in which the project **k** has a place **g** (i.e. $\mathbf{D_k}$ = **g**–**1**). Therefore we have:

$$r_{kl} = \sum_{u=1}^{m} \left|D_k - D_{k,u}\right| . \qquad (16)$$

In our case for the data from an example losses matrix **R** is presented in (Table 2).

Table 2. Losses matrix R

		rank			
		0	1	2	3
Project	1	7	4	3	2
	2	3	2	3	6
	3	4	3	2	5
	4	4	3	4	5

2) to solve a problem about assignments to which search of Kemeni median is reduced to such form:

$$\begin{aligned} &\sum_{k=1}^{n}\sum_{l=1}^{n} r_{kl}x_{kl} \to \min, \\ &\sum_{k=1}^{n} x_{kl} \quad l = 1,...,n, \\ &\sum_{l=1}^{n} x_{kl} \quad k = 1,...,n, \\ &x_{kl} \in \{0,1\} \quad k,l = 1,...,n \end{aligned} \qquad (17)$$

where $\mathbf{x_{kl}}$ = **1** if the alternative k is appointed on a place l, and $\mathbf{x_{kl}}$ =0 otherwise. Matrix **X** = {$\mathbf{x_{kl}}$} at performance of conditions (17) corresponds (meets) to some ranging. After perfoming optimization accorging (17), we have matrix **X*** (Table 3).

Table 3. Matrix **X***

	0	1	2	3
1	**0**	**0**	**0**	**1**
2	**1**	**0**	**0**	**0**
3	**0**	**0**	**1**	**0**
4	**0**	**1**	**0**	**0**

In received matrix **X*** = {x^*_{kl}} we can restore a vector of group preference **P***, analyzing matrix **X*** in the lines: if in **X*** element $x^*_{kl} = 1$ we believe $P^*_l = k$. In an example $x^*_{14} = 1$; $x^*_{21} = 1$; $x^*_{33} = 1$; $x^*_{42} = 1$; hence, **P*** is in the form (Table 4).

Table 4. Vector P*

P* =	**2**	**4**	**3**	**1**

The illustration of the vector P* is shown (Figure 1).

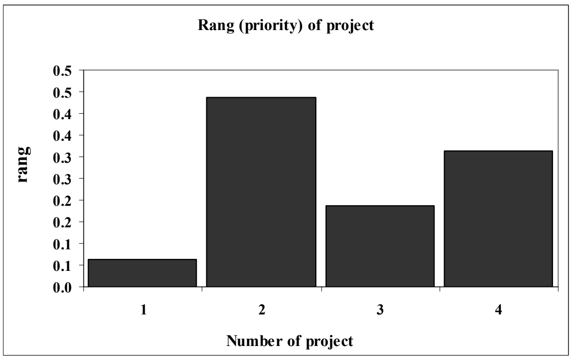

Fig 1. Ranking of the projects

Fig 1 gives the information about the best project (second project) under free parameters.

4. Conclusions

1. More adequate ranging on a degree of importance using a method of ranging by means of median distributions is received. Thus it is possible to use expert estimations of parameters of the considered projects received by a traditional expert method. The investigation of influence of separate groups of parameters on ranging of investment projects on a degree of importance is possible.

2. The offered algorithm of ranging of objects on set of parameters is constructive. It is enough easy to realize the offered algorithm in the environment of program MS Excel.

3. Manager (decision maker) can use a graphic illustration of a vector of priorities. By means of an offered graphic illustration it is easy to make a decision on importance of the considered investment project.

4. The authors show the possibility of using of median's distribution at formation of strategy of investment of the enterprise**.**

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