# COMPUTATION OF BRIDGE BEAM'S STRENGTHENING APPLYING CLASSICAL LAMINATE THEORY

**F. Bulavs, I. Radinsh, N. Tirans**
*Riga Technical University (Latvia)*

*Received ; accepted*

**Abstract.** The use of externally bonded carbon fiber reinforced polymers (CFRP) is a promising new technology of increasing flexural and shear capacities of reinforced concrete members. The calculation methods, which are given in building codes, contain simplifications and the safety reserves. There are difficult to determine the safety reserves, which can be in range from a tenth till fifteen percents (and in a special cases more). The proposed calculation method, which is based on classical laminate theory, gives more exact stress distribution diagrams in the whole reconstructed beams cross section. The method is successfully applied in the reinforced concrete bridge reconstruction above the channel Varkali in Latvija using externally bonded CFRP materials.

**Keywords:** beam, deflection, reinforced concrete, calculation methods, carbon plates, crack propagation, bridge beam

## 1. Introduction

Most of constructive building elements are heterogeneous and asymmetric. It is typically inherent in cases of strengthened reinforced concrete elements. Because of the asymmetric structure complicated stress and deformation relationships develop. There is necessity to develop a calculation method for determination of strength and stiffness of concrete elements, reinforced with steel rebars and composites, undergoing bending actions, considering the changing structural and deformative features of element during its loading, and uses this method for calculation of particular operating structure reinforcement.

Heterogeneous, yet longitudinally oriented reinforced concrete elements have a lamellar structure, therefore for calculations of such elements the calculation models of lamellar structure can be used. Hereby the hypotheses and solutions of classical laminate theory can be used in these calculations.

## 2. A stress distribution calculation in laminate beams

Classical laminate theory [1,2] practically usable equations can be obtained using multiple hypotheses for deformations of individual layers and the whole bar.

Geometric location of particular layers plays significant role in lamellar structure bar mechanics. To escape from a possibility for the bar to gain unfavorable deformations, for example, twist, the structure of the bars is frequently created symmetric against its central plane. To most efficiently use the potential of determined layers, asymmetric setup is frequently created, i.e. relatively to the central plane layers are placed with different deformative features. This situation can also be originated if the reinforced concrete element is additionally reinforced with bonded composites to the tension plane of the cross section (fig.1.) [3,4,5]. An important feature of composite reinforced beams is that formation structure is changing during their loading of the element. The origin for this change is cracks formation and development in concrete in tension zone.

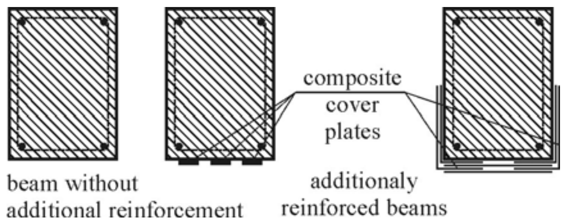



**Fig. 1.** Additional reinforcement from composite cover plates on reinforced concrete beams.

Because the crack originated on the tension plane is continuously developing of the tension stresses, into the direction of the compression zone bearing part of cross section is continuously diminishing. As a result of this process, continuous redistribution of stresses occurs both between the components of the bar and in concrete [6,7]. Due to the load increase the stresses significantly exceed the limit of linear deformations in the greatest part of concrete. Usage of many layers in the lamellar material mechanics bar model the precision of calculation results is magnified.

If equilibrium equations are written for each layer using their physical equations, the diaphragm, mixed and bending stiffness can be determined.

In case of undetermined number of layers n, the stiffness can be written as follows:

$$A=\sum_{i=1}^{n} b_i E_i h_i = \sum_{i=1}^{n} E_i F_i \qquad (1)$$

$$B=\frac{1}{2}\sum_{i=1}^{n} E_i F_i \left(y_i + y_{i-1}\right)\left(2y_i - h_i\right) - y_n A; \qquad (2)$$

$$D=\frac{1}{3}\sum_{i=1}^{n} E_i F_i \left(3y_i\left(2y_i - h_i\right) + h_i^{2}\right) - - y_n\left(2B + y_n A\right). \qquad (3)$$

These equations contain conventional signs: $F_i$ - the cross sectional area of layer i; $h_i$ $b_i$ - i-the width and depth of layer i; $y_i$ - the maximal coordinate of limiting surface of layer i, $y_n$ – the location of neutral axis of lamellar material set, $E_i$ – deformative constant of layer i.

Using the provision that in case of linear axial loading of the bar, there is no bending, the condition can be obtained that the mixed stiffness equals zero ( $B=0$ ). Location of neutral axis of lamellar material bending element can be determined using this condition:

$$y_n = \frac{\sum_{i=1}^{n} b_i E_i \left(y_i^{2} - y_{i-1}^{2}\right)}{2\cdot\sum_{i=1}^{n} b_i E_i \left(y_i - y_{i-1}\right)} \qquad (4)$$

The advantage of classical laminate theories calculation is the possibility to define individual deformative features for each layer, thus the nonlinear deformative nature of concrete can be taken into consideration. It is especially important because the distribution of stresses in cross section is changing both because of changing loads and cracking of tension concrete.

If we analyze the stress distribution and deflections of reinforced concrete beams with relatively low level of loads a linear continuous lamellar calculation model is valid, it is frequently used for forecasting of lamellar structure deformative feature determination. Unfortunately, usage of this model for calculations for reinforced concrete elements is restricted with very low limits due to the very early cracking of tension concrete. Cracks originate when the tension stresses rise till the tension resistance of concrete, which, depending on the class of concrete differs from 1/7 to 1/15 of the compression strength.

In combined beam case with bending plane parallel layer structure, created by different materials, the reduced cross section method is frequently used. According to this method, the composite beam cross section is replaced with an equivalent one material cross section. These two cross sections can be called equivalent if the location of neutral axis and the bending resistance are the same. The reduced cross section method usage for reinforced concrete elements strengthened with composites is limited. It can be used only in the elastic deformation stage of all materials, i.e. till the appearance of first cracks. Therefore the calculation model using classical laminate theory was created using the following assumptions – the layers reinforced with steel reinforcement bars are replaced with pure metal layers, concrete located between the bars is divided to both sides of steel layer.

The practical realization of method is carried out using numerical method by creating electronic spreadsheets. Necessary results are obtained with step by step iterations repeating the calculations with adjusted features of layers.

Practically the calculation for reinforced concrete beam with lamellar material mechanics model can be carried out in following way:

1. Particular cross section of the beam is replaced by parallel different width layer set. Number and thickness of layers is determined by the user. The stiffness $E_iF_i$ of the layers depends on the geometry (area) $F_i$ and deformative constant $E_i$.

The layers containing steel reinforcement bars are included in the set of layers and replaced with layers of such width that their area is equal to the total area of reinforcement bars.

2. The algorithm of classical laminate theories calculation is applied to the lamellar structure model of reinforced concrete beam and distribution of stresses in components is obtained.

The obtained result is relevant to the loading situation when none of the tension stresses in the concrete exceed the limits of linear deformation in tension. Results can be displayed in numerical or diagram format.

3. In cases when a stress larger than the limit of linear deformation in tension is reached in any layer, a repeated calculation is performed with adjusted deformation modulus for this layer. This calculation is repeated multiple times, until the required accuracy is obtained. This calculation is performed for a determined level of bending actions.

4. In cases that in any concrete layer the stress reaches its tension resistance, a crack develops. This situation is modeled with an immediate change in value of deformation module for this layer for the next iteration – the deformation modulus value is equaled to zero (the layer is excluded from the calculation) and a repeated calculation is performed.

5. In cases that stress in compressed concrete reach its resistance, a failure is recognized.

6. In cases that external stresses in reinforcement bars reach their yield strength the reinforced concrete beam is recognized as failed.

7. Stresses exceeding yield strength in reinforcement bars for reinforced concrete elements reinforced with composites loaded with static loads can not be called as a failure phase. If the calculation problem allows yielding of metal, a deformative model permitting yielding for metal can be included in the calculation. Ultimate resistance of beam in this case is reached when either composite material fails in tension or concrete fails in compression.

## 3. Critical conditions of a beam

Due to the crack formation, continuous displacement of the neutral axis and reduction of bending stiffness of the beam occur. Stresses in uncracked layers are increasing. Limits of the crack formation process are reached when develops critical condition in any of the components. It may occur when the stresses in the compressed zone of the beam reaches compression strength of a concrete. As a result, disintegration of the concrete practically goes on along the whole cross-section of the beam. Since $\sigma_{\max b} = \frac{E_{bi} M}{D_{pl}} y_n$, the critical bending moment value at which disintegration of the compressed concrete section begins, is determined by relationship

$$M_b = \frac{D_{pl}}{E_{bi} y_n} R_b^- \quad (5)$$

It is noteworthy that $E_{bi}$ is the characteristic value of deformative properties – stress and strain ratio in the extreme concrete layer. In the relationship (7) parameters $D_{pl}$ and $y_n$ have constant values applying to the bending moment value $M_b$. These can be defined in accordance with relationship (5) and (6) depending upon the structure of cross-section of the beam and mechanical properties of the components. Another possible critical condition of curved beam develops when the tensed steel reinforcement begins to yield.

The yielding may develop both in the tensed and compressed reinforcements. In either case the appropriate bending moment value is defined by equation:

$$M_T = \frac{D_{pl} R_T}{E_m y_n} \quad (6)$$

Beams with various structures, i.e., beam with various thicknesses of the strengthening layer, have different bending moment values at which the yielding of reinforcement begins. In beams with a thin reinforcement layer the tensed reinforcement is the one subjected to yielding, whereas if thickness of the supplementary layer is increased, critical condition of the beam develops either due to the yielding of the compressed reinforcement or rupture of the compressed concrete layer.

## 4. Results comparison with experimental data.

The presented calculation approach based on the model of laminated structure has been realized by means of a computing program, and it is applied for calculations of two types of reinforced concrete beams strengthened with carbon plastic layers. Cross-section of the first type beams is square with dimensions 200 x 200 mm, in which four steel reinforcements with ∅ 14 mm are enclosed. Characteristic values of mechanical properties of the components are shown in Table 1. 50 mm wide and 1,3 mm thick carbon plastic plates stuck to the surface of reinforced concrete beam by epoxy in three parallel layers are used for the strengthening of beams.

**Table 1.** Mechanical properties of the components

| Mechanical property | For first type beams | For second type beams |
|---|---|---|
| $E_b$, GPa | 25 | 25 |
| $R_b^+$, MPa | 2,6 | 1,9 |
| $R_b^-$, MPa | 33 | 30 |
| $E_s$, GPa | 200 | 200 |
| $R_{sT}$, MPa | 540 | 340 |
| $E_c$, GPa | 267 | 400 |
| $R_c^+$, MPa | 2900 | 3000 |
| Thickness of elementary layer of composite (mm) | 1,3 | 0,17 |

In accordance with the relationships (5) and (6) the stated critical bending moment values at which critical condition in one of the components of the combined beam develop, are summarized in Table 2. Parameter $n_c$ - estimates the number of elementary layers in the strengthening composite layer of the beam.

**Table 2.** Critical bending moment values for 1-st type beams.

| $n_c$ | $M_+$ kNm | $M_{T+}$ kNm | $M_c$ kNm | $M_{T-}$ kNm | $M_-$ kNm | $D_+$ kNm$^2$ | $D_k$ kNm$^2$ |
|---|---|---|---|---|---|---|---|
| 0 | 4,2 | 24 | - | - | - | 3,8 | 1 |
| 1 | 4,5 | 44 | - | - | 58 | 4,1 | 1,56 |
| 2 | 4,9 | 66 | - | - | 64 | 4,43 | 2 |
| 3 | 6,1 | 92 | - | - | 72 | 4,73 | 2,4 |
| 5 | 7 | - | - | 122 | 83 | 5,3 | 3,1 |
| 10 | 9,7 | - | - | 134 | 102 | 6,7 | 4,3 |

For the first type of unstrenghened beams, tension steel reinforcement yielding begins at the bending moment value $M_+$ = 24 kNm, which is followed by inadmissibly large deflections of beams (and consequently, the loss of load bearing capacity). With the same type of beams with one strengthening layer ($n_c$ = 1) and its thickness 1,3 mm, the tension steel reinforcement yielding starts at the bending moment value $M_{T+}$ = 44 kNm. However, it is not yet the loss of load bearing capacity of a beam. The beam carries on to take up the load, and further the strengthening layer is loaded more intensively. When the bending moment value M_= 58 kNm is reached, failure of the compressed concrete part of the beam occurs, and which is the reason of its load bearing capacity loss.

For beams with two strengthening layers ($n_c$ = 2), failure of the compressed zone of the beam at the bending moment value 64 kNm, and practically also a simultaneous ($M_{T+}$ =66 kNm) yielding of the tension steel reinforcement occur. As a result, load bearing capacity loss of the beam as well entails the failure of the compressed concrete. By increasing the number of strengthening layers ($n_c$ > 2), the reason of load bearing capacity loss of the beam does not change. So the increase of the number of strengthening layers for this type of beams leads up to over reinforcement effect.

Bending moment value $M_+$, at which the first cracking in the tensed concrete zone of the combined beam occurs, monotonously grows up along with the increment of the thickness of the composite layer. However, a conclusion can be drawn that increment of the crack resistance is rather trivial. So, for example, by increasing the thickness of the strengthening layer by 10 times the crack resistance increases only two times. The calculations show that practically critical condition of a beam at any possible number of strengthening layers develop owing to the rupture of the compressed concrete section at bending moment value M. It can also be counted as the loss of load carrying capacity of the beam. In case of small thickness of the strengthening layer ($n_c$=1) yielding of the tensed steel reinforcement occurs, which is followed by redistribution of stresses in the beam section. As a result, rupture of the compressed concrete begins which can be as well treated as the loss of load carrying capacity of the beam. The character of modification of stresses in the components of bent beams under the loading is demonstrated in Fig.2.

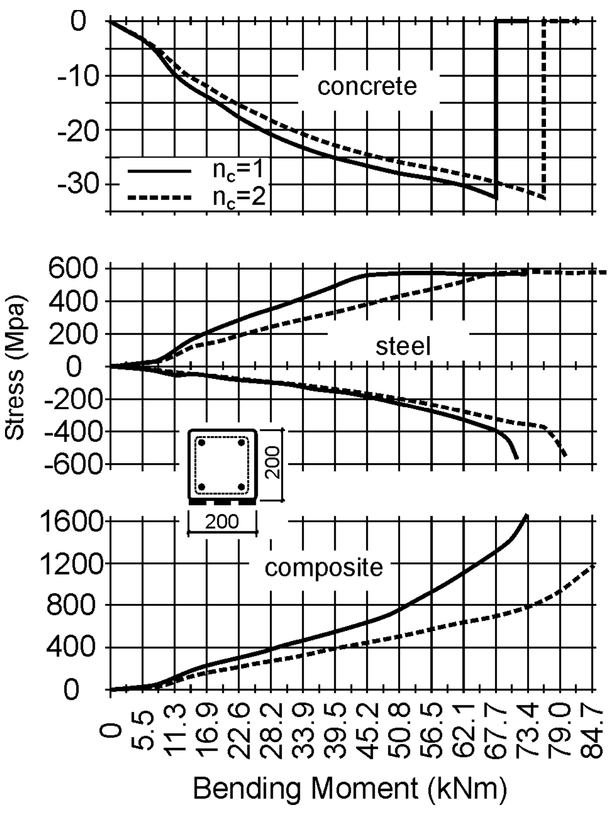



**Fig. 2** Character of modification of stresses in the components of curved (bent) beam

In compliance with the stated results, one can realise that composite material layers at the moment of rupture of the compressed concrete have particularly high strength reserve. Thickness of the strengthening layer may be regarded as optimal, if critical condition in the combined beam develops practically synchronously in some or even all the components. Also in case of thickness of two carbon plates the character of the loss of load carrying capacity of the beam is similar to the preceding one. Along with break-up of the compressed concrete section (M=64kNm), yielding of the tensed steel reinforcement develops, which is followed by yielding of the compressed reinforcement. The results represented testify that by increasing the thickness of the strengthening layer over five layers the critical bending moment value changes just slightly. Comparing the results of calculations with those obtained experimentally [8], which were carried out by loading of 2 m long beams in 4-point bending, we can draw the conclusion that the prognosticated critical value of of bending moment has not been achieved. In the cases of beams strengthened both with one and two carbon plate layers, delamination at the ends of strengthening layers occurred at the bending moment values that are smaller than the prognosticated strength. The critical bending moment value of beams strengthened with one carbon plate layer reached only 39 kNm, whereas that with two tape layers - 32 kNm. The attempt by the authors of the experiment to prevent delamination of the ends of plates by developing steel type tie bars did not provide an essential increment of critical bending moment value.

This value increased up to 42 kNm in a two-layered-plate variant. So, the critical bending moment value of reinforced concrete beams – 23 kNm - has additionally risen by 19 kNm. It makes up quite a small part from the prognosticated bending strength. Increase of the load carrying capacity of reinforced concrete beams strengthened with carbon plates can be achieved, if a solid monolith of the beam and strengthening layers is provided.

Cross-section dimensions of the 2-nd type reinforced concrete beams are 400 x 300 mm, in which five steel reinforcements with Ø 13 mm are enclosed; two of them are positioned in the compressed part, but three – in the tension part of the beam. Unidirectional carbon plate layers with 0,17 mm thick elementary layer along the beam′s surface have been used for its strengthening.. Mechanical properties of component parts of the beam are shown in Table 1. The prognosticated critical bending moment values are summarized in Table 3. The bending moment, at which the tension steel reinforcement yielding begins, may be assumed as the critical bending moment value (45 $kNm^2$) for unstrengthened second type reinforced concrete beam. Large beam deflections are observed at this bending moment value. Critical value $D_k$ =7,14 kNm of the bending stiffness is an indicator to the starting point of tension steel reinforcement yielding. If compared to the beam stiffness during the first concrete crack formation ($D_k$ =42,7 $kNm^2$), it has decreased by six times.

By using of unidirectionally reinforced high modulus carbon fiber reinforced plastic for strengthening of the second type of reinforced concrete beams, and depending on the number ($n_c$) of strengthening elementary layers, three different variants of load bearing capacity loss of beams are observed. Steel reinforcement yielding of beams begins with one, two and three composite layers ($n_c$ = 1...3) at the bending moment values $M_{T+}$ = 59,6kNm, $M_{T+}$ = 74,5kNm and $M_{T+}$ = 90kNm, accordingly. As a result, steel reinforcement yield point has increased by 1,3 to 2 times due to the strengthening composite layers. It is important to point out that steel reinforcement yielding does not yet entail an immediate loss of load bearing capacity of beams. A uniform deformation in beams proceeds up to the bending moment value $M_c$ , at which failure of a composite material occurs. These values are as follows: 107, 158, and 200 kNm, and can be assumed to be critical for beams.

The reason of load bearing capacity loss of beams changes, when thickness of composite material is increased, i.e., with the number $n_c$ of elementary layers from 4 to 10. In accordance with the data illustrated in table 3, the tension steel reinforcement yielding starts at such bending moment values - 106, 118 and 192 kNm. Beams continue to take up the load and only at the bending moment values 240, 260 and 310, when failure of the concrete concrete begins, beams are losing their load bearing capacity. Practically, failure of the concrete and the compressed steel reinforcement yielding are simultaneous.

**Table 3.** Critical bending moment values for 2-nd type beams

| $n_c$ | $M_+$ kNm | $M_{T+}$ kNm | $M_c$ kNm | $M_{T-}$ kNm | $M_-$ kNm | $D_+$ kNm$^2$ | $D_k$ kNm$^2$ |
|---|---|---|---|---|---|---|---|
| 0 | 17 | **45** | - | **-** | - | 42,7 | 7,14 |
| 1 | 17,4 | 59,6 | **107** | - | - | 43,5 | 4,5 |
| 2 | 18, | 74,5 | **158** | - | - | 44,3 | 6,6 |
| 3 | 19,2 | 90 | **200** | - | - | 45,1 | 8,5 |
| 4 | 20 | 106 | - | 250 | **240** | 46 | 13,2 |
| 5 | 20,7 | 118 | **-** | 278 | **260** | 46,7 | 16,5 |
| 10 | 22,5 | 192 | **-** | 320 | **310** | 50,4 | 19,4 |
| 20 | 27 | 345 | **-** | **350** | 370 | 57,2 | 31,9 |
| 30 | 30 | - | **-** | **412** | 430 | 63,5 | 39,8 |
| 40 | 40 | - | **-** | **420** | 460 | 69,2 | 46 |

## 5. Results comparison with results according building codes.

The stress calculations results according developed methodology and according Latvian building code LBN 203-97 are compared.

According Latvian building code LBN 203-97 reinforced concrete sections stress calculations are provided regarding following approximations:

- The stress diagram for compressed concrete is assumed rectangular.
- The stress limit (design strength) in concrete for calculation is decreased by different factors.
- Tensioned concrete in calculation is not considered.

According to stress balance equation $R_sA_s = R_cbx$, the depth of concrete compressed part is calculated from equation:

$$x = \frac{R_s A_s}{R_b b} \qquad (7)$$

where: x – the depth of concrete compressed part – distance between beams upper part and neutral axis; $R_s$ - design strength of reinforcement; $A_s$ - cross sectional area of reinforcement; $R_b$ - design strength of concrete; $b$ - width of beam.

As it is evident from equation (7), the depth of compressed concrete does not depend from load level. The assumption about rectangular stress diagram for compressed concrete is initially adopted.

The assumptions in building codes deform a bit the stress distribution in cross section of reinforced concrete, but in this result the calculations according building codes are simple and convenient for practical calculations. The deformed stress distribution is recompensed with safety factors.

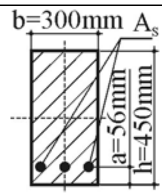
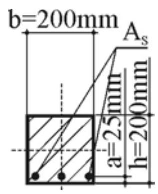
For all analyzed reinforced concrete cross sections following materials are used:

- concrete B30, $R_b$=17MPa according LBN 203 -97;
- reinforcement steel AIII (A400), $R_s$=365MPa according LBN 203-97.

The beams cross sections and the loads on these sections are given in table 4.

According developed laminate structures methodology and Latvian building code there are computed the depth of concrete compressed part and stress value in tensioned steel bars. The results are given in table 5.

**Table 4.** Beams cross sections and loads

| No. | Cross section b x h, mm reinforcement, mm | Illustration | Loading No. bending moment, kNm |
|---|---|---|---|
| 1. | 300 x 400 $A_s$ – 2Ø12 |   | 1.1. M=15,5 kNm |
| | | | 1.2. M=31,0 kNm |
| 2. | 300 x 400 $A_s$ – 5Ø12 | | 2.1. M=37,5 kNm |
| | | | 2.2. M=75,0 kNm |
| 3. | 300 x 400 $A_s$ – 3Ø32 | | 3.1. M=135 kNm |
| | | | 3.2. M=270 kNm |
| 4. | 200 x 200 $A_s$ – 2Ø10 |   | 4.1. M=4,5 kNm |
| | | | 4.2. M=9,0 kNm |
| 5. | 200 x 200 $A_s$ – 4Ø10 | | 5.1. M=9,0 kNm |
| | | | 5.2. M=18,0 kNm |
| 6. | 200 x 200 $A_s$ – 3Ø20 | | 6.1. M=21,0 kNm |
| | | | 6.2. M=42,0 kNm |

According the building code there are greater stresses in tensioned bars and deepest cracks.

To compare results, we chose the proportion of reinforcement in terms of cross section (%).

As it is seen in fig.3. the big disparity is proper for cross sections with low reinforcemnt ratio.

**Table 5.** Computation results and comparison

| loading No. | Computation results | | | | Results comparison | |
|---|---|---|---|---|---|---|
| | According developed methodology (SS) | | According LBN | | | |
| | $x_{SS}$, mm | $\sigma_{sSS}$, MPa | $x_{LBN}$, mm | $\sigma_{sLBN}$, MPa | $x_{LBN}/x_{SS}$ | $\sigma_{sSS}/\sigma_{sLBN}$, % |
| 1.1. | 56 | 182,05 | 16 | 177,57 | 3,50 | -2,5 |
| 1.2. | 57 | 364,67 | 16 | 355,14 | 3,56 | -2,7 |
| 2.1. | 88 | 178,96 | 41 | 177,42 | 2,15 | -0,9 |
| 2.2. | 87 | 364,37 | 41 | 354,85 | 2,12 | -2,7 |
| 3.1. | 159 | 163,70 | 173 | 181,87 | 0,92 | 10,0 |
| 3.2. | 169 | 334,17 | 173 | 363,73 | 0,98 | 8,1 |
| 4.1. | 38 | 172,87 | 17 | 171,99 | 2,24 | -0,5 |
| 4.2. | 38 | 352,94 | 17 | 343,98 | 2,24 | -2,6 |
| 5.1. | 51 | 179,78 | 34 | 181,12 | 1,50 | 0,7 |
| 5.2. | 52 | 364,60 | 34 | 362,32 | 1,53 | -0,6 |
| 6.1. | 78 | 150,30 | 101 | 179,10 | 0,77 | 16,1 |
| 6.2. | 83 | 306,65 | 101 | 358,20 | 0,82 | 14,4 |

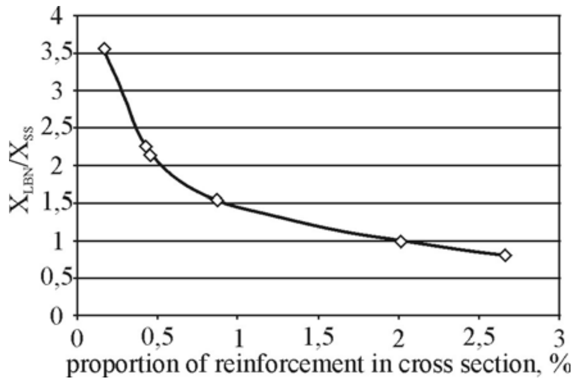



**Fig.3.** The depth of concrete compressed part correlation depending from reinforcement ratio

Increasing the reinforcement ratio, the results according developed methodology and building code come nearest. It can be explained with a condition, that building code does not appraise compressed concrete parabolic stress diagram. For cross sections with minimal reinforcement ratio this parabolic diagram is more like triangular not rectangular diagrams.

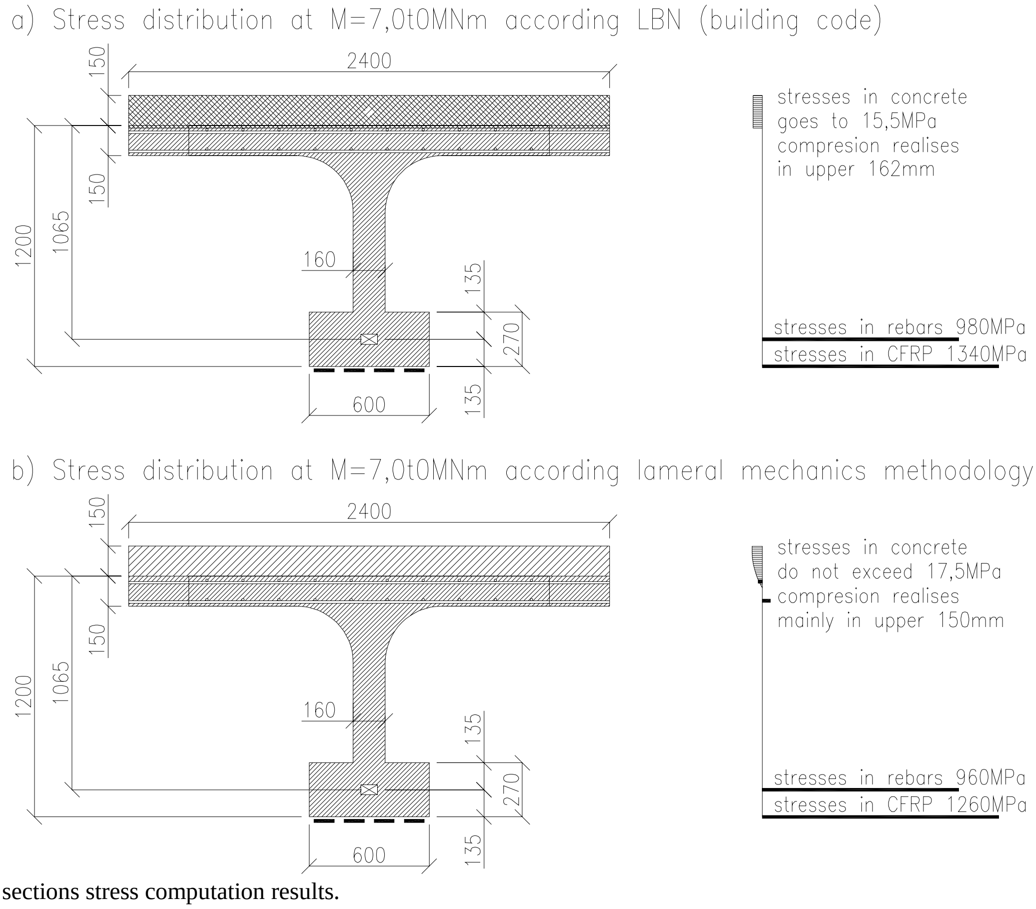



**Fig.4.** Cross sections stress computation results.

According building codes proposed methodology, the stresses are higher, the assumptions in building codes include additional not predictable safety factor.

The currently exploited beam cross section can withstand bending moment $M_{exist.}$=4,28 MNm . Because of required load bearing capacity means the bending moment is $M_{req}$=7,00 MNm, the beams are by 63% overloaded. Thus an increase in bending moment bearing capacity of $M_{reinf.}= M_{req} - M_{exist}$ = 7,00 - 4,28 = 2,72 MNm must be secured.

Using the lamellar material mechanics calculation model a required amount of additional reinforcement was determined. Eight carbon fiber polymer strips, h=1,4mm, width 120mm, have to be bonded in two layers in four parallel rows 5cm distant from one other.

The cross sections stress computation results according Latvian building code and according developed lamellar material mechanics calculation method in strengthened reinforced concrete beam are shown in fig.4.

Developed methodology gives result, that in compressed concrete stresses will be 12,9% higher, but in rebars and composite cover plates 6% smaller, comparing with results according LBN (building code).

## 6. Conclusions

Using composite cover plates, there are good possibilities to strengthen reinforced concrete beams.

Depending from cross sections parameters (materials, reinforcement ratio ect.) the cross sections calculation methodology include non predictable safety factor, which can be in spectrum from a few till 15 and in some cases more.

A new calculation method for computing reinforced concrete building constructions with external reinforcement has been created using basic principles of classical laminate theory for package of layers with significantly differing physically mechanical features, which gives possibility to calculate most efficient amount of cover plates for strengthened beams.

The practical realization of method is carried out using numerical method by creating electronic spreadsheets, the results obtained theoretically are compared to experimental values for reinforced concrete beams with and without external reinforcement.

The methodology allows predicting redistribution of stresses in combined beam components during the development of cracks and deformations.

It is possible to determine the most rational amount of additional reinforcement for a particular size concrete beam with particular steel reinforcement bars. Strength margin can be determined in particular level of loading; failure of composite element can be predicted.

## References


1. R. M. Jones, Mechanics of composite materials [USA], Scripta Book Company, Washington (1975).
2. V.V.Vasiliev, Mechanics of constructions of composite materials [in Russian], Engineering, Moscow (1988).
3. DotDuthich, Monica Starnes. Strength and Ductility of Concrete Beams Reinforced with Carbon FRP and Steel. US Department of Commerce NISTIR, 2001, 44 p.
4. N.F.Graca, G.A. Sayed, A.K.Soliman, K.R.Salch. Strengthening Reinforced Concrete Beams Using Fiber Reinforced Polymer (FRP) Laminates. ACI Structural Journal, 1999, pp. 865 – 874.
5. F.Bencardino, G.Spadea, R.N.Swamy. Strength and Ductility of reinforced Concrete beams Externally Reinforced with Carbon Fiber Fabric. ACI Structural Journal, 2002, pp. 163 – 171..
6. A.Skudra, F.Bulavs, I.Radinsh, N.Tirans. Regulation of Deflections of Homogeneous beams. Latvian Journal of Physics and Technical Sciences, 2003, p. 51 – 55.
7. F.Bulavs, I.Radinsh, N.Tirans. Method of prediction of the deflections of reinforced concrete beams considering cracking. Scientific proceedings of Riga Technical University. Architecture and construction science. RTU, Riga, 2003.
8. M. Arduini, A. Di Tommaso, A. Nanni. “Brittle failure in FRP plate and sheet bonded beams”, ACI Structural Journal, V. 94, No 4, July – August 1997., pp. 363- 370.



Felikss Bulavs is a professor of the Department of Structural Analysis of Riga Technical universitv. His scientific interests are related with composite materials mechanics, and application of these materials in quality improvement of the building produce.

Ivars Radinsh is the head of the Department of Structural Analysis of RTU, an associate professor. His scientific interests are related with composite materials mechanics.

Normunds Tirans is a doctoral student of RTU and prepared to defend thesis on the subject "Load-bearing capacitv improvement possibilities (options) of reinforced concrete beams by strengthening them with carbon-filled plastic tapes”.

**Felikss Bulavs**, Dr.habil.sc.ing., professor

Riga Technical University, Faculty of Civil Engineering

Address: Azenes street 16, LV 1048, Latvia

Phone: + 3717089262; Fax + 3717089121

**Ivars Radinsh,** Dr.sc.ing., asoc.professor

Riga Technical University, Faculty of Civil Engineering

Address: Azenes street 16, LV 1048, Latvia

Phone: + 3717089262; Fax + 3717089121

**Normunds Tirans**, master of civil engineering

Riga Technical University, Faculty of Civil Engineering
Address: Azenes street 16, LV 1048, Latvia
Phone: + 3717089262; Fax + 3717089121