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THE OPTIMIZATION OF GEOMETRIC PARAMETERS FOR MANSARD DESIGN

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ABSTRACT

Efficient use of attic area or mansard is determined by proper usage of the slope angle of roof planes. The paper deals with the determination of values of geometric parameters for optimal design of mansard in the buildings with gable roof during both building renovation and planning a new design. The optimization analysis regarding the useful floor area or available mansard volume is performed with respect to the angle of the slope of roof planes. Obtained nomograms will allow architects and customers to make the final decision on building's roof concept at the early design stage based on both economic considerations and architectonic impressions.

KEYWORDS: Roof Construction, Mansard Design, Optimization

INTRODUCTION

The shape and form of the roof plays an important role especially in planning the design and construction of residential buildings. The type of the roof and its geometric parameters are selected based on architectural requirements, climate, purpose of the mansard room, and roofing material properties. Choosing the right roof form depends on the number of roof planes and their slope angles [1-2].

The idea to effectively use attic area in buildings dates back to the year 1630, when it was offered and popularized in France by a French architect *François Mansart* [3-4]. In fact, the reason for this was that French houses were taxed by the number of floors below the roof. Therefore the feature

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had an added benefit of exempting the top floor from taxation.

1. OPTIMIZATION OF ROOF DESIGN

Rational planning of mansard in buildings with gable roof (two single slope planes) presumes correct selection of the slope angle [5]. Geometric parameters are considered in this study while the selection is presented in Fig. 1.

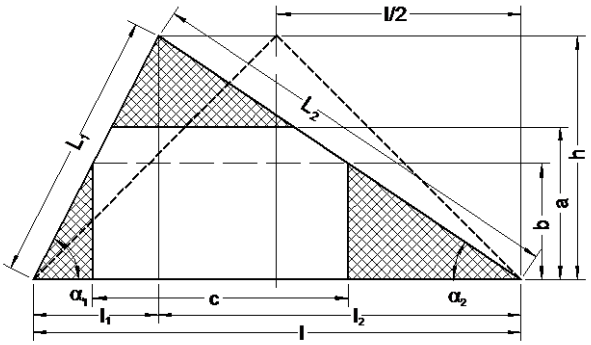

Fig. 1. Geometric parametrs of mansard

Regardless of the mansard depth the useful volume under the gable roof will be characterized by efficient cross section F:

$$F = \left(a - \frac{a^2 - b^2}{2h} \right) \cdot l,$$

the floor area by its width c

$$c = \left(1 - \frac{b}{h} \right) \cdot l,$$

but the roof area by the sum of the length of two slope planes L_1 and L_2

$$L = L_1 + L_2 = \left(\sqrt{1 + \cot^2 \alpha_1} + \sqrt{1 + \cot^2 \alpha_2} \right) \cdot h .$$

In these equations $h = l_1 \tan \alpha_1$ or $h = l_2 \tan \alpha_2$.

As follows from these expressions, both the useful volume and floor area of the mansard do not vary with respect to the relocation of the roof ridge to the right or to the left from the axis of symmetry at *l*/2 (Fig. 1). In turn, the least roof area will result when the roof ridge is in the middle or $\alpha_1 = \alpha_2$.

Let us first determine the roof plane slope angle when the most useful mansard volume is obtained. For the generalization of the calculation let us introduce the following parameters

$$v_i = \frac{b_i}{a} \text{ and } u_i = \frac{l_i}{a},$$

where $i = 1$ refers to the left side of the roof and $i = 2$ to the right side of the roof. Let us consider the ratio between the useful area and the total area of the mansard, and carry on the derivation of α_1 to locate the maximum. The slope angle α_1 may be obtained from

$$\cot \alpha_1 = \frac{u_i}{1 + v_i^2}. \quad (1)$$

This expression may be used for the determination of the optimal slope angle of both the right and left sides of the roof planes according to the useful volume at $i = 1$ and $i = 2$. Depending on the selected design parameters v_i and u_i presented in Fig. 2 there is a nomogram for the determination of α_1.

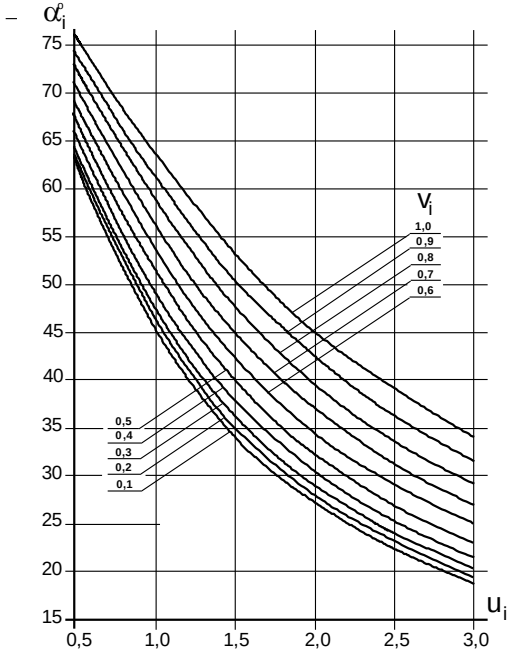

Fig. 2. Nomogram for the determination of the optimal roof plane slope angle according to useful mansard volume

Looking for the optimal calculation scheme of the mansard, the ratio between the mansard floor and roof areas should be considered as well. Finding the maximum for this ratio an equation of order 3 is obtained:

$$\cot^3 \alpha_i + 2\cot \alpha_i - \frac{u_i}{v_i} = 0. \qquad (2)$$

Using Cardano's cubic formula we find [6] that

$$\cot \alpha_i = \sqrt[3]{\sqrt{\frac{8}{27} + \left(\frac{u_i}{2v_i}\right)^2} + \frac{u_i}{2v_i}} - \sqrt[3]{\sqrt{\frac{8}{27} + \left(\frac{u_i}{2v_i}\right)^2} - \frac{u_i}{2v_i}}. \qquad (3)$$

In this case between parameters v_i and u_i there is a relation

$$v_i = u_i\left(\cot^3 \alpha_i + 2\cot \alpha_i\right).$$

Fig. 3 represents a nomogram for the determination of optimal α_i value according to maximal ratio between the floor and roof area for the mansard.

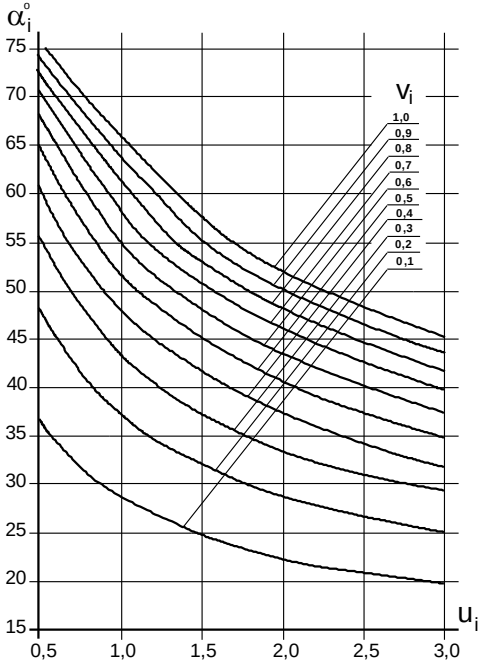

Fig. 3. Nomogram for the determination of slope angle α_i for roof planes according to optimal floor and roof area ratio

To find out those geometric parameters of the mansard design which result in at the same time the least roof area of mansard and the maximal useful volume and floor area, we combine equations (1) and (2):

$$\frac{u_i^3}{\left(1+v_i^2\right)^3}+2\frac{u_i}{1+v_i^2}-\frac{u_i}{v_i}=0 \qquad (4)$$

and calculate most reasonable values (Table 1).

Table 1. The values of design parameters v_i and u_i.

u_i	0,500	1,000	1,500	2,000	2,500	3,000
v_i	0,721	0,444	0,272	0,179	0,126	0,093

Then optimal slope angle α_1 for the roof planes may be calculated according to equations (1) and (3). Obtained nomogram (Fig. 4) allows architects and customers to make the final decision [7] based on the roof concept of a building at an early design stage from economic point of view.

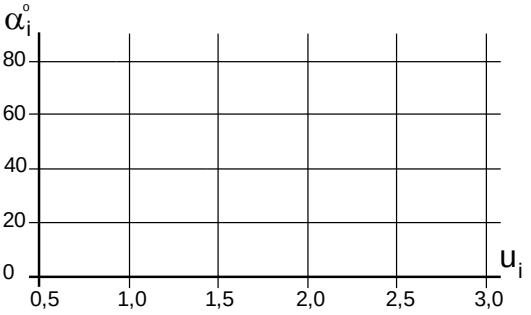

Fig. 4. Nomogram for the determination of optimal slope angle α_1 of the roof plane according to useful volume and ratio between the floor and roof area in mansard

2. CONCLUSIONS

- The suggested method allows the determination of optimal slope angle of roof planes according to selected geometric parameters of the mansard at the early design stage.
- The suggested method allows the determination of optimal geometric parameters of the mansard during renovation of the existing building and while planning a new design of mansard.
- The useful mansard volume and floor area do not depend on relocation of the roof ridge from the axis of symmetry to either side.
- The least roof area is a result of the symmetric location of roof ridge.

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