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GEOMETRIC PARAMETERIZATION OF A TETRAPOD-SHAPED STRUCTURAL ELEMENT

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ABSTRACT

Two discoveries allow fundamentally diverse structural systems with hexagon-type lattice to be offered for the synthesis of minimum mass and maximum stiffness constructions. The highest efficiency of both properties could be achieved using hexagon-type spatial rigid bar structures with tetrapod-shaped joints, referred to as superelements. Geometric parameterization, required for shape optimization as the chosen means for realising the task set, will be considered.
The main tasks for defining the parameterization include exact definition of the geometry to be parameterized, choosing the most appropriate independent shape parameters and determination of the parameter constraints to adhere.

INTRODUCTION

Two discoveries allow fundamentally diverse structural systems with hexagon-type lattice to be offered for the synthesis of minimum mass and maximum stiffness constructions [1-3]. The first lies within structural mechanics for the solving of an optimal lattice problem and the second within structural geometry for the solving of an optimal two dimensional multiscale tessellation problem. The highest efficiency of both mechanical and geometrical properties could be achieved using hexagon-type spatial rigid bar structures with tetrapod-shaped joints, further jointly referred to as superelements. Due to lightweight considerations, the superelements could be implemented as shell structures.

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1. INITIAL CONSIDERATIONS

In order to realize the goals set above, a mechanically determined geometry optimization of the superelements should be carried out. For this task, one option beside topology and parameter optimization in general is the shape optimization [5], the parameterization for which will be considered in this work.

Shape optimization requires certain assumptions regarding the expected optimum shape of the geometry to be optimized. Since the assembled structure of the superelements is supposed to be a continuous structure, the first shape assumption is that there are no discontinuities on the frame surfaces present [6]. This implies in-between tangentially of the superelement outer surfaces at element contact points. Another assumption is that each tetrapod leg has a revolution surface shape because no prior loading directions are assumed. Additional assumption regarding the shape of legs is that the outer leg diameter should become larger towards the element center because of supposedly increased stress concentration in the joint part of the elements.

2. SHAPE SIMPLIFICATION CONSIDERATIONS

Generally, all the shape assumptions can be met by implementing NURBS curves and surfaces. However, no specific information regarding the optimum superelement shape being available at the moment, it was chosen to describe the element shape by possibly the simplest geometric shapes [4], which would allow easy parameterization and would conform to the initial shape assumptions. The chosen shapes include a sphere in the center of the element, which is joined through constant radius fillets, corresponding to toroidal surfaces, with the four legs, represented by conical surfaces, which are likewise being joined with the legs of other elements by constant radius fillet transitions (Fig. 1). Such element geometry parameterization model corresponds to a set of five independent parameters subject to constraints imposed by the initial shape assumptions.

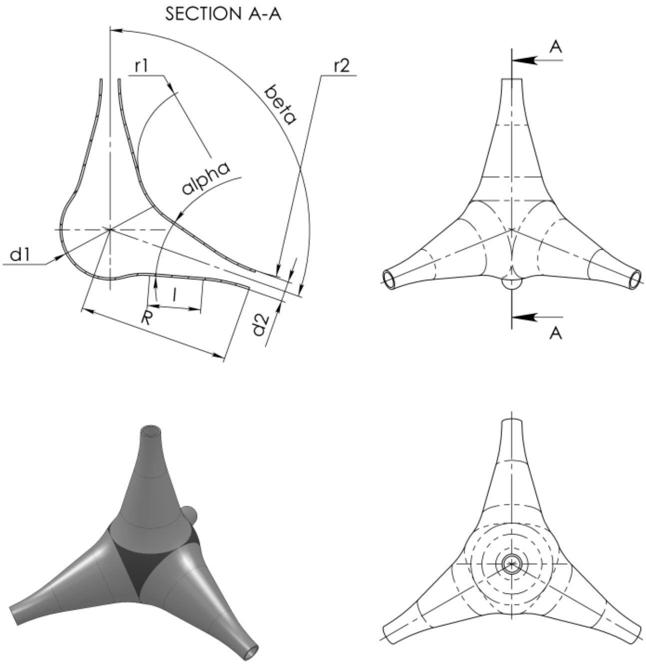

Fig. 1. Superelement shape approximation model

3. PARAMETER CHOICE AND DEPENDENCIES

Since the choice of parameters was arbitrary, it was decided to include parameters which would have either mechanical or structural meaning or explanation (Fig. 1). The primary structural dimension is the length of tetrapod legs, termed R. The next important dimension is the diameter of tetrapod legs d_2 at the contact point between two superelements, which determines in great respect the mechanical stiffness in the element transition region. The initial guess value d_2 could be defined as a function of R. Further, from the viewpoint of mechanical stress concentration important are both transition fillet radiuses r_1 and r_2, corresponding to sphere-cone and cone-cone, i.e., element-element, transitions, respectively. The initial guess values r_1 and r_2 could be expressed as a function of d_2. The last independent parameter could be either the central sphere diameter d_1, or the aperture of the conical surface α, both having to fulfil the requirement for diameter increase in element center direction. To provide a mechanically meaningful interpretation the parameter α was chosen as the fifth parameter.

4. PARAMETER CONSTRAINTS

Beside the proposed interrelations of chosen geometry parameters there are geometry driven constraints, which have to be fulfilled in order to make the parameter values compatible and mechanical constraints due to shape assumptions. In general case these result in three conditions.

The first one concerns the size of the truncated portion of the conical surface generatrix l, which should be greater or equal to zero (Fig. 1). It is expressed by the following equation:

$$l = -\left(\sin(\alpha)\cdot r_2 \sin\left(\frac{1}{2}\beta\right)\right) - \sin(\alpha)\cdot r_1 \sin\left(\frac{1}{2}\beta\right) + \cos\left(\frac{1}{2}\beta\right)\cdot d_2 + \\ + \cos\left(\frac{1}{2}\beta\right)\cdot r_2 - \cos\left(\frac{1}{2}\beta\right)\cdot r_2 \cos(\alpha) + \cos\left(\frac{1}{2}\beta\right)\cdot r_1 \cos(\alpha) - \\ - R\sin\left(\frac{1}{2}\beta\right)\Big) \Big/ \left(\cos(\alpha)\sin\left(\frac{1}{2}\right)\beta + \cos\left(\frac{1}{2}\right)\beta \sin(\alpha)\right). \tag{1}$$

The second one requires α being nonnegative in order to satisfy the requirement for outer leg diameter increase in element center direction. In fact, the minimum α value case corresponds to r_2 being equal to infinity (Fig. 2), so that the contact point tangentiallity condition would be met. This value of α also allows for another limit case when l is being equal to infinity.

The third constraint relates to the maximum α value, which is determined by the symmetry angle between the tetrapod legs β ($2\arccos(1/\sqrt{3}) \approx 109.5°$) and corresponds to r_1 being equal to infinity. In this case the conical surface is tangential to the central sphere (Fig. 3), allowing the maximum α value to be expressed as follows:

$$\alpha_{\max} = 90^\circ - \frac{1}{2}\beta. \tag{2}$$

Another limit case corresponds to l being equal to zero and r_1 and r_2 having equal values r, which results in a single fillet transition between the central spheres of two elements. This provides the following parameter coupling relation:

$$d_2 = 2\left(\tan\left(\frac{1}{2}\beta\right)R - r\right). \tag{3}$$

The chosen geometry parameters, their constraints and interdependencies are summarized in Table 1.

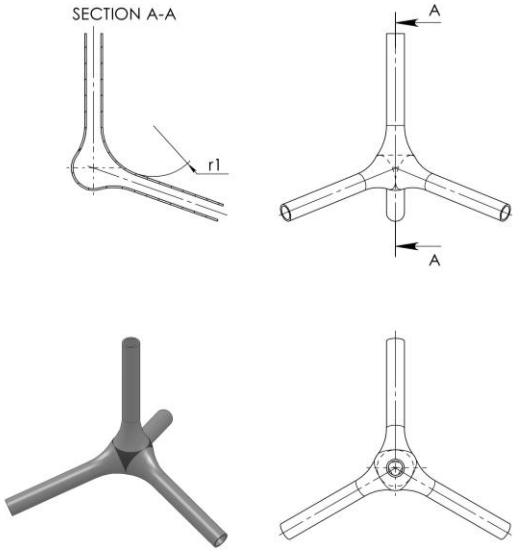

Fig. 2. Superelement shape corresponding to the minimum α value

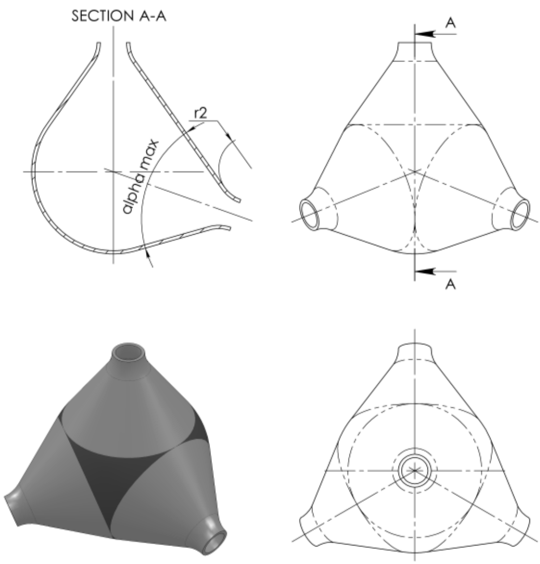

Fig. 3. Superelement shape corresponding to the maximum α value

Table 1. Constraints and interdependencies of superelement geometry parameters

Parameter	Description	Constraints
R	Length of tetrapod legs	$R > 0$
d_2	Diameter of tetrapod legs at the contact point	$l \geq 0$
α	Aperture of the conical surface	$0 < \alpha < \alpha_{max}(\beta)$
r_1	Sphere-cone transition fillet radius	$l \geq 0$
r_2	Cone-cone transition fillet radius	$l \geq 0$
l*	Length of conical surface generatrix	-
	* dependent parameter	

5. CONCLUSIONS

A structural tetrapod-shaped superelement, inspired by the rotationally symmetric four-legged structures, has been found as an option for lightweight structures. In order to obtain an optimized rigid shell structure, certain assumptions regarding its geometry with reference to mechanical considerations have to be made. The assumptions can be met by a parameterized model, built of elementary geometric shapes. In addition, the exact parameters can be selected according to their mechanical importance, finally arriving at a set of five independent parameters, which are bound by geometric and mechanical constraints.

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