*Proceedings of the 9th International Conference "Reliability and Statistics in Transportation and Communication" (RelStat'09), 21–24 October 2009, Riga, Latvia, p. 55-62. ISBN 978-9984-818-21-4*
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# ON A MODEL OF NONRECURRENT FLOW OF CLAIMS

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The paper describes a model of nonrecurrent flow and its main characteristics. The model allows describing flows with positive or negative correlation between adjacent inter-arrival times. A single server queueing system with such an input flow has been considered as application of the suggested model. Numerical results show that the dependence between inter-arrival times exercises great influence on efficiency characteristics of the service.

***Keywords:*** *nonrecurrent flow, dependence, queueing system*

## 1. Introduction

An important element of a probabilistic model is a description of considered random variable distributions. It is supposed usually that all random variables are independent. But numerous statistical data prove the opposite. For example, it has been experimentally stated that characteristics of Internet flows are dependent ones [6, 7, 9]. Analogously, flows of insurance claims for damages have dependent structure [2-5, 10]. In the first case, a correlation between inter-arrivals of the claims is described by so called *Batch Markovian Arrival Process*, where a claim circulates in some markovian network before an arrival [11]. In the second case, *copulas* are used usually for a description of the dependence [3, 5, 8].

In our paper another approach is used. We suppose that inter-arrival times of flows claims correspond to a certain Markov chain with a set of possible values ***I*** = [0, 1]. Note that the last condition is not very restricted because a suitable time scaling can fulfil it.

The paper proceeds as follows. Section 2 establishes the proposed model of claim flows and describes their properties. This model allows us to get flows with positive or negative correlation between adjacent inter-arrival times. In Section 3 this model is used as an input flow for a single server queueing system. Section 4 provides numerical results to compare various flows influence on the service efficiency. It is shown that this influence is very remarkable. Final remarks are given in Section 5.

## 2. Flows description and analysis

A considered flow of claims is received by random variables, which describe interval lengths between claims arrivals. We suppose that this sequence forms a Markov's chain. If $Z$ and $X$ are lengths of previous and current intervals, then a corresponding transition probability density is $q(x/z)$. We assume the following form of the density:

$$q(x|z) = \frac{1}{1-\exp(-\lambda)}\left(\delta(z)\lambda e^{-\lambda x} + \left(1-\delta(z)\right)\lambda e^{-\lambda(1-x)}\right),\ 0 < z, x < 1, \tag{1}$$

where $\lambda > 0$ is a parameter,

$\delta(z)$ has one of two possible forms $\delta(z) = z$ or $q(z) = 1 - z$.

One can see that there is a mixture of two densities. With probability $\delta(z)$ we have random variables, which are close to zero, and with opposite probability $1 - \delta(z)$ – random variable, which is close to one. By that, $z$ is the value of the length of the previous interval. So, the case $\delta(z) = 1 - z$ gives a positive correlation between the adjacent intervals, and the case $\delta(z) = z$ gives a negative correlation.
The conditional distribution function of $X$ given $Z = z$ is the following:

$$Q(x|z) = P\{X \le x | Z = z\} = \frac{1}{1-\exp(-\lambda)}\left(\delta(z)\left(1-e^{-\lambda x}\right) + \left(1-\delta(z)\right)e^{-\lambda}\left(e^{\lambda x}-1\right)\right), \quad (2)$$

$$0 < z, x < 1,$$

*Lemma 1.* Conditional mean value *X* of interval's length between arrivals given *Z* = *z* equals to

$$E(X|z) = \frac{1}{\lambda(1-\exp(-\lambda))}\left(\delta(z)\left(1-e^{-\lambda}-\lambda e^{-\lambda}\right) + \left(1-\delta(z)\right)e^{-\lambda}\left(1-e^{\lambda}+\lambda e^{\lambda}\right)\right). \quad (3)$$

Proof:

$$E(X|z) = \int_0^1 \left[1-Q(x|z)\right]dx =$$

$$= \int_0^1 \left[1 - \frac{1}{1-\exp(-\lambda)}\left(\delta(z)\left(1-e^{-\lambda x}\right) + \left(1-\delta(z)\right)e^{-\lambda}\left(e^{\lambda x}-1\right)\right)\right]dx =$$

$$= 1 - \frac{1}{1-\exp(-\lambda)}\left(\delta(z)\left(1-\frac{1}{\lambda}\left(1-e^{-\lambda}\right)\right) + \left(1-\delta(z)\right)e^{-\lambda}\left(\frac{1}{\lambda}\left(e^{\lambda}-1\right)-1\right)\right) =$$

$$= 1 - \frac{1}{\lambda(1-\exp(-\lambda))}\left(\delta(z)\left(\lambda-\left(1-e^{-\lambda}\right)\right) + \left(1-\delta(z)\right)e^{-\lambda}\left(\left(e^{\lambda}-1\right)-\lambda\right)\right) =$$

$$= \frac{1}{\lambda(1-\exp(-\lambda))}\left\{\lambda - 1 + e^{-\lambda} - \delta(z)\left(\lambda-\left(1-e^{-\lambda}\right)-e^{-\lambda}\left(\left(e^{\lambda}-1\right)-\lambda\right)\right)\right\} =$$

$$= \frac{1}{\lambda(1-\exp(-\lambda))}\left\{\lambda - 1 + e^{-\lambda} - \delta(z)\left(\lambda-2\left(1-e^{-\lambda}\right)+e^{-\lambda}\lambda\right)\right\}.$$

Let us find the stationary probability density *f(x)* for the interval's length *X*. For that, we have an equation

$$f(x) = \int_0^1 f(z)q(x|z)dz, \quad x \ge 0. \quad (4)$$

Substitution of (1) gives the following equation:

$$f(x) = \int_0^1 f(z)\frac{1}{1-\exp(-\lambda)}\left(\delta(z)\lambda e^{-\lambda x} + \left(1-\delta(z)\right)\lambda e^{-\lambda(1-x)}\right)dz =$$

$$= \frac{1}{1-\exp(-\lambda)}\lambda e^{-\lambda(1-x)} + \frac{1}{1-\exp(-\lambda)}\left(\lambda e^{-\lambda x} - \lambda e^{-\lambda(1-x)}\right)\int_0^1 \delta(z)f(z)dz, \quad 0 \le x \le 1. \quad (5)$$

Let us denote $a = \int_0^1 \delta(z)f(z)dz.$ In order to calculate this value we multiply both sides of (5) by $\delta(z)$ and integrate with respect to *z* from 0 to 1. Then, for $\delta(z) = z$

$$a = \frac{1}{1-\exp(-\lambda)}e^{-\lambda}\frac{1}{\lambda}\left(1-e^{\lambda}+\lambda e^{\lambda}\right)(1-a) + \frac{1}{1-\exp(-\lambda)}\frac{1}{\lambda}\left(1-e^{-\lambda}-\lambda e^{-\lambda}\right)a.$$

$$a\lambda\left(1-\exp(-\lambda)\right) = e^{-\lambda}\left(1-e^{\lambda}+\lambda e^{\lambda}\right)(1-a) + \left(1-e^{-\lambda}-\lambda e^{-\lambda}\right)a.$$

$$a\lambda = e^{-\lambda}\left(1-e^{\lambda}+\lambda e^{\lambda}\right) - ae^{-\lambda}\left(1-e^{\lambda}+\lambda e^{\lambda}\right) + \left(1-e^{-\lambda}\right)a.$$

$$a\lambda = \left(e^{-\lambda}-1+\lambda\right) - a\left(e^{-\lambda}-1+\lambda\right) + \left(1-e^{-\lambda}\right)a.$$

$$2a\lambda = \left(e^{-\lambda}-1+\lambda\right) + 2\left(1-e^{-\lambda}\right)a.$$

So $a = \frac{1}{2}$. We have the same result for $\delta(z) = 1 - z$. Therefore, the following Lemma is true.

*Lemma 2.* The stationary probability density *f(x)* and distribution function *F(x)* have the forms

$$f(x)=\frac{\lambda}{2(1-\exp(-\lambda))}\left(e^{-\lambda x}+e^{-\lambda(1-x)}\right),\ 0\le x\le 1, \tag{6}$$

$$F(x)=\begin{cases}0, & x\le 0,\\ \frac{1}{2(1-\exp(-\lambda))}\left(1-e^{-\lambda x}-e^{-\lambda}\left(1-e^{\lambda x}\right)\right), & 0\le x\le 1,\\ 1, & x\ge 1.\end{cases} \tag{7}$$

Now we rewrite the joint probability density for previous *Z* and current *X* intervals between arrivals as

$$h(z,x)=f(z)q(x|z),\ 0\le z,x\le 1. \tag{8}$$

An integration of the last expression gives the final result.

*Theorem 1.* The joint distribution function for previous *Z* and current *X* intervals between arrivals is calculated by formula

$$H(z,x)=\frac{1}{1-\exp(-\lambda)}\{e^{-\lambda}\left(e^{\lambda x}-1\right)F(z)+\frac{1}{2\lambda(1-\exp(-\lambda))}\times \\ \times\left[1-e^{-\lambda x}-e^{-\lambda}\left(e^{\lambda x}-1\right)\right]\left[1-e^{-\lambda z}-\lambda z e^{-\lambda z}+e^{-\lambda}\left(1-e^{-\lambda z}+\lambda z e^{\lambda z}\right)\right]\},\ 0\le z,x\le 1. \tag{9}$$

Note that some estimation problems of Markov chains, defined via copula, have been considered in [1].

## 3. Queueing system analysis

A widely known queueing system is GI/M/1/∞: claims arrive at a single server station in accordance with a recurrent flow. Upon arrival, the claim is immediately served if the server is free. Otherwise, one has to wait occupying one of an infinite number of places. The service time is assumed to be independent and identically exponentially distributed with intensity $\mu$.

We consider the following modification of this system. At first, instead of the recurrent, the above introduced flow will be used. At second, a number of the waiting places are restricted and equal to *k*. If all places are busy, then an arrived claim is rejected. We shall determine the stationary distribution of the claim number in the system.

Let us consider a time moment when a new claim arrives at the system. Let *N* denote a number of the claims in the system, immediately before the very moment. Earlier we have denoted *X* as a length of an interval that is ended after the considered arrival. Let

$$P_i(x)=P\{N=i,X\le x\},\quad p_i(x)=\frac{\partial}{\partial x}P_i(x),\ \ x\ge 0,\ \ i=0,\ 1,\ \ldots.$$

A usual reasoning gives the following equations for $x\ge 0$:

$$p_i(x)=\int_0^1\sum_{j=i-1}^{k}p_j(z)q(x|z)\frac{(\mu x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!}\exp(-\mu x)dz,\ \ i=1,\ \ldots,k, \tag{10}$$

$$p_0(x)=\int_0^1\sum_{j=0}^{k}p_j(z)q(x|z)\left(1-\sum_{\nu=0}^{\Delta(j)}\frac{(\mu x)^{\nu}}{\nu!}\exp(-\mu x)\right)dz, \tag{11}$$

where $\Delta(j)=1$, if $j<k$ and $\Delta(j)=0$ otherwise.

We must solve this system with respect to $p_i(x)$. A substitution of (1) into (10) gives

$$p_i(x)=\int_0^1\sum_{j=i-1}^{k}p_j(z)\frac{1}{1-\exp(-\lambda)}\left(\delta(z)\lambda e^{-\lambda x}+(1-\delta(z))\lambda e^{-\lambda(1-x)}\right)\frac{(\mu x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!}\exp(-\mu x)dz.$$ Let

$$p_i = P\{N = i\} = \int_0^1 p_i(x)dx, \quad \tilde{p}_i = \int_0^1 \delta(z) p_i(z)dz.$$

Now we can rewrite for $i = 1, 2, \ldots, k$:

$$p_i(x) = \sum_{j=i-1}^{k} \frac{\lambda}{1-\exp(-\lambda)} \left\{ e^{-\lambda(1-x)} p_j + \left( e^{-\lambda x} - e^{-\lambda(1-x)} \right) \tilde{p}_j \right\} \frac{(\mu x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!} \exp(-\mu x). \tag{12}$$

An integration with respect to $x \in (0, 1)$ gives

$$p_i = \sum_{j=i-1}^{k} \frac{1}{1-\exp(-\lambda)} \left\{ A_{i,j} \left( p_j - \tilde{p}_j \right) + B_{i,j} \tilde{p}_j \right\}, \quad i = 1, \ldots, k, \tag{13}$$

where for $\mu > \lambda$

$$\begin{aligned} A_{i,j} &= \int_0^1 e^{-\lambda(1-x)} \lambda \frac{(\mu x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!} \exp(-\mu x) dx = \\ &= e^{-\lambda} \lambda \frac{(\mu)^{j+\Delta(j)-i}}{(\mu-\lambda)^{j+\Delta(j)-i+1}} \int_0^1 (\mu-\lambda) \frac{((\mu-\lambda)x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!} \exp(-(\mu-\lambda)x) dx = \\ &= e^{-\lambda} \lambda \frac{(\mu)^{j+\Delta(j)-i}}{(\mu-\lambda)^{j+\Delta(j)-i+1}} \left( 1 - \exp(-(\mu-\lambda)) \sum_{\nu=0}^{j+\Delta(j)-i} \frac{(\mu-\lambda)^{\nu}}{\nu!} \right), \end{aligned} \tag{14}$$

and for $\mu < \lambda$

$$\begin{aligned} A_{i,j} &= \int_0^1 e^{-\lambda(1-x)} \lambda \frac{(\mu x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!} \exp(-\mu x) dx = \\ &= e^{-\lambda} \lambda \frac{(\mu)^{j+\Delta(j)-i}}{(\lambda-\mu)^{j+\Delta(j)-i+1}} \int_0^1 (\lambda-\mu) \frac{((\lambda-\mu)x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!} \exp((\lambda-\mu)x) dx = \\ &= \lambda e^{-\lambda} \frac{(\mu)^{j+\Delta(j)-i}}{(\lambda-\mu)^{j+\Delta(j)-i+1}} \left( \exp(\lambda-\mu) \sum_{\nu=0}^{j+\Delta(j)-i} \frac{(\lambda-\mu)^{\nu}}{\nu!} (-1)^{j+\Delta(j)-i-\nu} + (-1)^{j+\Delta(j)-i+1} \right), \end{aligned}$$

and for both cases

$$\begin{aligned} B_{i,j} &= \int_0^1 \lambda e^{-\lambda x} \frac{(\mu x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!} \exp(-\mu x) dx = \\ &= \lambda \frac{\mu^{j+\Delta(j)-i}}{(\mu+\lambda)^{j+\Delta(j)-i+1}} \int_0^1 (\mu+\lambda) \frac{((\mu+\lambda)x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!} \exp(-(\mu+\lambda)x) dx = \\ &= \lambda \frac{(\mu)^{j+\Delta(j)-i}}{(\mu-\lambda)^{j+\Delta(j)-i+1}} \left( 1 - \exp(-(\mu+\lambda)) \sum_{\nu=0}^{j+\Delta(j)-i} \frac{(\mu+\lambda)^{\nu}}{\nu!} \right). \end{aligned} \tag{15}$$

Multiplying by $\delta(x)$ both sides of (12) and integrating as earlier, we can get

$$\tilde{p}_i = \sum_{j=i-1}^{k} \frac{1}{1-\exp(-\lambda)} \left\{ \tilde{A}_{i,j} \left( p_j - \tilde{p}_j \right) + \tilde{B}_{i,j} \tilde{p}_j \right\}, \quad i = 1, \ldots, k, \tag{16}$$

where for $\delta(x) = x$ and $\mu > \lambda$

$$\begin{aligned}
\tilde{A}_{i,j} &= \int_0^1 e^{-\lambda(1-x)} \lambda\delta(x) \frac{(\mu x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!} \exp(-\mu x) dx = \\
&= e^{-\lambda} \lambda \frac{(\mu)^{j+\Delta(j)-i} (j+\Delta(j)-i+1)}{(\mu-\lambda)^{j+\Delta(j)-i+2}} \int_0^1 (\mu-\lambda) \frac{((\mu-\lambda)x)^{j+\Delta(j)-i+1}}{(j+\Delta(j)-i+1)!} \exp(-(\mu-\lambda)x) dx = \\
&= e^{-\lambda} \lambda \frac{(\mu)^{j+\Delta(j)-i} (j+\Delta(j)-i+1)}{(\mu-\lambda)^{j+\Delta(j)-i+2}} \left( 1 - \exp(-(\mu-\lambda)) \sum_{\nu=0}^{j+\Delta(j)-i+1} \frac{((\mu-\lambda)^{\nu}}{\nu!} \right),
\end{aligned} \tag{17}$$

and for $\mu < \lambda$

$$\begin{aligned}
\tilde{A}_{i,j} &= \int_0^1 e^{-\lambda(1-x)} \lambda\delta(x) \frac{(\mu x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!} \exp(-\mu x) dx = \\
&= e^{-\lambda} \lambda \frac{(\mu)^{j+\Delta(j)-i} (j+\Delta(j)-i+1)}{(\lambda-\mu)^{j+\Delta(j)-i+2}} \int_0^1 (\lambda-\mu) \frac{((\lambda-\mu)x)^{j+\Delta(j)-i+1}}{(j+\Delta(j)-i+1)!} \exp((\lambda-\mu)x) dx = \\
&= \lambda e^{-\lambda} \frac{(\mu)^{j+\Delta(j)-i} (j+\Delta(j)-i+1)}{(\lambda-\mu)^{j+\Delta(j)-i+2}} \left( \exp(\lambda-\mu) \sum_{\nu=0}^{j+\Delta(j)-i+1} (-1)^{j+\Delta(j)-i+1-\nu} \frac{(\lambda-\mu)^{\nu}}{\nu!} + (-1)^{j+\Delta(j)-i+2} \right),
\end{aligned}$$

and for all cases

$$\begin{aligned}
\tilde{B}_{i,j} &= \int_0^1 e^{-\lambda x} \lambda\delta(x) \frac{(\mu x)^{j+\Delta(j)-i}}{(j+\Delta(j)-i)!} \exp(-\mu x) dx = \\
&= \lambda \frac{(\mu)^{j+\Delta(j)-i} (j+\Delta(j)-i+1)}{(\mu+\lambda)^{j+\Delta(j)-i+2}} \int_0^1 (\mu+\lambda) \frac{((\mu+\lambda)x)^{j+\Delta(j)-i+1}}{(j+\Delta(j)-i+1)!} \exp(-(\mu+\lambda)x) dx = \\
&= e^{-\lambda} \lambda \frac{(\mu)^{j+\Delta(j)-i} (j+\Delta(j)-i+1)}{(\mu+\lambda)^{j+\Delta(j)-i+2}} \left( 1 - \exp(-(\mu+\lambda)x) \sum_{\nu=0}^{j+\Delta(j)-i+1} \frac{((\mu+\lambda)x)^{\nu}}{\nu!} \right),
\end{aligned} \tag{18}$$

For $\delta(x) = 1 - x$ we must use $A_{i,j} - \tilde{A}_{i,j}$ and $B_{i,j} - \tilde{B}_{i,j}$ as $\tilde{A}_{i,j}$ and $\tilde{B}_{i,j}$.
Also we have the equation system (12) and (16). It is necessary to add some normalization conditions:

$$\begin{aligned}
p_0 &= 1 - \sum_{i=1}^{k} p_i, \\
\tilde{p}_0 &= \frac{1}{2} - \sum_{i=1}^{k} \tilde{p}_i.
\end{aligned} \tag{19}$$

Now the system (12), (16) and (19) can be solved by a numerical method. In particular, the probability to reject a claim equals to $p_k$. The mean value of the claims, which a new arrived claim finds in the system is calculated as

$$m = \sum_i i p_i. \tag{20}$$

Since probabilities $\{ p_i \}$ are known, it is possible to calculate the conditional distribution function of the claims waiting time to be served:

$$P\{W \le w\} = \frac{1}{1-p_k} \left\{ p_0 + \sum_{i=1}^{k-1} p_i \left[ 1 - \exp(-\mu w) \sum_{j=0}^{i-1} \frac{(\mu w)^j}{j!} \right] \right\}, \quad w \ge 0, \tag{21}$$

where $w^j = 1$ if $w = j = 0$.

## 4. Numerical comparing of queueing systems with various input flows

Now we perform a numerical analysis to evaluate various flows influence on queueing system efficiency. All three above described systems will be considered, namely: the system GI/M/1/∞ with the distribution (7) of the independent inter-arrival times (the first system); the analogous system with restricted number *k* of waiting places and dependent inter-arrival times: the case $\delta(z) = z$ (the second system) and the case $\delta(z) = 1 - z$ (the third system). We will denote these systems as GD/M/1/*k*.

As efficiency criteria, we will use the probability not to wait for a beginning of the service and the mean value of waited claims at the time moment of a new arrival.

Let us give some commentaries. At first, we have chosen rather big value of *k* that gives the same results for the systems GI/M/1/∞ and GI/M/1/*k* with the infinite and restricted number of waiting places. At second, all three systems differ from each other by the inter-arrival times. They are independent for the first system. And they are dependent for the rest ones. For the second system there is the following tendency: the next inter-arrival time will be distinguished from the previous inter-arrival. Note that it is a positive fact for the queueing systems because long and short inter-arrival times will be alternate.

For the third system there is an opposite tendency: the next inter-arrival time will be analogous to the previous inter-arrival. It has a negative influence on the queueing systems as it is highly probable to have long series of short inter-arrival times.

At first we remind some necessary expressions for the system GI/M/1/∞. The probability $p_i$ that an arrived claim finds *i* other claims in the system is calculated by the formula

$$p_i = (1-u)u^i, \ \ i = 0, \ 1, \ldots, \qquad (22)$$

where $u \in (0, 1)$ is the unique root of the equation

$$u - \int_0^\infty e^{-\mu x(1-u)} f(x)dx = 0.$$

Further, we give numerical results for the following input date. The intensity of the service times $\mu = 4$. Because the mean inter-arrival time *a* equals to 0.5, then the load coefficient of the server $\rho = 1/(a\mu) = 0.5$. The number of the waiting places for the systems with the dependent inter-arrival times $k = 12$. The parameter $\lambda$ of the distribution (1) will be taking the following values: 0.1, 0.5, 1, 2, 3, 5, 6, and 7.

The next tables contain the probabilities of queueing system states $\{p_i\}$ and the mean number of claims *m* (the last row of the Table) that has been calculated by formula (20).

**Table 1.** The efficiency indices for the system GI/M/1/∞

| $\lambda$ | 0.1 | 0.5 | 1 | 2 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|---|---|---|
| $p_0$ | 0.639 | 0.638 | 0.634 | 0.621 | 0.602 | 0.563 | 0.531 | 0.507 |
| $p_1$ | 0.231 | 0.231 | 0.232 | 0.235 | 0.240 | 0.246 | 0.249 | 0.250 |
| $p_2$ | 0.083 | 0.084 | 0.085 | 0.089 | 0.095 | 0.108 | 0.117 | 0.123 |
| $p_3$ | 0.030 | 0.030 | 0.031 | 0.034 | 0.038 | 0.047 | 0.055 | 0.061 |
| $p_4$ | 0.011 | 0.011 | 0.011 | 0.013 | 0.014 | 0.021 | 0.026 | 0.030 |
| $p_5$ | 0.004 | 0.004 | 0.004 | 0.005 | 0.006 | 0.009 | 0.012 | 0.015 |
| $p_6$ | 0.001 | 0.001 | 0.002 | 0.002 | 0.002 | 0.004 | 0.006 | 0.007 |
| $p_7$ | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.002 | 0.003 | 0.004 |
| $p_8$ | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.001 | 0.003 | 0.002 |
| $p_9$ | 0 | 0 | 0 | 0 | 0 | 0 | 0.001 | 0.001 |
| *m* | 0.564 | 0.567 | 0.577 | 0.611 | 0.660 | 0.776 | 0.883 | 0.971 |

**Table 2.** The efficiency indices for the system GD/M/1/*k*, case $\delta(z) = z$

| $\lambda$ | 0.1 | 0.5 | 1 | 2 | 3 | 5 | 7 | 9 |
|---|---|---|---|---|---|---|---|---|
| $p_0$ | 0.640 | 0.643 | 0.645 | 0.641 | 0.631 | 0.606 | 0.583 | 0.564 |
| $p_1$ | 0.232 | 0.236 | 0.242 | 0.255 | 0.271 | 0.301 | 0.327 | 0.347 |
| $p_2$ | 0.082 | 0.080 | 0.077 | 0.073 | 0.071 | 0.068 | 0.067 | 0.066 |
| $p_3$ | 0.029 | 0.027 | 0.025 | 0.022 | 0.020 | 0.019 | 0.018 | 0.018 |
| $p_4$ | 0.010 | 0.009 | 0.008 | 0.006 | 0.006 | 0.005 | 0.004 | 0.004 |
| $p_5$ | 0.004 | 0.003 | 0.003 | 0.002 | 0.002 | 0.001 | 0.001 | 0.001 |
| $p_6$ | 0.001 | 0.001 | 0.001 | 0.001 | 0 | 0 | 0 | 0 |
| $m$ | 0.559 | 0.539 | 0.523 | 0.508 | 0.506 | 0.521 | 0.539 | 0.556 |

Let us discuss the represented results. The system GI/M/1/∞ with the recurrent input flow is a "neutral" one. The rest two systems are the opposite ones: the case $\delta(z) = z$ improves the efficiency characteristics of the service, the case $q(z) = 1 - z$ – makes them worse. For example, if $\lambda = 2$, then the probability of the service without waiting $p_0$ and the mean number of claims in the system $m$ consist of: for the system GI/M/1/∞ $p_0 = 0.621$, $m = 0.611$; for the system GD/M/1/12 (the case $\delta(z) = z$) $p_0 = 0.641$, $m = 0.508$; for the system GD/M/1/12 (the case $\delta(z) = 1 - z$) $p_0 = 0.595$, $m = 0.794$. Note the efficiency characteristics for the case $\delta(z) = 1 - z$ deteriorate catastrophically, when the value $\lambda$ increases and long series of short inter-arrival times is probable. It seems to us that precisely such a situation appears in the various telecommunication and financial systems.

**Table 3.** The efficiency indices for the system GD/M/1/*k*, case $\delta(z) = 1 - z$

| $\lambda$ | 0.1 | 0.5 | 1 | 2 | 3 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $p_0$ | 0.638 | 0.632 | 0.622 | 0.595 | 0.561 | 0.496 | 0.472 | 0.457 |
| $p_1$ | 0.230 | 0.225 | 0.220 | 0.208 | 0.191 | 0.149 | 0.130 | 0.113 |
| $p_2$ | 0.084 | 0.087 | 0.091 | 0.100 | 0.105 | 0.101 | 0.093 | 0.084 |
| $p_3$ | 0.031 | 0.034 | 0.038 | 0.049 | 0.061 | 0.072 | 0.072 | 0.068 |
| $p_4$ | 0.011 | 0.013 | 0.016 | 0.024 | 0.035 | 0.052 | 0.056 | 0.056 |
| $p_5$ | 0.004 | 0.006 | 0.007 | 0.012 | 0.020 | 0.038 | 0.043 | 0.046 |
| $p_6$ | 0.002 | 0.002 | 0.003 | 0.006 | 0.012 | 0.027 | 0.033 | 0.037 |
| $p_7$ | 0.001 | 0.001 | 0.001 | 0.003 | 0.007 | 0.020 | 0.026 | 0.030 |
| $p_8$ | 0 | 0 | 0 | 0.004 | 0.004 | 0.014 | 0.020 | 0.025 |
| $p_9$ | 0 | 0 | 0 | 0.001 | 0.002 | 0.010 | 0.016 | 0.021 |
| $p_{10}$ | 0 | 0 | 0 | 0 | 0.001 | 0.008 | 0.013 | 0.019 |
| $p_{11}$ | 0 | 0 | 0 | 0 | 0.001 | 0.006 | 0.012 | 0.019 |
| $p_{12}$ | 0 | 0 | 0 | 0 | 0 | 0.006 | 0.013 | 0.025 |
| $m$ | 0.571 | 0.600 | 0.648 | 0.794 | 1.018 | 1.692 | 2.083 | 2.459 |

## 5. Conclusions

In this paper, we have considered a model of a nonrecurrent flow, for which inter-arrival times can be positive or negative correlated. Such flows often appear in various telecommunication and financial systems. This flow has been used as an input flow for a single server queueing system with exponential distributed service times. Numerical results show that the dependence between inter-arrival times exercises great influence on efficiency characteristics of the service.

The elaborated approach gives a general tool for a construction of wide flows class with correlated inter-arrival times.

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