The System's Limitations Costs Determination Using the Duality Concept of Linear Programming

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Abstract ***- The duality concept in the issued competitive electricity market optimization task allows determining the electricity nodal prices. The direct task can be solved with the linear programming simplex method. The nodal price can be realized by finding the dual variables at the accepted limits. The calculations have shown that the property of electricity as a good does not allow considering the concept of a balanced price of electricity as the sole price in the market taking into account the technological limitations on EPS functioning.***

Keywords : ***electric power, linear programming, dual task, optimization, nodal price***

I. INTRODUCTION

If we ignore limitations for the system and technical losses, which are related with electricity transmissions, then the calculation of the competitive equilibrium price for the market participants is relatively simple [1-5].

Consumer and supplier applications form the demand and supply step curve (Fig.1.), this intersection determine the equilibrium price, which correspond the demand and supply volumes equations as: $W_{supply} = W_{demand}$.

The consumers (buyers) put their requirements about the volume of the electricity supply W_{dj} by the price as c_{dj} ($j \in D$). But the suppliers (sellers) are prepared to sell at market the amount of electricity W_{gi} by the prices as c_{gi} ($i \in G$).

In the adopted model the electricity volume determine by multiplying the value of active power with the adopted time interval value (1 hour), it means $W_i = P_i \cdot \Delta t$, where $\Delta t = 1$ hour.

The task objective function or welfare (profit) function can write down with the form as:

$$F = \max\left(\sum_{j=1}^{D} c_{dj} \cdot P_{dj} - \sum_{i=1}^{G} c_{gi} \cdot P_{gi}\right), \quad (1)$$

where c_{dj} , c_{gi} – buyers / sellers bid prices;

P_{dj} , P_{gi} – an applied electricity hourly loaded/generated capacities;

d – the consumptions in the node;

g – the generation in the node;

G, D - the generation and loads node's number.

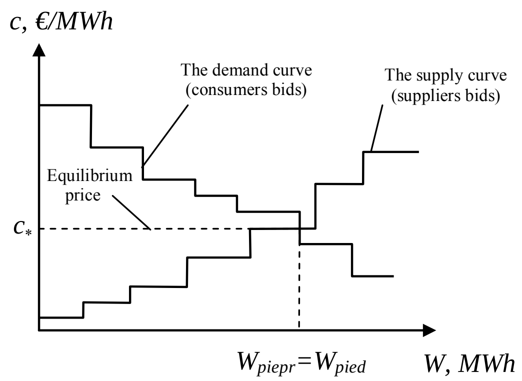

Fig. 1. The equilibrium price notice in a flexible market

II. THE COMPETITIVE MARKET TASK OF OPTIMIZATION AND NODAL PRICES

At the energy power system (EPS) there are two factors that affect the price formation (for the pricing). They are the system limitations and technical losses. The systems limitations include network limitations, which are raised by certain network split throughput capacity limits. Those limits are calculated so as to prevent the system's overload and stability losses. The technical losses in the electrical power networks arise because of the electricity transmission.

To solve the power equilibrium price and volume of this task, and take into account the system's limitations and losses, is necessary to use appropriate software. The use of it allows to calculate the optimal hourly equilibrium prices, production and consumption volumes, with considering the system's limitations and technical losses.

When we are solving the divided auction of calculated results on the total task, first of all is need to find out the function's (1) maximum, considering the power flow P_{ij} from node i to node j along the line ij limitations or flow sum limitations of controlled splits S

$$P_s^{\min} \le \sum_{(i,j)\in S} P_{ij} \le P_s^{\max} , \quad (2)$$

as well as the balance of the limitations at nodes where are

$$\sum_j P_{ij} + \sum_G P_{gi} - \sum_D P_{di} = 0 , \quad (3)$$

for each node i. Calculation of sum can make after all generators G and consumers D which are connected to the i- node and all nodes j which are connected with the i-node.

After calculating we obtain that at each network node will form the individual equilibrium price – the nodal price. The nodal prices are the most precise cost assessment instrument for the electricity supplying in a certain network point. They are defined as the ratio of total costs, which are necessary to satisfy the consumer needs at the consumption changes in the specific delivery points. There are observed the technical losses in a nodal price, which are connected with the electricity supply in a certain network points and with necessary regime deviations from optimal, to prevent the system's limitation overrunning.

From the software view the market estimates are not needed the additional nodal price calculation, because it is the EPS regime optimization products.

III. THE LINEAR PROGRAMMING

The linear optimization tasks at which can add under considered electricity market output (direct) task with the objective function (1) and limitations (2) and (3), are most effectively solved with the linear programming simplex method. This calculation allow to find out the minimum value of the objective function (1), the generating equipment structure and the load, the power flow on the controlled lines, but need to consider all limitations.

The duality concept use in the linear programming allows increasing significantly the information's volume of the obtained optimization's task result. The duality in the issued competitive electricity market optimization task allows determining the electricity nodal prices [6].

The basic duality theory is that each of the linear programming tasks have the another linear programming task, and theirs solution is closely connected with the output task of solution. They are direct and dual tasks. The both tasks have the same objective function optimal value (if any exist). The both tasks are symmetrical to each other and determined solutions allow economic interpret the dual variable values. The optimal points of dual variables are defined as the direct linear programming task's additional variable relative assessment.

Between the direct and the dual task's solutions are series of important relevance, which are useful for the common characteristics optimal solution research and to verify acceptable solution optimum. An optimal dual solution can be interpreted as the limitations resource assessment group. The final case plays an important role in the sensitivity analysis, i.e. at the output task parameters changes. In the dual task variables are called by the "shadow prices".

According to the Kuna-Tacker theorem each of the direct optimization's task variables correspond dual variables. If this variable corresponds to or is included in the essential (active) limit, then its value is different from zero. If this variable corresponds to negligible (passive) limit then its value equals to zero. If carefully examining the definition of dual task then is seen the resistance between the direct and dual task, it is: if we have more stringent limitations on one of the tasks, so then in the second task its limitations are more liberal. This balance's expression is the Sleiter condition (complementary flexibility condition), which is necessary and sufficient for the direct and dual variables giving the same (after the objective function value) optimal solution.

The nodal price can be realized by finding the dual variables at the accepted limits. The nodal prices objectively reflect the conditional resources valuations (generation, system constraints and losses). These ratings determine the degree of resource shortage. So fully used resource valuations are different from zero, but not fully used resources valuation equals to the zero value.

The dual variables are determined by solving the dual task in the ratio to the direct task. The dual task allows to assess the direct task's calculation. For each variant of production plan corresponds their own resource utilization, whom is estimated its significance (dual variation). This reflects the resource influences degree to the result.

IV. EXAMPLE – DIRECT TASK

So we will solve the electrical power systems, which scheme is established at the Fig.2. and system optimization tasks with the work with a full staff of the equipment. The solution of this task we will make with the linear programming method. At 1st Table are given the system's capacities of generators and their bid prices. We will suppose that the network is homogeneous and all line lengths equal 100 km. At the calculation we will take into account that line l_9 has the capacity flow limit $P_{l9}^{\max} = 130$ MW.

Tab.1

The information about generation

Nr	1	2	9
$P_{gi}^{\max}$, MW	100	300	350
$P_{gi}^{\min}$, MW	10	10	10
c_{gi}, €/MWh	50	60	30

For the network nodes 4, 6 and 8 are connected appropriate loads, as: $P_{d4} = 90$ MW, $P_{d6} = 100$ MW, $P_{d8} = 125$ MW. The total consumer load P_D is 315 MW.

We will consider that the power losses are included in the load.

The presented scheme's regime of linear programming optimization in direct order (a case) the mathematic model's aim is to minimize the function

$$F = 50 \cdot P_{g1} + 60 \cdot P_{g2} + 30 \cdot P_{g9} + c_4 \cdot P'_{g4} + c_6 \cdot P'_{g6} + c_8 \cdot P'_{g8}$$

where P'_{g4}, P'_{g5} and P'_{g6} - are an additional variables that describe the possible generation of load nodes.

Since in a given task are free prices loads, then bid prices c_4, c_5 and c_6 can be regarded for non-limits of high.

The task solution is to be satisfied the system power balance limitation

$$P_{g1} + P_{g2} + P_{g9} + P'_{g4} + P'_{g6} + P'_{g8} = P_{d4} + P_{d6} + P_{d8} = 315$$

And the generator active power limits ($P_{gi} \geq P_{gi}^{\min}; P_{gi} \leq P_{gi}^{\max}$) on all nodes in the EPS:

$$\begin{array}{llllllll}
P_{g1} & & & & & & \geq & 10, \\
-P_{g1} & & & & & & \geq & -100, \\
 & P_{g2} & & & & & \geq & 10, \\
 & -P_{g2} & & & & & \geq & -300, \\
 & & P_{g9} & & & & \geq & 10, \\
 & & -P_{g9} & & & & \geq & -350, \\
 & & & P'_{g4} & & & \geq & 0, \\
 & & & & P'_{g5} & & \geq & 0, \\
 & & & & & P'_{g6} & \geq & 0.
\end{array}$$

To obtain the power flows through the lines, we will use the Kirchhoff's laws in a matrix form and draft a matrix A with the form as:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{M} \\ \boldsymbol{N} \cdot \boldsymbol{Z}_d \end{bmatrix},$$

where $\boldsymbol{M}$ – the first incidence (the independent node connection) matrix;

$\boldsymbol{N}$ – the second incidence (the independent rectangular contour connection) matrix;

$\boldsymbol{Z_d}$ – diagonal, the quadratic branches resistance matrix (since the network is homogeneous, the line's resistances will be replaced with the line's lengths).

If we invert the matrix A, then we can obtain the current distribution coefficients for the given EPS scheme. In our case the matrix $\boldsymbol{\alpha}$ of the current distribution coefficients is formed with the matrix $\boldsymbol{A}^{-1}$ and its first 8hts column's help as:

$$\boldsymbol{\alpha} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -1 & -1 & -1 & -1 & -1 & -1 & -1 \\
-0{,}33 & -0{,}67 & 0 & -0{,}83 & -0{,}67 & -0{,}5 & -0{,}33 & -0{,}17 \\
0{,}33 & 0{,}67 & 0 & -0{,}17 & 0{,}67 & 0{,}5 & 0{,}33 & 0{,}17 \\
-0{,}33 & 0{,}33 & 0 & 0{,}17 & 0{,}33 & -0{,}5 & -0{,}33 & -0{,}17 \\
0{,}33 & -0{,}33 & 0 & -0{,}17 & -0{,}33 & -0{,}5 & 0{,}33 & 0{,}17 \\
0{,}67 & 0{,}33 & 0 & 0{,}17 & 0{,}33 & 0{,}5 & 0{,}67 & -0{,}17 \\
-0{,}67 & -0{,}33 & 0 & -0{,}17 & -0{,}33 & -0{,}5 & -0{,}67 & -0{,}83
\end{bmatrix}.$$

According to the current distribution coefficients, which are calculated with the relation to the 9th generator (generator with a minimum bid price), the power flow through the line l9 can be recorded as:

$$P_{l9} = -0{,}67 \cdot P_{g1} - 0{,}33 \cdot P_{g2} - 0{,}17 \cdot \left(P'_{g4} - 90\right) - \\ -0{,}5 \cdot \left(P'_{g6} - 100\right) - 0{,}83 \cdot \left(P'_{g8} - 125\right)$$

Since the technological requirements must be executed $P_{l9} \leq 130$, and taking into account this expression $P'_{g4} = P'_{g6} = P'_{g8} = 0$, the systems limit inequalities must be supplemented with the inequalities:

$$0{,}67 \cdot P_{g1} + 0{,}33 \cdot P_{g2} \geq 39{,}05.$$

Simplex method use in the output task with a full staff of generators allows to determine example's (given load values in the node) the most economical required generation. The optimal regime's generation values on the nodes are:

$$P_{g1} = 53{,}75 \text{ MW}, \ P_{g2} = 10 \text{ MW}, \ P_{g9} = 251{,}25 \text{ MW}.$$

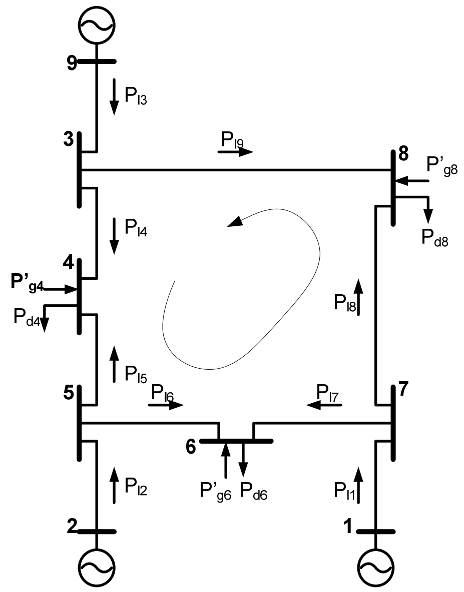

Fig. 4. The electrical power system's scheme

It means that the most expensive generator G2 works with the minimum possible load, but the generator G1 and cheapest generator G9 are not fully loaded.

The capacities through the line will be, as:

$$\mathbf{P_l} = \boldsymbol{\alpha} \cdot \boldsymbol{P_m} = \begin{bmatrix} 53{,}75 \\ 10 \\ 251{,}25 \\ 121{,}25 \\ -31{,}25 \\ 41{,}25 \\ 58{,}75 \\ -5 \\ 130 \end{bmatrix}$$

In this case, the objective function's value will be:

$$F = 50 \cdot 53{,}75 + 60 \cdot 10 + 30 \cdot 251{,}25 = 10825.$$

V. EXAMPLE – DUAL TASK

There is need to solve the electrical power system's nodal price, which there is established in the Fig.2., for price determination is need to solve the linear programming dual task.

The problem direct optimization solution, considering the system and node limitations, approved all EPS generators the need of participation into the market. Moreover, it was certainly founded the active and passive limitations, which are firstly given with the inequality form. According to the nonlinear programming theory for the active limitations belong those in which one of the parameter is adopted as the limit value. All other limitations of form inequalities are passives. If previously would be known, which limitations of inequalities form are passive and which are active, then the passive could be excluded from consideration, but the active can be observed as constraints equation form.

In the issued example the limit values amount are reached such variables as P_{g2} and P_{l9}. In this way, in the direct linear programming task with the capacity balance limitations were enough to observe the active limitations

$$P_{g1} + P_{g2} + P_{g9} + P'_{g4} + P'_{g6} + P'_{g8} = 315$$

$$P_{g2} \geq 10$$

$$-0,67 \cdot P_{g1} - 0,33 \cdot P_{g2} - 0,17 P'_{g4} - $$

$$- 0,5 P'_{g6} - 0,83 P'_{g8} + 169,05 \leq 130$$

with the coefficient matrix at the variables

$$A = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0{,}67 & -0{,}33 & 0 & -0{,}17 & 0 & -0{,}5 & 0 & -0{,}83 & 0 \end{pmatrix}.$$

Then we are making the linear programming dual task's objective function and limitations is need to observe, that the vector b and c are clearly defined from the task direct of the linear programming. In the issued example they will be, as:

$$\mathbf{b} = (315 \quad 10 \quad -39{,}05)^T,$$

$$\mathbf{c} = (50 \quad 60 \quad 0 \quad c_4 \quad 0 \quad c_6 \quad 0 \quad c_8 \quad 30)^T$$

According the linear programming dual task the mathematical model may be written with such form as:

$$F_D = (315 \cdot y_b + 10 \cdot y_2 - 39{,}05 \cdot y_{l9}) \rightarrow \max$$

$$\begin{aligned} & y_b \qquad\quad - 0{,}67 \cdot y_{l9} \leq 50, \\ & y_b + y_2 - 0{,}33 \cdot y_{l9} \leq 60, \\ & y_b \qquad\qquad\qquad\quad \leq 30, \end{aligned}$$

where the variable vector's components $\mathbf{y} = (y_b; \quad y_2; \quad y_{l9})^T$ of the dual tasks correspond to:

y_b - the capacity balance equation of the similarity form;

y_2 - the open active limitations of the second node;

y_{l9} - the line's l_9 capacity flow limitation.

The dual task limitation system does not include inequalities with variables c_4, c_6 and c_8 this is because of variable uncertainties.

If we will solve the dual task, we obtain the following results:

$$y_2 = 30; \quad y_b = 20; \quad y_{l9} = -30.$$

The maximum objective function's value in this case will be:

$$F_D = 315 \cdot 30 + 10 \cdot 20 - 39{,}05 \cdot (-30) = 10821{,}5.$$

This result is a little beat differ from the previously obtained output (direct) task of the minimum objective function value, because of the calculations allowed rounding.

Because of the nodes and lines limitations is need to differentiate the consumer nodal price after the expression

$$\boldsymbol{c}_n = \boldsymbol{A}^T \cdot \boldsymbol{y},$$

where in the column type vector $\mathbf{c}_n$ are defined in addition variables c_4, c_6 and c_8, which determine the price in the load nodes.

In this way obtained solution result of the linear programming dual task allows to determine nodal price vector of the given EPS scheme:

$$\mathbf{c}_n = (c_{n1} \quad c_{n2} \quad c_{n3} \quad c_{n4} \quad c_{n5} \quad c_{n6} \quad c_{n7} \quad c_{n8} \quad c_{n9})^T =$$

$$= (50 \quad 60 \quad 0 \quad 35 \quad 0 \quad 45 \quad 0 \quad 55 \quad 30)^T.$$

Consequently in the generation nodes the nodal prices correspond to the bid prices. The nodal prices in the load nodes serve as the electricity supply cost assessment. The obtained result shows that, if we observe the capacity flow through the line (line throughput), then the 4th node's electricity price for consumers will be 35 €/MWh, 6th node's electricity price for consumers will be 45 €/MWh, but the 8th node's will be the highest electricity price for the consumer it will be 55 €/MWh.

The consumer nodal prices are included not only in electricity supplying costs, but also in the additional costs, which are arisen when the regime deviate from the optimum line l_9 and it is closely connected with limits satisfaction of throughput.

VI. CONCLUSIONS

1. A common balanced price can be formed in all units of an electrical network if the EPS has no technological limitations. When technological limitations exist that affect the price formation, at different nodes differing nodal prices form
2. For the sequence with solving the linear programming of direct and dual tasks, we can calculate the system's limitation costs as well as nodal prices of the electricity.

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