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EVALUATION OF THE NUMBER OF NON-NEGATIVE SOLUTIONS OF DIOPHANTINE EQUATIONS

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We consider the Diophantine equation

$$a_1x_1^m + a_2x_2 + \cdots + a_nx_n = N, \tag{1}$$

where m, N, $a_1, a_2, \ldots, a_n$ are non-negative integer numbers. The generating function of equation (1) has the form $\varphi(t) = [(1-t^{a_2})(1-t^{a_3})\cdots(1-t^{a_n})]^{-1}\sum_{p=0}^{\infty} t^{a_1p^m}$. The number of non-negative solutions $J(N)$ of equation (1) is given by the formula

$$J(N) = 1/(N!)\varphi^{(N)}(0). \tag{2}$$

We suggest two methods of calculation $J(N)$ using formula (2).

Example 1.

$$3x + 5y = N. \tag{3}$$

Expanding the generating function $\varphi(t) = [(1-t^3)\,(1-t^5)]^{-1}$ into the partial fractions and using formula (2), we obtain the solution of the equation (3) for all N in the form

$$J(N) = \frac{1}{45}\quad 3N + 12 + 18\sqrt{\tfrac{2}{5-\sqrt{5}}}\cos\tfrac{\pi(8N+17)}{10} + 18\sqrt{\tfrac{2}{5+\sqrt{5}}}\cos\tfrac{\pi(4N+1)}{10} + 10\sqrt{3}\sin\tfrac{2\pi(N+1)}{3}\quad . \tag{4}$$

It follows from (4) that, for example, $J(100) = 7$. Formula (4) was obtained also by using the computer package MATHEMATICA. The similar formula is obtained also for the equation $x + 2y + 3z + 4u = N$.

Example 2.

$$x^3 + 2y = N. \tag{5}$$

We use the trick (see [1, p.386]) which allows us to evaluate the recursion formula for the number $b_N = J(N)$ of all non-negative solutions of (5). Expanding the generating function $\varphi(t) = (1-t^2)^{-1}\sum_{p=0}^{\infty} t^{p^3}$ in the Maclaurin series and multiplying by $(1-t^2)$, we obtain $1 + t + t^8 + \cdots + t^{n^3} + \cdots = (1-t^2)(1 + b_1t + b_2t^2 + \cdots + b_nt^n + \cdots)$. Equating the coefficients of t^n we find the recursion formula for the terms b_n: $b_{n+2} - b_n = \delta_{0,l}$, $b_1 = b_2 = 1$, where $\delta_{i,k}$ - the Kronecker symbol, $l = \{\sqrt[3]{n+2}\}$, $\{x\}$ denotes the fractional part of x and $[x]$ is the integer part. The next two cases exhaustively solve the problem for all non-negative integer numbers N: 1) $J(N) = [\frac{\sqrt[3]{N}}{2}] + 1$, if N is even, and the solutions are: $(x = 2m,\ y = \frac{N}{2} - 4m^3)$, $m = 0, 1, 2, \ldots, [\frac{\sqrt[3]{N}}{2}]$; 2) $J(N) = [\frac{\sqrt[3]{N}-1}{2}]+1$, if N is odd, and the solutions are: $(x = 2m+1,\ y = \frac{N-(2m+1)^3}{2})$, $m = 0, 1, 2, \ldots, [\frac{\sqrt[3]{N}-1}{2}]$.

REFERENCES

[1] M.Ya. Antimirov, A.A. Kolyshkin and R. Vaillancourt. *Complex variables.* Academic Press, Orlando, 1998.