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USING REGULATED BROWNIAN MOTION FOR MODELING OF AN EXCHANGE RATE

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In the presented work a generalization of Krugmans Target zones model [1] for the non-stationary case is proposed. Non-stationarity in the model arises due to fixing of the terminal value of the exchange rate, which corresponds to joining a currency area.

In the model it is assumed that exchange rate $e(t)$ depends on fundamental $f(t)$, and on expected change in the exchange rate:

$$e(\tau) = f(\tau) + \alpha \mathbf{E}_\tau[de(\tau)/d\tau], \quad \alpha > 0.$$

We assume that there exist lower and upper bounds for the fundamental, such that the fundamental fulfills $\overline{f} < f(\tau) < \underline{f}$. At the edges of the band $[\overline{f}, \underline{f}]$ at the points $\overline{f}$ and $\underline{f}$ there are infinitesimal interventions $dL > 0$, $dU > 0$ to prevent the fundamental from moving outside the $[\overline{f}, \underline{f}]$ band. So the fundamental follows the equation

$$df(t) = \sigma dw - dU + dL.$$

This implies that the fundamental is a regulated Brownian motion process [2].

Using terminal condition $e(T, f) = 0$ and "smooth pasting conditions" $\frac{\partial e(f)}{\partial f} = \frac{\partial e(\overline{f})}{\partial f} = 0$ (see [1]) the following analytical solution of the problem was obtained: $e(t, f) = e^*(t, f) + \hat{e}(f)$, where

$$e^*(t, f) = \sum_{k=1}^{\infty} C_k \exp\left(-\left(\lambda_k + \frac{1}{\alpha}\right)t\right) \sin\left(\frac{\sqrt{2\lambda_k}}{\sigma} f\right), \quad \lambda_k = \frac{(\pi/2 + \pi k)^2 \sigma^2}{2\overline{f}^2},$$

$$C_k = \frac{(-1)^k 2\overline{f}}{(\pi/2 + \pi k)^2 + \frac{2\overline{f}^2}{\alpha\sigma^2}} - \frac{(-1)^k 2\overline{f}}{(\pi/2 + \pi k)^2}, \quad \hat{e}(f) = f - \frac{\operatorname{sh}\left(\sqrt{\frac{2}{\alpha\sigma^2}} f\right)}{\sqrt{\frac{2}{\alpha\sigma^2}} \operatorname{ch}\left(\sqrt{\frac{2}{\alpha\sigma^2}} \overline{f}\right)}.$$

The proposed model of exchange rate behavior is more adequate, when the moment of joining currency area approaches. The issue is actual for new EU member countries, which are going to introduce EURO instead of national currencies.

REFERENCES

[1] P.Krugman. Article in journal. *Quarterly Journal of Economics*, **106** 1991, 311 – 325.

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