LIMIT THEOREMS FOR MARKOV DYNAMICAL SYSTEMS

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The report deals with dynamical systems in $\mathbf{R^n}$ of impulse type (IDS) which dynamical characteristics are dependent on the step Markov process (MP). The phase motion $x(t) \in \mathbf{R^n}$ has jums in the moments of switchings of the above MP and satisfies the ordinary differential equation within the intervals of constancy of the MP. Both the quantities of jumps and the right part side of differential equation are proportional to small positive parameter ε and are dependent on phase coordinates and MP.

The proofs of the main results are based on the described in [3] special constructing family of distance functions in $\mathbf{R^n}$ which permits to apply for asymptotical analysis the second Lyapunov method combined with advance martingale technique for positive semimartingales. It is proven that the solutions $\{x(t/\varepsilon),\, 0 \leq t \leq T\}$ of the above IDS converge on probability to the corresponding solutions $\{\bar{x}(t),\, 0 \leq t \leq T\}$ of the averaged equation as $\varepsilon \to 0$ and if the averaged vector field is equal identically to zero then one can use the limit theorems for right continuous processes in Skorokhod space [2] for the family of the processes $\{x(t/\varepsilon^2),\, 0 \leq t \leq T\}$(diffusion approximation). As an implication of this result one proves that the family of the normalized deviations $\{[x(t/\varepsilon) - \bar{x}(t)]/\sqrt{\varepsilon},\, 0 \leq t \leq T\}$ weak converges to the solution of stochastic differential equation of Ornstein-Uhlenbeck type with zero initial

condition. The above results one can use not only for segment $[0, T]$ but also for analysis of the phase trajectories with $t \to \infty$. It is proven that if ε is sufficiently small, both the averaged equation and the diffusion approximation can be used for stability analysis of IDS. If the Markov system does not have perturbations of impulse type one can get the same results as in [1] but in more simple way. Having applied the above described method it has succeeded to obtain some interesting mechanical results [4, 5].

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References

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