AVERAGING AND STABILITY OF QUASI-LINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS WITH MARKOV IMPULSE SWITCHINGS

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Let $\{y(t),\ t \geq 0\}$ be the homogeneous ergodic step Markov process on the metric phase space $\mathbf{Y}$ with the unique invariant measure $\mu(dy)$. Process $y(t)$ has jumps at time moments $t \in \{\tau_j,\ j \in \mathbb{N}\}$ and time intervals between jumps under condition $y(\tau_j) = y$ are exponentially distributed with intensity $a(y)$.

Let us consider the n-dimensional process $u^\varepsilon(t)$ which satisfies quasi-linear functional differntial equation with the small parameter $\varepsilon \in [0,\ 1)$

$$\frac{du^\varepsilon(t)}{dt} = g(u_t^\varepsilon) + \varepsilon F(t,\ u_t^\varepsilon,\ y(t),\ \varepsilon) \tag{1}$$

for $\tau_{j-1} < t < \tau_j$ and has the jumps in time moments $t \in \{\tau_j,\ j \in \mathbb{N}\}$

$$u^\varepsilon(t) = u^\varepsilon(t_-) + \varepsilon H(t,\ u_{t_-}^\varepsilon,\ y(t_-),\ \varepsilon), \tag{2}$$

where u_t^ε is part of solution defined by the equality $u_t^\varepsilon := \{u^\varepsilon(t+\theta), \theta \in [-h,\ 0]\}$ with some positive number h, $g(\varphi)$ is the linear continuous mapping of the Skorokhod space $\mathbb{D}_n([-h, 0])$ [1], the perturbing terms $F(t,\ \varphi,\ y,\ \varepsilon)$ and $H(t,\ \varphi,\ y,\ \varepsilon)$ are the continuous mappings of the product $\mathbb{R}_+ \times \mathbb{D}_n([-h,\ 0]) \times \mathbf{Y} \times [0,\ 1)$ in the space $\mathbb{R}^n$, uniformly continuous as $\varepsilon \to 0$, $F(t,\ 0,\ y,\ \varepsilon) \equiv 0$, $H(t,\ 0,\ y,\ \varepsilon) \equiv 0$ and satisfying the Lipschitz condition by φ. Let $\mathbb{A}$ be the resolving ininitesimal generator [2] coresponding to (1) as $\varepsilon = 0$, having the spectrum in the form $\sigma(\mathbb{A}) = \sigma_0 \cup \sigma_\rho$, where $\sigma_0 \subset \{Re\, z = 0\}$ and $\sigma_\rho \subset \{Re\, z \leq -\rho < 0\}$. Let P_0 be the spectral projektive operator corresponding to σ_0, $\mathbf{X}_0 := P_0\mathbb{D}_n$, $V(\theta)$ be the matrix of the basis in $\mathbf{X}_0$ and A_0 be the matrix of the restriction of $\mathbb{A}$ on $\mathbf{X}_0$. Assuming that there exists the m-dimensional vector function of the argument $x \in \mathbb{R}^m$

$$\check{F}(x) := \lim_{T\to\infty} \frac{1}{T} \int_0^T \int_{\mathbf{Y}} [e^{-tA_0} \hat{\Psi} F(t,\ \mathbf{V}e^{tA_0}x,\ y,\ 0) +$$

$$+a(y)\,\hat{\Psi} H(t,\,\mathbf{V}x,\,y,\,0)]\mu(dy)dt$$

one can define the averaged differential equation

$$\frac{d\check{x}}{dt} = \check{F}(\check{x}). \tag{3}$$

where $\mathbf{V} := \{V(\theta), -h \leq \theta \leq 0\}$, marix-valued function $\{\mathbf{1}(\theta),\ -h \leq \theta \leq 0\}$ is defined by the equation

$$\mathbf{1}(\theta) := \begin{cases} 0, & \text{if } \ -h \leq \theta < 0, \\ I, & \text{if } \ \theta = 0, \end{cases}$$

I is unit $n \times n$-matrix,

$$\Gamma(\theta) := \frac{1}{2\pi i} \int\limits_{B} ((Jz - \mathbb{A})^{-1}\mathbf{1})(\theta)dz = \sum_{j=1}^{m} res\{U^{-1}(z)e^{z\theta}\}|_{z=z_j}$$

and matrix $\hat{\Psi}$ is defined by the equality $\Gamma(\theta) = V(\theta)\hat{\Psi}$.

Theorem 1. *Let in addition to the previous assumptions the functions $F(t,\,\mathbf{V}x,\,y,\,\varepsilon)$ and $H(t,\,\mathbf{V}x,\,y,\,\varepsilon)$ are uniformly continuous in zero as functions of the argument ε, have uniformly bounded continuous x-derivatives $DF(t,\,\mathbf{V}x,\,y,\,0)$ and $DH(t,\,\mathbf{V}x,\,y,\,0)$, belong to the domane $\mathcal{D}(Q)$ of the operator Q, have continuous bounded t-derivatives and there exists such a constant b that*

$$\sup_{\substack{y\in\mathbf{Y},T>0\\ s\geqslant 0}} \Big| \int\limits_{s}^{s+T} \int\limits_{\mathbf{Y}} [e^{-tA_0}\hat{\Psi}F(t,\,\mathbf{V}e^{tA_0}x,\,y,\,0)+$$

$$+\,a(y)\,\hat{\Psi}H(t,\,\mathbf{V}x,\,y,\,0)]\mu(dy)\,dt - T\,\bar{F}(x)\Big| \leq b|x|,$$

for any $x \in \mathbb{R}^m$. If the trivial solution of the averaged equation (3) is exponentially stable in large, then the trivial solution of the system (1)-(2) is exponentially p-stable in large for any sufficiently small positive numbers ε and p.

V.Minkēviča. Kvazilineāru funkcionāldiferenciālvienādojumu ar Markova impulsu pārslēgumiem vidējošana un stabilitāte .

References.

[1] Skorokhod. A. V., *Asymptotic Methods of Theory of Stochastic Differential Equations*, AMS, Providance, 1994.

[2] Hale, J. and Sjord, M. *Introduction to Functional Differential Equations*, Springer-Verlag, New York, Hong Kong, 1993