THE MODEL OF THE CAPITAL'S MOVING IN THE INSURANCE COMPANY.

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Let consider a whole life insurance process with one kind of polices. Life insurance company gets some requirements on polices - the Poisson flow of requirements with parameter λ. Service times $\{\tau_j,\ j=0,1,2,\ldots\}$ (the future lifetime of life) are exponential with parameter μ. $u(t)$ is the company's capital. $u(0)=u$ is statute capital. Differential equation

$$\frac{du(t)}{dt} = \delta(t)\,u(t) \tag{1}$$

is the equation of the capital increasing with interest rate δ, which is the step function. $\delta(t)$ can be describen by impulsive equation

$$\delta(s) = \delta(s-) + \varepsilon\delta_n, \quad s \in [s_n, s_{n+1}), \quad \delta_n = \begin{cases} r & \textit{with probability } p, \\ -r & \textit{with probability } q. \end{cases} \tag{2}$$

Let us consider simple example. A man buy a whole life insurance policy with net single premium. If death results by any means life insurance policy issued d USD. Let's c is net single premium of this life insurance policy. In the tyme of payment the capital value is

$$u(\tau) = u(\tau-) + \varepsilon g, \qquad g \in \{c, -d\} \tag{3}$$

Net single premium and insurance payment are small enough comparing with the company's capital. Due to this fact we can use values c and d with small parameter ε. System (1)-(3) describes the capital's value depending on time t. This system has two sequences of impulse times $\{\tau_j\}$ un $\{s_j\}$. And we have two embedded Markov chains with transition probabilities

$$p_1(u,y) = \begin{cases} \frac{\lambda(u)}{\lambda(u)+\mu}, & y = c, \\ \frac{\mu}{\lambda(u)+\mu}, & y = -d, \end{cases} \qquad p_2(\delta_n) = \begin{cases} p, & \delta_n = r, \\ q, & \delta_n = -r. \end{cases}$$

It is convenient to investigate the normalized deviation of the system (1)-(3) from the averaging equation. Let write the averaged equation.

$$\frac{d\rho(t)}{dt} = r(p-q)\rho(t) + \frac{\lambda(\rho)\,\mu(c-d)}{\lambda(\rho)+\mu}. \tag{4}$$

Let denote $f_1(\rho,\ x,\ y) = \frac{(d-c)\lambda(\rho)\mu}{\lambda(\rho)+\mu}$, $f_2(\rho,\ x,\ y) = \delta(t)x(t)$, $g_1(x,\ y,\ z) = y$. These substitutions lead us to system for normalized deviation $x(t)$

$$\frac{dx}{dt} = \frac{1}{\varepsilon} f_1(\rho, x, y(t)) + f_2(\rho, x, y(t)), \tag{5}$$

$$x(\tau) = x(\tau-) + \varepsilon g_1((x), y(t)), \tag{6}$$

$$\delta(s) = \delta(s-) + \varepsilon\delta_n. \tag{7}$$

Then it is possible to obtain, using the methods of [3,5] that the solution of this system converge to the solution of the diffusion process describen by the equation

$$dx(t) = r(p-q)x(t)\,dt + \sqrt{\frac{(c^2\lambda(\rho) - d^2\mu^2)(\lambda(\rho)-\mu)}{(\lambda(\rho)+\mu)\mu\lambda(\rho)}}\,dw(t).$$

This is a linear stochastic differential equation equation. So it is easy to write the solution of this equation (see [1,2]). We can find the characteristics of $x(t)$ also $(Mx(t), Dx(t))$ to know better the structure of this deviation.

The important question is if the value of capital does not less of some critical value. We can analyse if deviation of the capital from its averaged value belongs to some interval. Let denote $\tau_x[r_1, r_2] = inf\{t,\ x(t) \le r_1\ or\ x(t) \ge r_2\}$ - the moment of the first attainmemt of the boundary of the interval $[r_1, r_2]$. The process $x(t)$ is homogeneous to time, due to this fact the following theorem give us the mean time of the attainment of the critical values.

Theorem 1. If $(c^2\lambda(\rho) - d^2\mu^2)(\lambda(\rho)-\mu) > 0$ for $x \in [r_1, r_2]$, then the value $\tau_x[r_1, r_2]$ with probability equal to one is finite for $x \in [r_1, r_2]$ and $M\tau_x[r_1, r_2] = v(x)$, where $v(x)$ is the solution of the differential equation

$$\frac{(c^2\lambda(\rho) - d^2\mu^2)(\lambda(\rho)-\mu)}{2(\lambda(\rho)+\mu)\mu\lambda(\rho)}\,v''(x) + r(p-q)x(t)\,v'(x) = -1,$$

with conditions $v(r_1) = v(r_2) = 0$.

Due the following theorem we can find the variance of the time of the attainment of the boundary.

Theorem 2. If conditions of Theorem 1 is fulfilled $M\left(\tau_x[r_1, r_2]\right)^2 = v_1(x)$, where $v_1(x)$ is the solution of the differential equation

$$\frac{(c^2\lambda(\rho) - d^2\mu^2)(\lambda(\rho)-\mu)}{2(\lambda(\rho)+\mu)\mu\lambda(\rho)}\,v''(x) + r(p-q)x(t)\,v'(x) = -2v(x),$$

with conditions $v_1(r_1) = v_1(r_2) = 0$ and $v(x)$ is function from Theorem 1.

O. Pavļenko. Apdrošināšanas kompānijas kapitāla kustības modelis. Modelis aprakstīts ar impulsu sistēmu ar divām lecienu virknēm, kuras atrisinājums konverģē uz difuzijas procesu. Doti daži raksturojumi šim procesam.