LIMIT THEOREM AND STABILITY OF IMPULSE SYSTEM WITH MARKOV SWITCHINGS, DEPENDENT ON COORDINATES

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Consider the family of right continuous Feller Markov processes $\{y(t),\ t \geq 0\}$ with parameter $x \in \mathbf{R}^n$, on countable space $\mathbf{Y} \subset R$ with infinitesimal generator Q_x :

$$(Q_x v)(y) = a(x,y) \sum_{z \in \mathbf{Y}} [v(z) p_x(y,z) - v(y)] \tag{1}$$

where $a(x,y)$ is positive continuous uniformly bounded function with two bounded $x-$derivatives; $p_x(y,z)$ is the transition probability of embedded Markov chain, having two continuous bounded $x-$derivatives. For fixed x the Markov process with infinitesimal generator(1) is piecewise constant process. Operators (1) are defined on the space $\mathbf{B}(\mathbf{Y})$ of all $\mathcal{B}_{\mathbf{Y}}$-measurable bounded real functions $v(y)$ with norm $\|v\| = \sup_y |v(y)|$. We shall require additively following conditions on Q_x :- there exist $\frac{d}{dx} Q_x v(y)$ and $\frac{d^2}{dx^2} Q_x v(y)$, which are bounded operators on $\mathbf{B}(\mathbf{Y})$. With y fixed, p_x, $\frac{dp_x(y,z)}{dx}$, $\frac{d^2 p_x(y,z)}{dx^2}$ are countably additive functions of bounded variation (on $\mathbf{B}(\mathbf{Y})$).

Besides, we will assume that Markov process with generator Q_x is uniformly ergodic for each x and has invariant measure μ_x.

It is proved, that there exists unique solution of system, consisting of differential equation

$$\frac{dx}{dt} = \varepsilon f_1(x, y(t)) + \varepsilon^2 f_2(x, y(t), \varepsilon) \tag{2}$$

with the initial condition $x(0) = x$ and condition of jump

$$x(t) = x(t-0) + \varepsilon g_1(x(t-0), y(t-0)) + \varepsilon^2 g_2(x(t-0), y(t-0), \varepsilon) \tag{3}$$

for time moments $t \in \{\tau_j,\ j \in N\}$, where times of switching have conditional exponential distribution:

$$\mathbf{P}(\tau_{j-1} - \tau_j > t \ / \quad y(\tau_{j-1}),\ x(\tau_{j-1})) = \exp\left(- \int_{\tau_{j-1}}^{\tau_{j-1}+t} a(x(s), y(\tau_{j-1})) ds \right).$$

We assume that functions f_j and g_j, $j = 1,2$ are bounded on x, continuous on y and have two bounded continuous x-derivatives.

Having $\sum_{y\in \mathbf{Y}} (f_1(x,y,\varepsilon) + a(x,y)g_1(x,y,\varepsilon))\,\mu_x(y) = 0$, one can analyse an asymptotic of solutions of (2)-(3) with $\varepsilon \to 0$.

Theorem 1 *Under the above hypotheses the processes* $x(\frac{t}{\varepsilon^2})$ *weakly converge as* $\varepsilon \to 0$ *to solution* $\overline{x}(t)$ *of stochastic diffusion equation*

$$dx = b(x)dt + \overline{A}(x)dw(t) \tag{4}$$

for any $T > 0$, $x \in \mathbf{R}^n$ *and* $y \in \mathbf{Y}$,

where $w(t)$ is a standard Wiener process in $\mathbf{R}^n$, vector $b(x)$ is defined by

$$b(x) = \sum_{y\in \mathbf{Y}} F_2(x,y)\mu_x(y) + \sum_{y\in \mathbf{Y}} \frac{d}{dx}[\Pi_x F_1(x,y)] f_1(x,y)\mu_x(y) +$$

$$+ \sum_{y\in \mathbf{Y}} a(x,y) \sum_{z\in \mathbf{Y}} \frac{d}{dx}[\Pi_x F_1(x,z)] g_1(x,y) p_x(y,z)\, \mu_x(y),$$

positive symmetric matrix $\overline{A}(x) = \{\overline{a}_{i,j}(x)\}_{i,j=1}^{n}$ is defined by formula:

$$\frac{1}{2} tr\left[\overline{A}^2(x)h\right] = \sum_{y\in \mathbf{Y}} (hF_1(x,y)\ ,\Pi_x F_1(x,y))\,\mu_x(y) -$$

$$- \sum_{y\in \mathbf{Y}} \left(hg_1(x,y)\ .\ \frac{a(x,y)}{2} g_1(x,y) + f_1(x,y) \right) \mu_x(y)$$

with arbitrary matrix h; $(\cdot,\cdot)$ is scalar product: $F_j(x,y) = f_j(x,y,\varepsilon) + a(x,y)g_j(x,y,\varepsilon)$, and

$$(\ \Pi_x F)(x,y) = \int_0^\infty \sum_{z\in \mathbf{Y}} (P_x(t,y,z) - \mu_x(z)) F(x,z)dt, \quad j = 1,2.$$

Theorem 2 *If the trivial solution of (4) is exponentially p-stable with some* $p > 0$, *then trivial solution of (2)-(3) is stable in probability for any* $\varepsilon \in (0,\varepsilon_0)$ *with some positive* ε_0.

N. Siņenko. Impulsu sistēmas ar Markova pārslēgumiem, kas atkarīgi no koordinātes, robežteorēma un stabilitāte. Apskatīta impulsu sistēmas stabilitāte, balstoties uz difūziju aproksimāciju.