ONE SIMPLE ANALYTICAL MODEL OF FINANCIAL DYNAMICS

An. Matveev

Riga Technical University

Let us consider a present value $X(t)$ of a bank account with interest rate $i(t)$. The equation of the account's growth at the moment t is

$$dX(t) = X(t)i(t)\,dt, \tag{1}$$

where $i(t)$ - process which can be described as stochastic differential equation of Markov type.

$$di(t) = a(t, i(t))\,dt + b(t, i(t))\,dw(t), \tag{2}$$

with some Winer process $w(t)$.

One of the most complicated problem [2] is to know the values of the parameters $a(t, i(t))$ and $b(t, i(t))$ of the diffusion process $i(t)$. If we know the evolution of the interest rate, we can described the quantities changing in connection with the present value of a bank account. We can conclude, that $X(t)$ depend on case only through $i(t, w)$, such way that to suppose the sufficiency smoothness $X(i(t))$ by t and i by using the Ito formula we can obtain

$$dX(i(t)) = X(i(t))\big[\alpha(i(t))\,dt + \beta(i(t))\,dw(t)\big]. \tag{3}$$

Here we reduce the system (1) - (2) to the linear stochastic differential equation (3), so we can solve it [1].

$$X(i(t)) = X(i(0))\,exp\Big\{\int_0^t \big[\alpha(i(s)) - \frac{1}{2}\beta^2(i(s))\big]\,ds + \int_0^t \beta(i(s))\,dw(s)\Big\}. \tag{4}$$

The equation (3) help us to analyze how the present value of a bank account depend on parameters assigned by interest rate. Now we can raise different proposals about koefficients such as a and b in equation (2). For example [4], we can consider the situation, when the interest rate i(t) is described by the process of Ornstein-Uhlenbeck with

$$di(t) = a(i(0) - i(t))\,dt + \sigma\,dw(t), \tag{5}$$

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where σ is the koefficient of changeability.

An. Matvejevs. Finansu dinamikas viens analitiskais modelis. Ar stohastisko diferenciālvienādojumu palīdzību tiek aprakstīta sistēma, kura raksturo bankas rēķina stāvokli. Tiek veidota šis sistēmas analīze.

References.

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