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# ITO FORMULA FOR QUADRATIC FUNCTIONALS

JEVGENIJS CARKOVS[1] AND KĀRLIS ŠADURSKIS[2]

*Riga Technical University*

Meža iela 1/4, LV-1048, Rīga, Latvia

E-mail: [1]`carkovs@latnet.lv`, [2]`skarlis@latnet.lv`

The paper deals with stochastic functional differential equations written in a quasilinear form

$$dx(t) = \{Ax_t + a(t, x_t)\}dt + \sum_{k=1}^{n} b_k(t, x_t)dw_k(t), \tag{1}$$

where $x_t$ is part of trajectory $\{x(t),\, t \in \mathbf{R}\}$ defined by equality $x_t := \{x(t+\theta), -h \leq \theta \leq 0\}$, $A$ is linear continuous mapping of the space $\mathbf{C} := C([-h, 0], \mathbf{R}^n)$ to $\mathbf{R}^n$, $\{w_k(t),\, k = 1, 2, ..., n\}$ are independent Brown motion processes, and mappings $a(t, \varphi)$, $b_k(t, \varphi)$ are continuous mappings of $\mathbf{R} \times \mathbf{C}$ to $\mathbf{R}^n$. As it has been proven in [1] many problems of dynamical analysis for equation (1) can be successfully solved if one succeeded in finding in an explicit form the differential $dv(x_t)$ for Lyapunov-Krasovsky type [2] functional $v(\varphi)$, that is, sufficiently smooth positive defined functional satisfying inequalities $c_1|\varphi(0)|^2 \leq v(\varphi) \leq c_2||\varphi||^2$. Our paper proves that there are sufficiently wide class of the above mentioned functionals which may be successfully used for Ito substitution in (1). These functionals are defined on the tensor product $\mathbf{C} \otimes \mathbf{C}$ by formula $< \mu\varphi, \varphi > := \int\limits_{-h}^{0}\int\limits_{-h}^{0} \varphi^T(\theta_1)\mu(d\theta_1, d\theta_2)\varphi(\theta_2)$, where $\mu(d\theta_1, d\theta_2)$ is such a symmetric count additive positive defined matrix valued measure on square $[-h, 0] \times [-h, 0]$ that quadratic functional $v(\varphi) :=< \mu\varphi, \varphi >$ has Lyapunov derivative [2] by virtue of corresponding to (1) linear equation. Let us denote $y(t, \varphi)$ the solution of Cauchy problem $y_0 = \varphi$ for linear equation $dy(t) = Ay_t$.

Theorem 1. *If* $\lim\limits_{t \to 0} \frac{1}{t}[< \mu y_t(\varphi), y_t(\varphi) > - < \mu\varphi, \varphi >] :=< \nu\varphi, \varphi >$ *for any* $\varphi \in \mathbf{C}$ *then stochastic process* $< \mu x_t, x_t >$ *has stochastic Ito differential*

$$\begin{aligned} d < \mu x_t, x_t > \;&=\; < \nu x_t, x_t > dt + \left\{ 2 < \mu \mathbf{1} a(t, x_t), x_t > + \sum_{k=1}^{n} < \mu \mathbf{1} b_k(t, x_t), \mathbf{1} b_k(t, x_t) > \right\} dt \\ &+\; 2\sum_{k=1}^{n} < \mu \mathbf{1} b_k(t, x_t), x_t > dw_k(t) \end{aligned}$$

*for any solution* $x(t)$ *of equation* (1), *where* $\mathbf{1}(\theta) = 0$ *for* $\theta \in [-h, 0)$ *and* $\mathbf{1}(0)$ *is* $n$*-dimensional matrix unit.*

### REFERENCES

[1] E. Tsarkov. *Random perturbations of functional differential equations.* Zinatne, Riga, 1989. (in Russian)

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