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ASYMPTOTICAL ANALYSIS OF CONTINUOUS TIME STOCHASTIC REGRESSION MODELS

JOLANTA GOLDSTEINE and OKSANA PAVLENKO

University of Latvia and Riga Technical University

Kalku iela 1, LV-1658, Rīga, Latvia

E-mail: oks@tvnet.lv, jolanta_g@osi.lv

Let consider two piece-wise processes $x(t)$, $y(t)$ with jumps in random time moments $\{\tau_j,\ j \in \mathbf{N}\}$, the same for both processes. These switching moments are the Poisson flow. The process $x(t)$ is described by the auto-regressive equation of the first order with mean which is dependent on conditional variance error, i.e. Generalized Auto-Regressive Conditional Heteroskedastic model GARCH-M (Engle, Lilien, Robins, 1987) with some additional conditions. It differs from usual ARMA and GARCH processes because switching moments are random, not deterministic. $y(t)$ is the conditional variance of $x(t)$, which is described by the equation with one auto-regressive term and one moving average term. I.e. the model can be described by two dynamical systems of equations (1)-(2) and (3)-(4):

$$\frac{dx}{dt} = 0 \tag{1}$$

$$x(t) = A(y(t))x(t-) + \sigma_t \tag{2}$$

$$\frac{dy}{dt} = 0 \tag{3}$$

$$y(t) = a_0 + a_1\sigma_{t-1}^2 + by(t-1) \tag{4}$$

related by the following equation:

$$\sigma_t = \nu_t\sqrt{y_t} \tag{5}$$

As in usual GARCH-M model, let take $E\nu_t = 0$, $E\nu_t^2 = 1$. Let introduce the additional restriction, that ν_t is a Markov process and it can obtain only finite number of values. Due to the condition of the stationarity all the coefficients of the equation (4) need to be positive and $a_1 + b < 1$. Let introduce some small positive parameter ε in the model (1)-(5). One can see, that the system (3)-(4) doesn't dependent on x directly. So we analyse the solution of (3)-(4) at first, and then (1)-(2). One can use the averaging principle for both systems, i.e. write the averaging equations, that the solutions of impulse systems tend to the solutions of these averaging equations, as $\varepsilon \to 0$. One can use the diffusion approximation, when the right parts of the averaging equations are equal to zero. After some mathematical operations, which are not difficult, but time-consuming, we obtain the diffusion equations, such, that the solutions of initial impulse systems tend to the solutions of these equations, as $\varepsilon \to 0$. The solutions of these diffusion equations can be modeled in MATHEMATICA and MATLAB. The processes $x(t)$, $y(t)$ are modelled for given values of parameters according to (1)-(5), and the solutions of the obtained diffusion equations are modeled too. Then the hypothesis about equality of distributions are tested. One interesting example can be obtained, if we take $A(y) = 1 + \varepsilon A_1$ and the parameters of infinitesimal matrix are equal. Then the coefficients of drift in both diffusion equations are equal to zero. The diffusion equations are idependent in this case.