Medical object 3D modelling using B-spline surface

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Introduction. 3D visualization of a medical object is an important and actual task in medical engineering. At the same time, the input data for this task often is described in 2D layer form (for example, medical images acquired with Magnetic Resonance Imaging, Computer Tomography, etc). In this case, the most important part of the complex visualization task is the 3D modelling of a medical object. One approach, based on Bezier surfaces is described in [1]. This approach uses local surface interpolation and produces a mathematical model based on the set of Bezier surfaces as a result. In this work, the approach of using only one B-spline surface for 3D modelling is described. This approach is based on the global surface interpolation method.

Medical object's mathematical formulation using B-spline surface. The input data for developing the mathematical model of a medical object is an array of points, obtained from segmented regions (some approaches for solving this task are described in [2 and 3]). In this case, the task of medical image visualization can be reduced to the task of creating a modeled surface using one patch.

B-spline surfaces play an important role in the current surface designing methods. They are flexible and powerful mathematical tools for 3D surface modeling. Mathematically, B-spline surface may be described as [4]:

$$S(u,v) = \sum_{i=0}^{n} \sum_{j=0}^{m} P_{i,j} \cdot N_{i,p}(u) \cdot N_{j,q}(v), \quad (1)$$

where $P_{i,j}$ – control points; $N_{i,k\_u}(u)$, $N_{j,k\_v}(v)$ – the B-spline polynomials; p, q – polynomial degree coeficient; u, v – parameters.

Let's assume that the input data of medical object is a grid of $cr \times pt$ data points, where cr – crosscuts number and pt – control points number in each crosscut. For description of these points using B-spline surface it is necessary to satisfy the following factors:

1) B-spline surface must be closed;
2) for implementation 1) it is necessary to use a periodical in one parametric direction of B-spline surface.

The degree of surface in each parametric direction is acquired from the user (coefficients p and q). Coefficients n and m (that describe the size of control points array) may be described as:

$$n = cr - 1, \quad (2)$$

$$m = pt + q - 1. \quad (3)$$

In this case, the array's size is $(n+1)\times(m+1)$ and the array of control point for B-spline surface interpolation is:

$$[D^*] = \begin{bmatrix} \overbrace{\begin{matrix} Q_{0,0} & \cdots & Q_{0,pt-1} \\ \vdots & \ddots & \vdots \\ Q_{cr-1,0} & \cdots & Q_{cr-1,pt-1} \end{matrix}}^{pt\_columns} & \overbrace{\begin{matrix} Q_{0,0} & \cdots & Q_{0,kv-1} \\ \vdots & \ddots & \vdots \\ Q_{cr-1,0} & \cdots & Q_{cr-1,kv-1} \end{matrix}}^{kv\_columns} \end{bmatrix}, \quad (4)$$

where $Q_{i,j}$ – initial control points of medical object.

B-spline surface is an approximating mathematical model, but there is an approach that allows the creation of a surface by passing through all initial data control points [5]. For this, it is necessary to find the control point array [P] for the equivalent surface. This approach is known as the global surface interpolation.

Global surface interpolation. Problem statement: initial data is a grid of $(n+1)\times(m+1)$ data points D_{ij} ($0 \le i \le n$ and $0 \le j \le m$) and a degree (p, q), it is necessary to find a B-spline surface of degree (p, q) defined by $(n+1)\times(m+1)$ control points that passes all data points in the given order.

B-spline surface can be described as in (1). Since the surface passes all data points:

$$D_{i1,j1} = S(u_{i1}, v_{j1}) = \sum_{i=0}^{n}\sum_{j=0}^{m} P_{i,j} \cdot N_{i,p}(u_{i1}) \cdot N_{j,q}(v_{j1}), \quad (5)$$

In a matrix view the given equations can be described as follows:

$$[D] = [C] \cdot [P] \quad (6)$$

where [D] – matrix of surface points; [C] – matrix of coefficients; [P] – matrix of control points.

These matrixes can be described as follows:

$$[P] = [P_{00} \;\; \ldots \;\; P_{0j} \;\; P_{10} \;\; \ldots \;\; P_{1j} \;\; \ldots \;\; P_{i0} \;\; \ldots \;\; P_{i,j}]^T, \quad (7)$$

$$[D] = [D_{00} \;\; \ldots \;\; D_{0j} \;\; D_{10} \;\; \ldots \;\; D_{1j} \;\; \ldots \;\; D_{i0} \;\; \ldots \;\; D_{i,j}]^T, \quad (8)$$

$$\begin{aligned} [C] = [&C_r(u_0, v_0) \;\; \ldots \;\; C_r(u_0, v_j) \;\; C_r(u_1, v_0) \;\; \ldots \\ &C_r(u_1, v_j) \;\; \ldots \;\; C_r(u_i, v_0) \;\; \ldots \;\; C_r(u_i, v_j)]^T \end{aligned}, \quad (9)$$

where $C_r(u,v)$ – row of matrix [C] what can be described as follows:

$$C_r(u,v) = \begin{bmatrix} N_{0,p}(u,v)\cdot N_{0,q}(u,v) \\ \dots \\ N_{0,p}(u,v)\cdot N_{j,q}(u,v) \\ N_{1,p}(u,v)\cdot N_{0,q}(u,v) \\ \dots \\ N_{1,p}(u,v)\cdot N_{j,q}(u,v) \\ \dots \\ N_{i,p}(u,v)\cdot N_{0,q}(u,v) \\ \dots \\ N_{i,p}(u,v)\cdot N_{j,q}(u,v) \end{bmatrix}^T, \qquad (10)$$

The system of linear equations (6) can be solved using different methods. In this work Gauss – Jordan elimination is used.

Experimental result. In this work the described method, as well as the methods described in [1] were implemented. Control points array (size 29×18) obtained from [1] was used as initial data. The size of the images in experiment is 512*512 pixels. The experiments were carried out on the computer with CPU Intel Xeon 3,2 GHz, RAM 2 GB. The received images are shown in a Fig. 1.

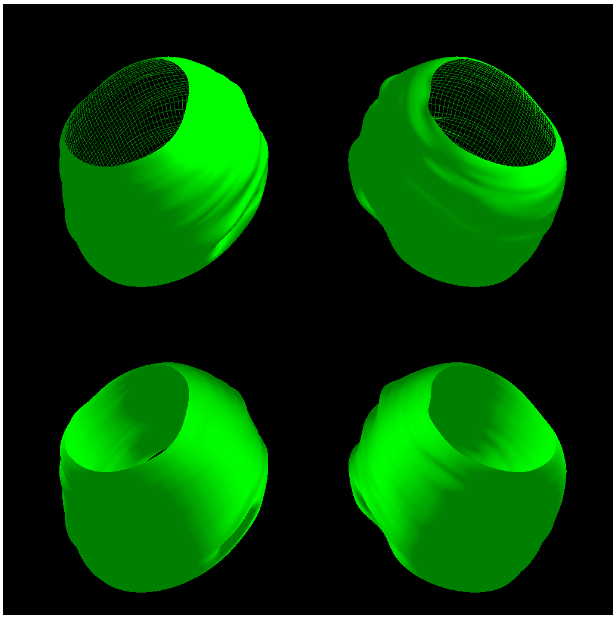

Fig. 1. Top row: models obtained using standard OpenGL library.
Bottom row: models obtained by ray tracing.

For the visualization of the medical object's resulting model by ray tracing, the methods of Bezier and B-spline surfaces visualization were used (described in [6 and 7]).

Conclusions. The described method is a flexible and powerful approach for medical object's 3D modelling and visualization. At the same time this method needs to do a great deal of calculations.

Medical object 3D visualization is based on standard graphic library OpenGL and on an alternative method of visualization – ray tracing. All the proposed methods were implemented in a practical tool for medical image processing. The developed program can be installed on any computer. That will allow the physicians to visualize the medical images outside the hospitals.

References

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One of the most important and actual tasks in medical engineering are 3D visualization of medical object. At the same time, the input data for this task often is described in 2D layer form (for example, medical images acquired with Magnetic Resonance Imaging, Computer Tomography, etc). In this case, the most important part of complex visualization task is medical object 3D modelling. In this work the approach of using of B-spline surfaces for 3D modelling is described. The approach is based on global surface interpolation method. This approach is a flexible and powerful tool for medical object 3D modelling, depending on the current medical task.