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RATIONAL STRUCTURE OF TRUSSED BEAM

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Abstract. Using of subdiagonals is one of the most common methods to decrease the materials consumption of the beams in the case of increased spans. Two variants of the trussed steel beam with the span in 12 m were considered. The considered beams are the main load bearing elements of the roof structure. The trussed beams consist of the top and bottom chords, struts and pillars.

Rational from the point of view of materials consumption distances from the support to the joint of the edge strut with the top and bottom chords, height of trussed beam and vertical dimension of the edge strut were evaluated by the response surface and inspection methods. The dependences of materials consumption and distances from the support to the joint of the edge strut with the top and bottom chords, height of trussed beam and vertical dimension of the edge strut were obtained as the second order polynomial equation on the base of numerical experiment.

Rational from the point of view of materials consumption structure of the trussed beam was suggested.

Keywords: steel trussed beam, inclined edge struts, materials consumption, numerical experiment, second order polynomial approximation.

1. Introduction

The problem of efficiency increase of steel structures is one of the most significant at the present moment (Shen *et al.* 2005). The different methods of optimization are used for determination of rational parameters of steel structures. The widely known methods are the Newton method, conjugate gradient method, response surface method, hybrid genetic algorithm and others (Fletcher 2009; Montgomery 2001; Peleshko *et al* 2009; Адлер *et al.* 1976; Баничук 1986). The trussed beams (Fig1) took a special position among the plane steel structures as a link between the beams and trusses.

The regulation of stress-strain state of bended steel structures in the design stage is one of the methods to increase the efficiency of structures (Трофимович and Пермяков 1981). The stress-strain state regulation for beams can be obtained by the using of subdiagonals. The beam is transformed in to continuous one with the complained supports in this case (Gogol 2009). More rational bending moment distribution for trussed beam, in comparison with the simple one, allows decreasing of materials consumption. But the more rational bending moment distribution and materials consumption can be obtained in the case when the trussed beam possesses a rational structure.

The most widely used types of trussed beams are beams with the central pillar, beams with the vertical edge and central pillars and beams with the inclined edge struts (Fig.1) (Gogol 2009; Михайлов 2002; Беленя 1991; Шмидт and Дмитриев 2002; Барабаш *et al.* 2008; Веденикoв 1998).

The comparison of the trussed beams types from the point of view of materials consumption under the action of the uniformly distributed load with intensity in 1 kN/m indicates that the most rational are types with the vertical edge struts and with the inclined edge struts. The trussed beams with the heights and spans equal to 1.5 and 12 m, correspondingly, were considered.

In this connection, the aim of the paper is to develop rational from the point of view of materials consumption structure of trussed beams with the vertical edge and central pillars and with the inclined edge struts and vertical central pillar. The dependences of the main geometric parameters of the trussed beams and materials consumption also must be obtained.

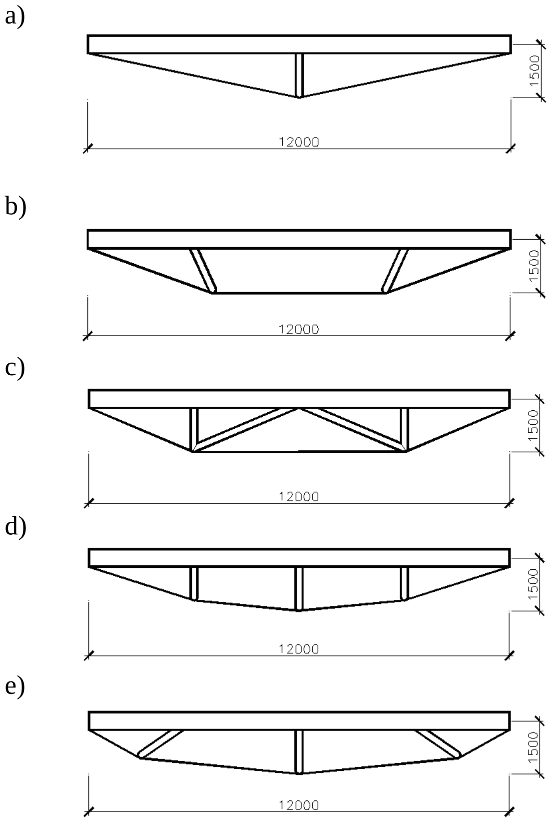

Fig 1. Types of the trussed steel beams: a) – type with the central pillar; b) – type with the inclined edge struts; c) – type with the inclined central struts and edge pillars; d) – type with the vertical edge and central pillars; e) – type with the inclined edge struts and vertical central pillar.

2. Solution of the problem

The trussed steel beams with the vertical edge and central pillars and with the inclined edge struts and vertical central pillar were considered as the objects of investigation. The spans of the beams were equal to 12 m (Gogol 2009). The development of rational structures of trussed beams was joined with the determination of it rational, from the point of view of materials consumption, geometric parameters. The main geometric parameters of trussed beam with the vertical edge and central pillars are heights of beam and edge pillar and distance from the support to the joint of edge pillar with the top chord (Fig 2).

The main geometric parameters of trussed beam with the inclined edge struts and vertical central pillar are distances from the support to the joint of the edge strut with the top and bottom chords, height of trussed beam and vertical dimension of the edge strut (Fig 2).

The beams were considered under the action of uniformly distributed load with intensity in 1 kN/m. The bottom chord is not prestressed. Steel with the point of yield in 280 MPa was considered as material of top chord, struts and pillars. Steel cable on the base of the wire with tensile strength in 1960 MPa and modulus of elasticity in $1.7 \cdot 10^5$ MPa was considered as the material of bottom chord.

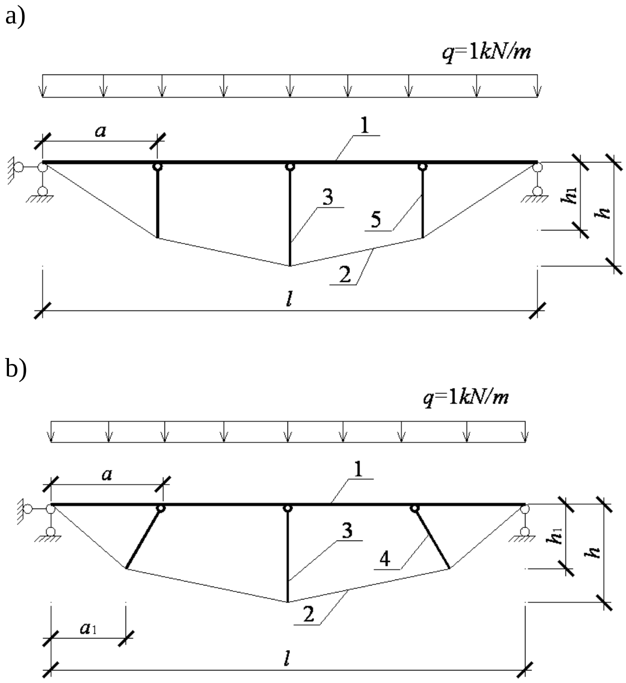

Fig 2. Design schemes of the trussed steel beams: *a*) – variant with the vertical edge pillar; *b*) – variant with the inclined edge struts; 1 – top chord; 2 – bottom chord; 3 – central pillar; 4 – edge strut; 5 – edge pillar; *a* – distance from the support to the joint of the edge strut with the top chord; a_1 – distance from the support to the joint of the edge strut with the bottom chord; *h* – height of the trussed beam; h_1 – vertical dimension of the edge strut or edge pillar.

Rational from the point of view of materials consumption main geometric parameters of trussed beams with the vertical edge and central pillars and with the inclined edge struts and vertical central pillar (Fig 2) were evaluated by the response surface method (Myers and Montgomery 2002) and by the enumerative technique. The dependence of the materials consumption on the heights of the beam, edge pillar and distance from the support to the joint of edge pillar with the top chord (Fig.2. a)), was evaluated in the form of second order polynomial equations (1) (Спиридонов 1981; Serdjuks and Rocens 2004):

$$P_a = b_0 + b_1 \cdot h + b_2 \cdot a + b_3 \cdot h_1 + b_{11} \cdot h^2 + b_{12} \cdot h \cdot a + \\ + b_{13} \cdot h \cdot h_1 + b_{22} \cdot a^2 + b_{23} \cdot a \cdot h_1 + b_{33} \cdot h_1^2 \tag{1}$$

The dependence of the materials consumption on the distances from the support to the joint of the edge strut with the top and bottom chords so as height of trussed beam and vertical dimension of the edge strut (Fig 2. b)) was evaluated by the same approach, as the equation (1):

$$P_b = b_0 + b_1 \cdot h + b_2 \cdot a + b_3 \cdot a_1 + b_4 \cdot h_1 + b_{11} \cdot h^2 + \\ + b_{12} \cdot h \cdot a + b_{13} \cdot h \cdot a_1 + b_{14} \cdot h \cdot h_1 + b_{22} \cdot a^2 + \\ + b_{23} \cdot a \cdot a_1 + b_{24} \cdot a \cdot h_1 + b_{33} \cdot a_1^2 + b_{34} \cdot a_1 \cdot h_1 + b_{44} \cdot h_1^2 \tag{2}$$

The coefficients of equations (1) and (2) were determined on the base of numerical experiment. The experiment was joined with the determination of internal forces, which acts in the elements of the truss when the uniformly distributed load is applied. The dimensions of elements cross-sections were determined then on the base of obtained internal forces.

Coefficients of the equation (1) b_1, b_2, b_3, b_{12}, b_{13}, b_{123}, b_{23} were determined by the equations (3-9), but the coefficients b_0, b_{11}, b_{22}, b_{33} were determined by the equation system (10), which were obtained on the base of least square method.

$$b_1 = \frac{\sum_{i=1}^{n} P_{a,i} h}{\sum_{i=1}^{n} h^2} \tag{3}$$

$$b_2 = \frac{\sum_{i=1}^{n} P_{a,i} a}{\sum_{i=1}^{n} a^2} \tag{4}$$

$$b_3 = \frac{\sum_{i=1}^{n} P_{a,i} h_1}{\sum_{i=1}^{n} h_1^2} \tag{5}$$

$$b_{12} = \frac{\sum_{i=1}^{n} P_{a,i} ha}{\sum_{i=1}^{n} h^2 a^2} \tag{6}$$

$$b_{13} = \frac{\sum_{i=1}^{n} P_{a,i} hh_1}{\sum_{i=1}^{n} h^2 h_1^2} \tag{7}$$

$$b_{123} = \frac{\sum_{i=1}^{n} P_{a,i} hah_1}{\sum_{i=1}^{n} h^2 a^2 h_1^2} \tag{8}$$

$$b_{23} = \frac{\sum_{i=1}^{n} P_{a,i} ah_1}{\sum_{i=1}^{n} a^2 h_1^2} \tag{9}$$

$$\begin{cases} nb_0 + \sum\limits_{i=1}^{n} h^2 b_{11} + \sum\limits_{i=1}^{n} a^2 b_{22} + \sum\limits_{i=1}^{n} h_1^2 b_{33} = \sum\limits_{i=1}^{n} P_{a,i} \\ \sum\limits_{i=1}^{n} a^2 b_0 + \sum\limits_{i=1}^{n} h^2 a^2 b_{11} + \sum\limits_{i=1}^{n} a^4 b_{22} + \sum\limits_{i=1}^{n} a^2 h_1^2 b_{33} = \sum\limits_{i=1}^{n} P_{a,i} a^2 \\ \sum\limits_{i=1}^{n} h^2 b_0 + \sum\limits_{i=1}^{n} h^4 b_{11} + \sum\limits_{i=1}^{n} h^2 a^2 b_{22} + \sum\limits_{i=1}^{n} h^2 h_1^2 b_{33} = \sum\limits_{i=1}^{n} P_{a,i} h^2 \\ \sum\limits_{i=1}^{n} h_1^2 b_0 + \sum\limits_{i=1}^{n} h^2 h_1^2 b_{11} + \sum\limits_{i=1}^{n} a^2 h_1^2 b_{22} + \sum\limits_{i=1}^{n} h_1^4 b_{33} = \sum\limits_{i=1}^{n} P_{a,i} h_1^2 \end{cases} \tag{10}$$

where: n – amount of the experiments; $P_{a,i}$ – value of materials consumption for i-th experiment.

Coefficients of the equation (2) were determined by the same method as coefficients of the equation (1).

The elements of the top chord, struts and pillars are subjected to the compression with the bending. The type of cross-sections of top chord, struts and pillars was rectangular to prevent influence of the assortment. The height/width relations were equal to 2 for the elements of top chord and to 1 for the struts and pillars.

The bottom chord is tensioned and its cross-sectional area was determined by the equation (11) (Ермолов 1991):

$$A \geq \frac{1.6 \cdot N}{k \cdot R_{un}} \tag{11}$$

where: A – cross-sectional area of the bottom chord; N – force acting in the bottom chord; R_{un} – tensile strength of the wire; k – coefficient, taking into account the drop in the breaking force of the cable, caused by the inhomogeneity of stress distribution; 1.6 – reliability index of the material.

The coefficient k, taking into account the drop in the breaking force of the cable, caused by the inhomogeneity of stress distribution, was equal to 0.75 (Трущев 1983).

Rational values of the heights of edge and central pillars and distance from the support to the joint of edge pillar with the top chord were determined by the system of equations (12) and then were precised by the inspection.

$$\begin{cases} \dfrac{\partial P_a}{\partial h} = b_1 + b_{12} \cdot a + b_{13} \cdot h_1 + 2 \cdot b_{11} \cdot h = 0, \\ \dfrac{\partial P_a}{\partial a} = b_2 + b_{12} \cdot h + b_{23} \cdot h_1 + 2 \cdot b_{22} \cdot a = 0, \\ \dfrac{\partial P_a}{\partial h_1} = b_3 + b_{13} \cdot h + b_{23} \cdot a + 2 \cdot b_{33} \cdot h_1 = 0. \end{cases} \tag{12}$$

Rational values of the heights of the distances from the support to the joint of the edge strut with the top and bottom chords, height of trussed beam and vertical dimension of the edge strut were evaluated by the inspection.

Minimum materials consumption was determined on the base of obtained rational values of trussed beams geometric parameters. Second limit state was not taken into account.

3. Numerical results

3.1. Numerical Results for Trussed Beam with the Vertical Edge Pillars

Twenty seven variants of trussed beam with the vertical edge pillars were analised using the computer program LIRA 9.4 (Городецкий and Евзоров 2005). The variants were differed by the values of heights of the beam and central pillars and distance from the support to the joint of edge pillar with the top chord. The heights of the edge pillar change within the limits from 1 to 4 m. The relation of the edge pillars height to the beams height, changes within the limits from 0.65 to 0.85. The distance from the support to the joint of edge pillar with the top chord of the trussed beam changes within the limits from 2.6 to 3.2 m.

Coefficients of second order polynomial equation, which describe the dependences of material consumption on the heights of the beam and edge pillars and distance from the support to the joint of edge pillar with the top chord, are given in the Table 1. The coefficient of multiple determination R^2, which describes the model adequacy, is equal to 0.85, that satisfy current engineering task.

Table 1. Coefficients of the equation (1)

Designation of coefficients	Values of coefficients
b_0	387,0540
b_1	-3,6141
b_2	16,2759
b_3	-905,5000
b_{11}	2,4378
b_{12}	-2,4778
b_{13}	-4,4444
b_{22}	53,2778
b_{23}	-422,7780
b_{33}	1404,3300

The dependence of materials consumption on the height of the beam and distance from the support to the joint of edge pillar with the top chord of the trussed beam is shown in Fig 3. The height of edge pillar is equal to three quarter of central pillars height.

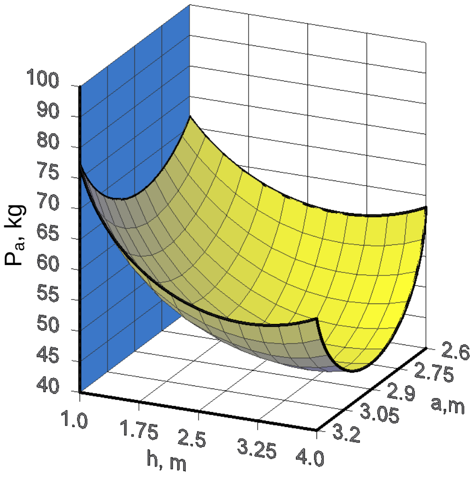

Fig 3. The dependence of materials consumption on the heights of the beam and distance from the support to the joint of edge pillar with the top chord of the trussed beam: P_a – materials consumption of trussed beam with the vertical edge pillar; h – height of the beam; a – distance from the support to the joint of edge pillar with the top chord. The height of edge pillar is equal to three quarter of the beams height.

The dependence indicates that materials consumption changes within the limits from 44.270 to 77.170 kg.

The dependence of materials consumption on the heights of the beam and edge pillar is shown in Fig 4. The dependence was obtained when the distance from the support to the joint of edge pillar with the top chord of the trussed beam was constant and equal to 2.9 m. The shapes of obtained dependences indicate that the rational values of the heights of edge and central pillars and distance from the support to the joint of edge pillar with the top chord can be determined by the system of equations (12) and the obtained values will be within the limits of the experiments field.

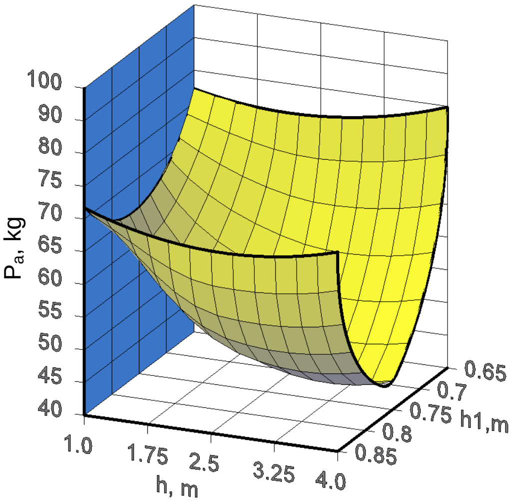

Fig 4. The dependence of materials consumption on the heights of the beam and edge pillar: P_a – materials consumption of trussed beam with the vertical edge pillar; h_1 – height of the edge pillar. Other designations as in Fig 3. The distance from the support to the joint of edge pillar with the top chord of trussed beam is equal to 2.9 m.

The dependence indicates that materials consumption changes within the limits from 44.270 to 85.650 kg.

The rational from the point of view of materials consumption values of height of the beam and distance from the support to the joint of edge pillar with the top chord are equal to 2.560 and 2.740m, respectively. The rational value of edge pillars height is equal to 1.820 m. Minimum value of materials consumption is equal to 42.97 kg.

3.2. Numerical Results for Trussed Beam with the Inclined Edge Struts

Eighty one variants of trussed beam with the inclined edge struts were analised using the computer program LIRA 9.4. The variants were differed by the values of heights of the beam, distance from the support to the joint of the edge strut with the top chord of trussed beam, distance from the support to the joint of the edge strut with the bottom chord of trussed beam and vertical dimension of the edge strut. The height of the beam change within the limits from 1.9 to 2.5 m. The distance from the support to the joint of the edge strut with the top chord change within the limits from 2.5 to 2.9 m. The distance from the support to the joint of the edge strut with the bottom chord change within the limits from 1.2 to 2.0 m. The vertical dimension of the edge strut change within the limits from 1.25 to 1.65 m.

Coefficients of second order polynomial equation, which describe the dependence of the materials consumption on the distances from the support to the joint of the edge strut with the top and bottom chords so as height of trussed beam and vertical dimension of the edge strut, are given in the Table 2. The coefficient of multiple determination R^2, which describes the model adequacy, is equal to 0.86, that satisfy current engineering task.

Table 2. Coefficients of the equation (2)

Designation of coefficients	Values of coefficients
b_0	215.358
b_1	-175.258
b_2	-171.295
b_3	-48.6105
b_4	455.119
b_{11}	68.1405
b_{12}	54.55
b_{13}	34.5403
b_{14}	-235.703
b_{22}	26.7259
b_{23}	19.1656
b_{24}	-99.8139
b_{33}	3.9044
b_{34}	-62.5264
b_{44}	156.977

The rational from the point of view of materials consumption values of main geometric parameters of trussed beam with the inclined edge struts were evaluated by the inspection. The distances from the support to the joint of the edge strut with the top and bottom chords changes with the step 0.02 m from 2.5 to 2.9 m and from 1.2 to 2 m, respectively. The heights of the beam and the vertical dimension of the edge strut changes with the step 0.02m, from 1.9 to 2.5 m and from 1.25 to 1.65 m, respectively.

The graphs of the dependence (2) are shown in Figs 5 and 6. The dependence of materials consumption on the height of the beam and distance from the support to the joint of the edge strut with the top chord of trussed beam is shown in Fig 5. The graphs were obtained at the fixed values of vertical dimension of the edge strut and distances from the support to the joint of the edge strut with the bottom chord. The vertical dimension of the edge strut was constant and equal to 1.45 m. The distances from the support to the joint of the edge strut with the bottom chord was fixes at the level of 1.20 m.

The dependence of materials consumption on the distances from the support to the joint of the edge strut with the top chord and vertical dimension of the edge strut is shown in Fig 6.

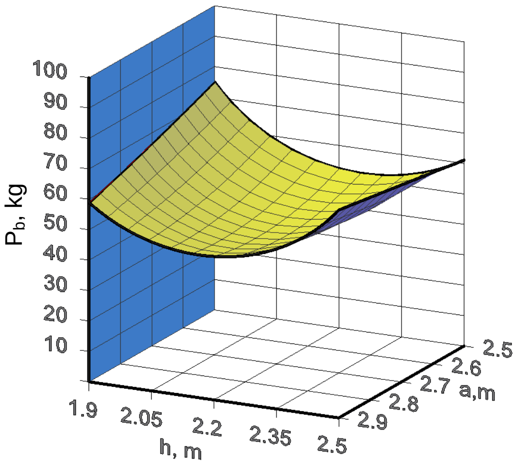

Fig 5. The dependence of materials consumption on the height of the beam and distance from the support to the joint of the edge strut with the top chord of trussed beam with the inclined edge struts: P_b – materials consumption of trussed beam; h – height of the beam; a – distance from the support to the joint of the edge strut with the top chord of trussed beam; The vertical dimension of the edge strut was equal to 1.45 m. The distances from the support to the joint of the edge strut with the bottom chord was equal to 1.20 m.

The rational, from the point of view of materials consumption, values of distances from the support to the joint of the edge strut with the top and bottom chords are equal to 2.840 and 1.910 m, respectively. The rational values of beams height and vertical dimension of the edge strut are equal to 2.480 and 1.630 m, respectively. Minimum value of materials consumption is equal to 40.919 kg.

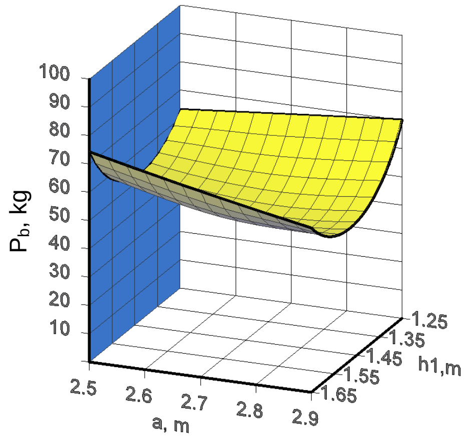

Fig 6. The dependence of materials consumption on the distances from the support to the joint of the edge strut with the top chord and vertical dimension of the edge strut: h_1 – vertical dimension of the edge strut. Other designations as in Fig 5. The distance from the support to the joint of the edge strut with the bottom chord of trussed beam is equal to 2 m. The height of the beam was equal to 2.2 m.

So, using of inclined edge struts instead of vertical pillars allows to decrease materials consumption of trussed beam by 5 %. The inclined edge struts allows to decrease more effectively the values of bending moments, which act in the top chord of trussed beam under the action of vertical uniformly distributed load.

Conclusions

The rational from the point of view of materials consumption structure of trussed beams with the vertical edge and central pillars and with the inclined edge struts and vertical central pillar was suggested.

It was shown, that the rational from the point of view of materials consumption values of height of the beam and distance from the support to the joint of edge pillar with the top chord of trussed beams with the vertical edge and central pillars, are equal to 2.560 and 2.740 m, respectively. The rational value of edge pillars height is equal to 1.820 m. Minimum value of materials consumption is equal to 42.97 kg.

The rational from the point of view of materials consumption values of distances from the support to the joint of the edge strut with the top and bottom chords of trussed beams with the inclined edge struts and vertical central pillar, are equal to 2.840 and 1.910 m, respectively. The rational values of the beams height and vertical dimension of the edge strut are equal to 2.480 and 1.630 m, respectively. Minimum value of materials consumption is equal to 40.919 kg.

It was stated, that using of inclined edge struts instead of vertical pillars allows to decrease materials consumption of trussed beam by 5 %.

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