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BEHAVIOUR OF MULTILAYER SHEET WITH TECHNOLOGICAL IMPERFECTION

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Abstract. In the civil engineering and mechanical engineering are widely used multilayer composite sheets that are designed with symmetrical structure with respect to sheet mid-surface. As a result of technological imperfections and tolerances the actual structure of the sheet is asymmetrical. The reasons of asymmetry may be many the most important are deviation of orientation angle for each layer, deviation of thickness of each layer and deviation of elastic characteristics. Using Monte-Carlo simulation is numerically analyzed the correlation between standard deviation of different technological tolerances and curvature, torsion of the sheet. Two kind of analysis is done- by taking into account geometrical non-linearity and without it.

Keywords: technological imperfections, multilayer wood composite, geometrical non-linearity, curvature, torsion.

Introduction

It is very popularly to use multilayer sheets in civil and mechanical engineering. Building process can be simplified and speed up by using of multilayer composite sheets. The sheet has to keep its initial shape. Unfortunately it cans change in process of environment changing because of technological imperfection.

Usually the sheets are made with symmetrical structure respect to its mid-surface. These sheets are beneficial in many cases because its behaviour under external load or environment influence is easy to understand. The sheet with asymmetrical structure deforms in very unusual way, sometimes it is beneficial, sometimes not. As a result of technological imperfections the sheets, which are designed with symmetrical structure, obtains some asymmetry. The reasons of asymmetry usually are geometrical imperfections and imperfections of elastic characteristics of the sheet. The geometrical imperfections arise from deviation of orientation angle of each layer and deviation of layer thickness.

The most popular example of composite sheet with multilayer structure is wood veneer. Usually it is manufactured from birch wood, therefore in this article are especially analysed birch wood veneer sheets.

The stress-strain field in multilayer sheet could be analysed by a few mathematical models. The most common analytical model is described in (Reissner and Stavsky 1961, Rocens and Steiners 1976, Brauns and Rocens 1997). This model is based on assumptions of small deformations and linear material behaviour, but same very simple method for taking into account large curvature and elastic characteristic change in moisture content changing process is described in (Sliseris and Rocens 2009). The most popular numerical method is Finite element method (FEM) (Ochoa and Reddy 1992, Reddy 1997). It could be modified for small deformations and also for large deformations.

Large displacement analysis of multilayer sheet

In general the components of strain is expressed by Green-Lagrange strain tensor (Decolon 2002) that takes into account large displacement influence on results

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i} \cdot \frac{\partial u_k}{\partial x_j}\right); \tag{1}$$

where: i=1..3; j=1..3; k=1..3; ε_{ij} – components of strains; u_i – displacement; x_i – global Cartesian coordinate system.

The Eq.(1). provide a linear part:

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \tag{2}$$

and nonlinear part

$$\frac{1}{2}\frac{\partial u_k}{\partial x_i}\cdot\frac{\partial u_k}{\partial x_j}=\frac{1}{2}\left(\frac{\partial u_1}{\partial x_i}\cdot\frac{\partial u_1}{\partial x_j}+\frac{\partial u_2}{\partial x_i}\cdot\frac{\partial u_2}{\partial x_j}+\frac{\partial u_3}{\partial x_i}\cdot\frac{\partial u_3}{\partial x_j}\right) \quad (3)$$

If the transversal displacement u_3 of the sheet is higher then in-plane displacements u_1 and u_2, then the non-linear part reduces as following:

$$\frac{1}{2}\frac{\partial u_k}{\partial x_i}\cdot\frac{\partial u_k}{\partial x_j}=\frac{1}{2}\left(\frac{\partial u_3}{\partial x_i}\cdot\frac{\partial u_3}{\partial x_j}\right). \quad (4)$$

Green-Lagrange strain tensor could be reduced to so-called Von Karman (Амбарцумян 1961) strains:

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)+\frac{1}{2}\frac{\partial u_3}{\partial x_i}\cdot\frac{\partial u_3}{\partial x_j}. \quad (5)$$

By using Kirchhoff-Love theory (Christensen 1991, Jones 1975, Whitney 1987) the strain field reduces to following equations:

$$\varepsilon_{11}=\frac{\partial u_1^0}{\partial x_1}-x_3\frac{\partial^2 u_3^0}{\partial x_1^2}+\frac{1}{2}\left(\frac{\partial u_3^0}{\partial x_1}\right)^2, \quad (6)$$

$$\varepsilon_{22}=\frac{\partial u_2^0}{\partial x_2}-x_3\frac{\partial^2 u_3^0}{\partial x_2^2}+\frac{1}{2}\left(\frac{\partial u_3^0}{\partial x_2}\right)^2, \quad (7)$$

$$\varepsilon_{12}=\frac{1}{2}\left(\frac{\partial u_1^0}{\partial x_2}+\frac{\partial u_2^0}{\partial x_1}\right)-x_3\frac{\partial^2 u_3^0}{\partial x_1\partial x_2}+\frac{1}{2}\frac{\partial u_3^0}{\partial x_1}\cdot\frac{\partial u_3^0}{\partial x_2}, \quad (8)$$

$$\varepsilon_{13}=\varepsilon_{23}=\varepsilon_{33}=0. \quad (9)$$

In the geometrical linear theory the relationship between curvatures, torsion and displacement component u_3 is defined by following matrix equations (Skudra A.M. and Skudra A.A. 2002, Brauns and Rocens 2004, Rocens 1983)

$$\begin{bmatrix} k_1^0 \\ k_2^0 \\ k_6^0 \end{bmatrix}=-\begin{bmatrix} \dfrac{\partial^2 u_3^0}{\partial {x_1}^2} \\ \dfrac{\partial^2 u_3^0}{\partial {x_2}^2} \\ 2\dfrac{\partial^2 u_3^0}{\partial x_1\partial x_2} \end{bmatrix}. \quad (10)$$

In the publication (Sliseris and Rocens 2009) is proposed a simplified analytical method for calculation values of curvature in moisture changing process when the sheet is asymmetrical, orthogonal structure. In the technological imperfection analyze is used following relationship:

$$\begin{bmatrix} \kappa_1^0(w) \\ \kappa_2^0(w) \\ \kappa_6^0(w) \end{bmatrix}=\sum_{l=1}^{n}\begin{bmatrix} k_1^0(w)_l\cdot\dfrac{I_{0,2}}{I_{l,2*}} \\ k_2^0(w)_l\cdot\dfrac{I_{0,1}}{I_{l,1*}} \\ k_6^0(w)_l \end{bmatrix}, \quad (11)$$

where: n – the total number of moisture increment steps; $\kappa_1^0(w)$ and $\kappa_2^0(w)$ – corrected value of curvature; $\kappa_6^0(w)$ – the value of torsion; $I_{0,1}$ and $I_{0,2}$ – the moments of inertia for a flat plate; $I_{l,1*}$ and $I_{l,2*}$ – the moments of inertia for the curved plate after step l – calculated from eq. (18); $1*$ and $2*$ – central axis for curved plate after l step.

Shape analysis could be done by the simplified method faster and easier in comparison with the Finite Element Method. Therefore it is used in numerical experiment to assess the technological imperfection influence on shape.

Numerical experiments

Technological imperfection influence on shape

The imperfection of real manufacturing process is modelled by using Monte-Carlo simulation (Ржаницин 1978). There are two main imperfections in manufacturing process. The first is geometrical deviation and second is deviation of elastic characteristic of material (wood). A few numerical experiments are done to assess imperfection influence on the shape of sheet if relative humidity of surrounding air changes.

The sheet is made from the birch wood layers with following mechanical properties at 12% moisture of wood (Белянкин 1957, Уголев 1971):

$$E_a(12\%)=16400MPa;$$

$$E_t(12\%)=530MPa;$$

$$G_{ta}(12\%)=890MPa;$$

$$\nu_{at}=0.04;$$

$$\nu_{ta}=0.45.$$

Elastic characteristics for different moisture content might be calculated by following equations (Уголев 1971):

$$E_a(W)=E_a(12\%)-200\cdot(W-12), \quad (12a)$$

$$E_t(W)=E_t(12\%)-25\cdot(W-12), \quad (12b)$$

$$G_{ta}(W)=G_{ta}(12\%)-30\cdot(W-12), \quad (12c)$$

where: a – axial direction of wood; t – tangential directions; W – moisture content of wood in percent.

The calculation of the sheet's shape is done by using of the method which is described in (Reissner and Stavsky 1961, Brauns and Rocens 1994, Vinson and Sierakovski 1986) with additional procedures that is described in (Sliseris and Rocens 2009).

The standard deviation of orientation angles (S(φ)) of each layer is modelled in the first experiment. For each standard deviation is analysed 300 random cases. The analysis is done for a square shaped sheet, that is projected with 7-layer, 9-layer, 11-layer (majority of layers are oriented in x2 direction) symmetrical structure. The length of edge of the sheet is 1500 mm and total thickness – 10.5, 13.5 and 16.5 mm. Geometrical imperfections are assumed to be governed by normal distribution. The standard deviation of layer orientation angle (S(φ)) varies from 0.5 to 6 degrees in the experiment. The results of analysis are shown in Figs 1, 2 and 3.

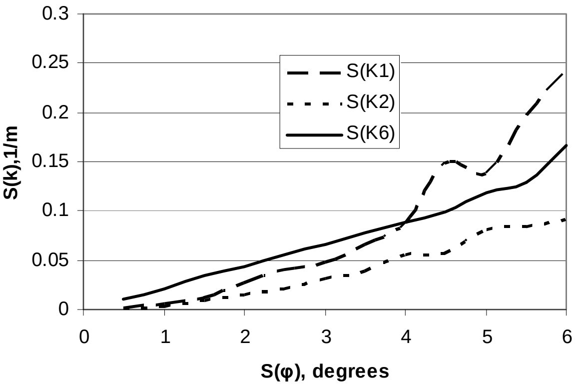

Fig 1. The dependence of curvature standard deviation (S(k)) on the standard deviation of layer's orientation in case of 7-layer structure.

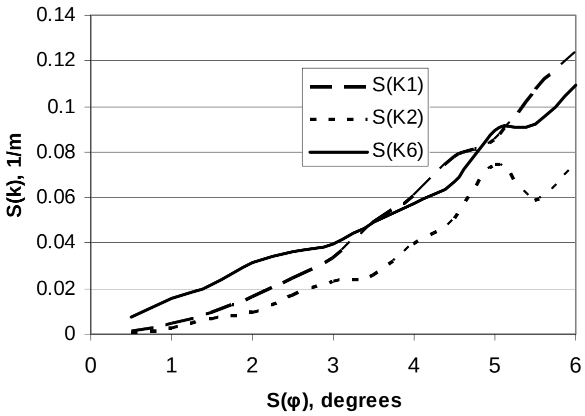

Fig 2. The dependence of curvature standard deviation (S(k)) on the standard deviation of layer's orientation in case of 9-layer structure.

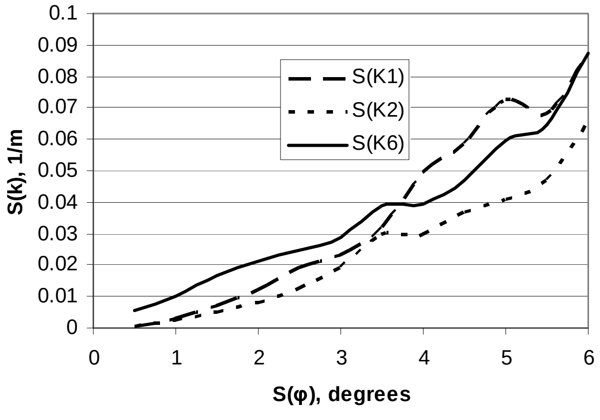

Fig 3. The dependence of curvature standard deviation (S(k)) on the standard deviation of layer's orientation in case of 11-layer structure.

To assess the relationship between the standard deviation of deformations and standard deviation of layer orientation's angle, the correlation coefficient is analysed. The correlation coefficient is defined by following relationship (Shao 2008):

$$r = \frac{\sum_j \left(X_j - \bar{X} \right)\left(Y_j - \bar{Y} \right)}{\sqrt{\sum_j \left(X_j - \bar{X} \right)^2 \sum_j \left(Y_j - \bar{Y} \right)^2}}, \qquad (13)$$

and the standard deviation

$$S(X) = \sqrt{\frac{\sum_j \left(X_j - \bar{X} \right)^2}{m-1}}, \qquad (14)$$

where: m – total number of experimental values; $\bar{X}$ – average value of experimental results.

The results are shown in Table 1.

Table 1. The correlation coefficient of standard deviation of deformations and layer orientation angle.

	7-layer	9-layer	11-layer
S(K1)	0.952183	0.981139	0.978825
S(K2)	0.985144	0.963409	0.976182
S(K6)	0.986032	0.983376	0.970272

According to results of correlation coefficient in this case there are linear relationship between standard deviation of values of curvature, torsion and standard deviation of layer orientation angle:

$$S(k) = a \cdot S(\varphi) + b \qquad (15)$$

The linear relationship coefficients are obtained by least square method (see Table 2).

Table 2. The coefficients of linear relationship (14) of standard deviation of deformations and layer orientation angle.

S(φ)	S(K1)		S(K2)		S(K6)	
	a	b	a	b	a	b
7-layer	0.0418	-0.0514	0.0179	-0.0173	0.0256	-0.0075
9-layer	0.0226	-0.0239	0.0144	-0.0149	0.0177	-0.0062
11-layer	0.0163	-0.0169	0.011	-0.0109	0.0131	-0.0055

In the second experiment is modelled the deviation of thickness (S(t)) of layer, it varies from 7.5 μm to 75 μm. The analysis is done for a square shaped sheet, that is projected with 7-layer, 9-layer and 11-layer symmetrical structure. The length of edge of the sheet is 1500 mm and the average total thickness – 10.5, 13.5 and 16.5 mm. 300 random cases are analysed. Geometrical imperfections are assumed to be governed by normal distribution. In Figs 4 and 5 are shown the results of analysis. The analysis shows that deviation of layer thickness doesn't affect the deviation of torsion angle.

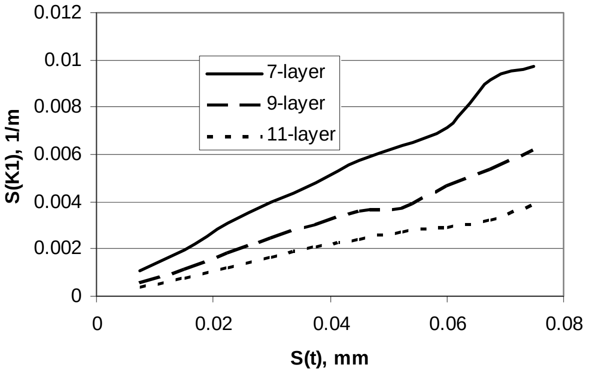

Fig 4. The standard deviation of curvature S(K1) dependence of deviation of thickness of layer.

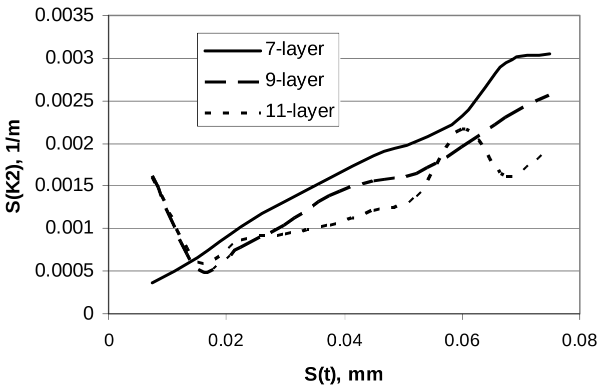

Fig 5. The standard deviation of curvature S(K2) dependence of deviation of thickness of layer.

The analysis of correlation coefficient is shown in Table 3. In the case of 7-layer structure there is linear correlation between standard deviation of curvature and standard deviation of layer thickness, but in case of 11-layer structure the linear correlation of smaller curvature doesn't exist. The linear relationship coefficients *a* and *b* is calculated and shown in Table 4.

Table 3. The correlation coefficient of standard deviation of deformations and thickness of layer.

	7-layer	9-layer	11-layer
S(K1)	0.995037	0.995901	0.995696
S(K2)	0.994854	0.827647	0.664852

Table 4. The coefficients of linear relationship (14) of standard deviation of deformations and thickness of layer.

S(t)	S(K1)		S(K2)	
	a	b	a	b
7-layer	0.1266	-8E-05	0.0397	8E-05
9-layer	0.08	-5E-05	No linear relationship	
11-layer	0.0492	9E-05		

In the third experiment is modelled the deviation of Young's modulus and shear modulus of each layer. The analysis is done for a square shaped sheet which is made with 11-layer symmetrical structure. The length of edge of the sheet is 1500 mm and the total thickness– 16.5 mm. 300 random cases are analysed. Physical imperfections are assumed to be governed by normal distribution. Obtained results shows that there isn't correlation between standard deviation of curvature and standard deviation of elastic characteristics ($r<0.3$), when the Young's modulus in direction of wood fibres deviate from 82 MPa to 820 MPa, with average value– 16400 MPa (at 12% moisture content), Young's modulus in tangential direction of wood fibres deviate from 2.65 MPa to 26.5 MPa, with average value– 530 MPa (at 12 % moisture content), and shear modulus of elasticity in axial-tangential direction deviates from 4.45 MPa to 44.5 MPa with average value 890 MPa (at 12% moisture content) .

Deformation analysis

In the previous numerical experiments was not taken into account torsion interaction with curvatures and strains- full geometrical nonlinearity (Karkauskas 2007, Grigorenko and Yaremchenko 2009). In this numerical experiment is analysed fully geometrical nonlinear problem. Analysis is done by using Finite Element Method (FEM)- computer program Ansys v.10.

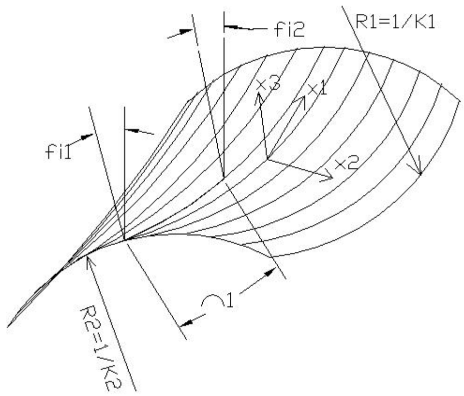

Fig 6. Geometrical interpretation of curvatures k_1, k_2 and torsion $k_6 = 2(fi1 - fi2)$ (radians/m).

In the numerical experiment are analyzed curvatures and torsion of the square shape sheet with length of edge – 1.5 m and 9-layer structure. Layers are made from birch tree. Its mechanical properties are described previously. Various 9-layer structures are analyzed by taking into account geometrical nonlinearity and without geometrical nonlinearity. The initial moisture content in all cases is 10% and final moisture content 12%. In the first case the sheet is made with 9-layer structure. The layer orientation angles with *x*1 axis are following: α/0/90/0/90/0/90/0/90. In the second case the sheet is made with 9-layer structure: α/0/α/0/90/0/90/0/90. In the third case the sheet is made with 9-layer structure:

α/0/α/0/α/0/90/0/90. The obtained results are shown in Figs 7, 8, 9.

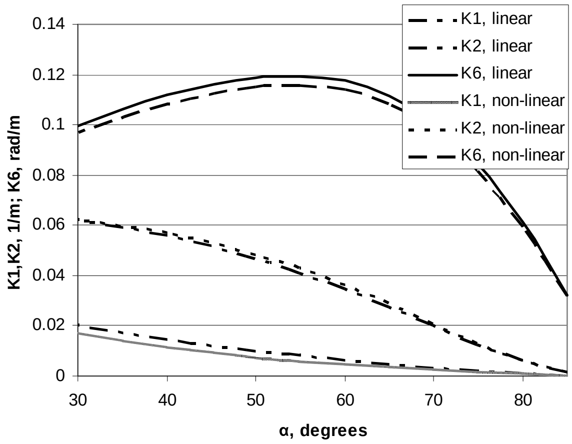

Fig 7. Curvature and torsion dependence on orientation angle α by taking into account geometrical nonlinearity and without it, in case of α/0/90/0/90/0/90/0/90 structured sheet.

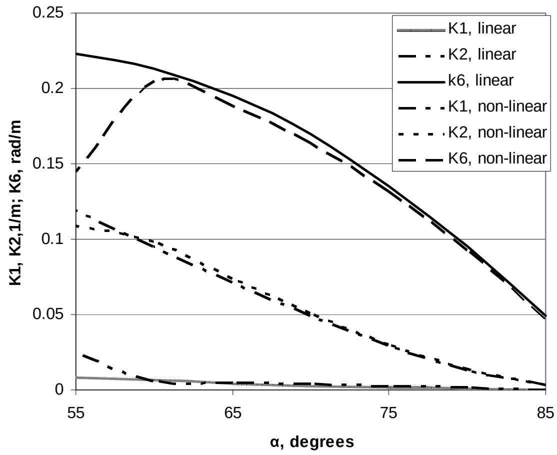

Fig 8. Curvature and torsion dependence on orientation angle α by taking into account geometrical nonlinearity and without it, in case of α/0/α/0/90/0/90/0/90 structured sheet.

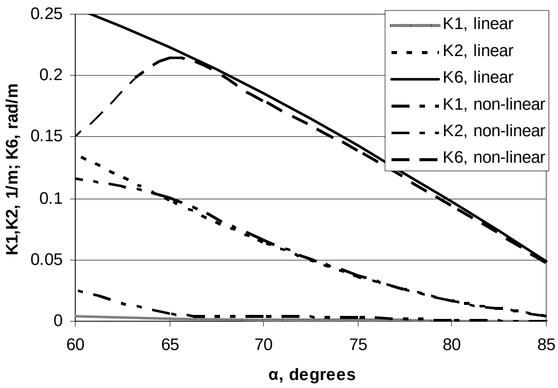

Fig 9. Curvature and torsion dependence on orientation angle α by taking into account geometrical nonlinearity and without it, in case of α/0/α/0/α /0/90/0/90 structured sheet.

Discussion of results

The obtained results in numerical experiments show that there is linear relationship between the standard deviation of layer orientation angle and standard deviation of curvature, correlation coefficient is very close to one. The sheet with less number of layers obtains curvature and torsion with smaller standard deviation. For example if the standard deviation of layer orientation angle is 5 degrees, than the standard deviation of deformations- K_1, K_2 and K_6 in case of 9-layer structure is approximately 38 %, 8 % 24 % less than in case of 7-layer structure, but in case of 11-layer structure – 47 %, 49 % and 50 %. The obtained results show that the standard deviation of curvatures is more affected by standard deviation of layer orientation of layers that are placed more away from the mid surface of the sheet.

The standard deviation of thickness of layer doesn't affect torsion of the sheet. The analysis shows that the correlation coefficient of standard deviation of layer thickness and standard deviation of curvature is close to one in case of 7-layer structure, but if the structure is 9-layer or 11-layer then the correlation coefficient for the dominant curvature is 0.83 and 0.66, therefore linear relationship disappear. The standard deviation of deformations is more affected by standard deviation of layer thickness for the sheet with less number of layers. The standard deviation of curvatures is more affected by standard deviation of layer thickness of the layers that are placed more away from the mid surface of the sheet.

The standard deviation of Young's modulus in axial and tangential direction of wood fibre and shear modulus in axial-tangential direction doesn't significantly affect the value of torsion. The correlation coefficient analysis shows that there isn't linear relationship between the standard deviation of elastic characteristics and standard deviation of deformations, correlation coefficient is less than 0.3.

Numerically was analysed the geometrical nonlinearity influence on deformations by using finite element method. For the 9-layer square shaped sheet with structure α/0/90/0/90/0/90/0/90 the most difference in geometrical linear and nonlinear analysis was obtained when α = 55 degrees, it was less than 5 %. In case of structure α/0/α/0/90/0/90/0/90 and α/0/α/0/α /0/90/0/90 the crucial value of α was 60 degrees, the difference in linear and nonlinear analysis was increased very significantly – more than 40 %.

Conclusions

Various technological imperfection's influence on the sheet's deformations was numerically analysed By using Monte-Carlo simulation.

The results of Monte-Carlo simulation show that there is linear relationship between standard deviation of layer orientation angle and deformations- curvatures and torsion. Deformation analysis was done by the simplified method. The sheet with less number of layers obtains greater standard deviation of deformation. In the analysis assumed technological imperfections to be governed by normal distribution.

The deviation of thickness of layer and elastic characteristics of the sheet doesn't affect the torsion of the sheet. In the area of the numerical experiment the correlation analysis didn't show any relationship between the standard deviation of elastic characteristic and standard deviation of deformation.

Using finite element method was analysed 9-layer sheet with three kinds of structures α/0/90/0/90/0/90/0/90, α/0/α/0/90/0/90/0/90 and α/0/α/0/α/0/90/0/90, each layer thickness – 1.5 mm. The obtained results shows that in first structure there isn't such a value of α where the difference between geometrically linear and nonlinear deformation solution is more than 5%. In case of second and third structure of the sheet the critical value of α is 60 degrees, where the difference between geometrically linear and nonlinear deformation solution becomes more than 40 %.

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