Cable material consumption depending on the geometrical parameters of hierarchic roof

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ABSTRACT: The formation of hierarchic cable roofs and calculation principles have been investigated. The structure is formed by negative Gauss bending type saddle shaped cable roofs by suspending separate saddle roof's corners to a higher level cable structure. These structures are characterized by all the same advantages, which as well apply to a saddle shaped cable roof, however, with a better ratio between the covered volume and the area. These systems can be intercombined to form wider roof areas. It is suitable to use hierarchic cable structures for long span roofs as well as for completely or partially dismounted provisional coverings. In order to reduce the complexity and the amount of calculation, and to avoid the calculation matrix convergence problem, calculation of stresses and displacements of complicated hierarchically subordinated structures with various geometrical parameters of hierarchic roof could be accomplished by the method of substructuring. The effect of interdependencies of separate structural elements and higher level cable structure can be determined by the iterative approximation method. By using of the proposed principle, the hierarchic cable structure is treated under the different wind and accidental snow load conditions.
KEYWORDS: hierarchic cable roof, prestressed orthogonal anticlastic structure, substructuring, lightweight cable net structures.

1 INTRODUCTION

Weight reduction and increase of spans of structures are topical tendencies in the development of load-bearing structures, which can be accomplished by application of new high-strength materials. As a result it shows as reduction of ratio between the dead and live loads of a structure from ancient massive structures to contemporary lightweight structures. This ratio can be reduced more than 100 times by the most effective exploitation of the properties of special high-strength materials with much higher specific strength in combination with structural systems where tensile stresses are dominant. Tension structures are characterized by non-linear geometric hardening which results in a less proportional increase of stress in elements in relation to increased external loads. This provides an increased nominal safety factor evaluated at ultimate limit state of structures. Cable roofs with axially tensioned elements and with identical tensile stresses acting in all the cross-section points represent one type of these structures, and it opens possibilities for rational application of advanced materials.

The main advantages of cable structures are as follows: new options of architectonic expression, possible translucency, small weight of structures, efficiency of high – strength material application in the production of cables, reduction of the construction time, advantages of transportation (in rolls), good seismic resistance, cheaper fire protection methods by using foaming polymers, low cost maintenance and may be the most essential is the possibility to cover extremely large spans without intermediate columns, which is impossible by using ordinary structures. There are also some drawbacks, i.e., increased deformability mainly of kinematic nature, relatively poor anticorrosion resistance in some cases, the need of supporting structures in taking up the abutment shear, and because of light weight and deformability they require special stabilization arrangements to provide geometric shape invariabilities.

The most perspective from the constructional point of view and most expressive regarding architectonical aspect, is a prestressed cable net with negative Gauss bending saddle-shaped hyperbolic surface known as anticlastic form covered with fabric. The structure is formed by orthogonally intersected load-bearing concaved and stressing convexed cables. This model is characteristic by kinematic rigidity and there is no requirement of additional loading for stabilization of structure. The construction has only one statically stable shape, preventing such destructive dynamic instabilities, as flutter or flapping. The model can be constructed on a rectangular supporting contour, which is one of its advantages, and to reach total rigidity of the structure, relatively small radiuses of curvature are required, enabling to provide the minimum material consumption both of the cable net and its supporting contour. Comparing to different cable structures, deformability under exposure of external loading is relatively smaller.

From the point of view of material consumption (valuable utilization of load-bearing capacity of high-strength materials) the most rational are saddle-shaped roofing with a compliant supporting contour.

Basing on previously accomplished investigations (Rocens et al. 1999, Serdjuks et al. 1999, 2003, Pakrastinsh et al. 2001), where have been determined the rational geometric characteristic values of separate saddle–shaped cable roofing with the compliant supporting contour and sizes in plan from 10 to 50 m, it is shown that the ratio between the roofed volume and the area is unacceptable at larger spans as it causes inexpedient increase of the heated volume. This problem can be solved by application of the saddle – shaped roofing with the compliant supporting contour as standard elements, and by suspending their corners to a higher level cable structure thus obtaining a hierarchic, intersubordinated large–span cable roofing (Pakrastinsh & Rocens 2001) (Fig.1).

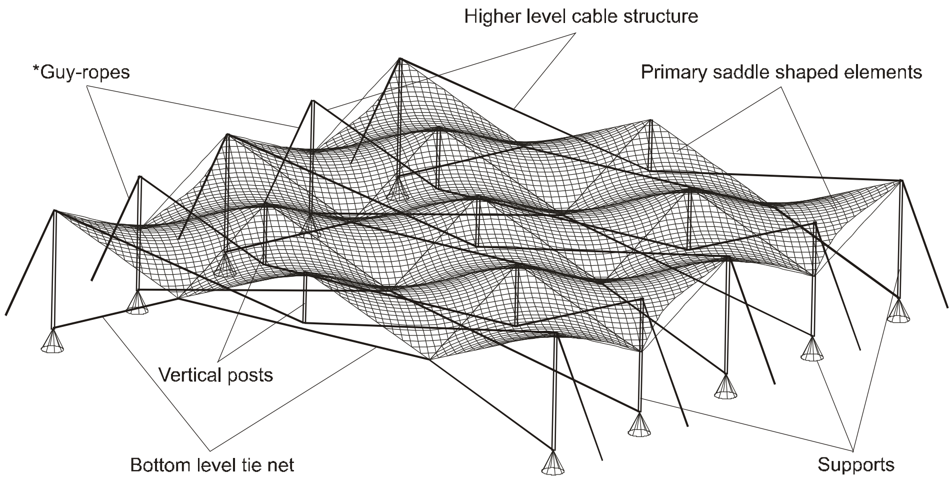

Figure 1. Simplified variant of hierarchic cable structure module
*Guy-ropes in longitudinal direction of module have not been conventionally displayed

2 FORMATION OF STRUCTURE AND CALCULATION MODEL

In comparison with the conventional structures cable structures can be economically suitably used for spans exceeding 40 m. Therefore, for further performance the following dimensions of primary elements have been assumed: 35 x 35 m, 45 x 45 m and 55 x 55 m, thus achieving full structural spans of 140m, 180m and 220m, accordingly.

A prestressed cable net with a negative Gauss' bending saddle – shaped hyperboloid surface covered with fabric has been assumed as the structure of primary element. The primary element is formed by orthogonally crossing curved load–bearing and stressing cables. Lest the roofing should lose rigidity it must be provided that under disadvantageous load combinations stresses in stressing cables shall not be reduced to zero because cable slack formation can provide damages of roofing or destructive flatter may occur.

Primary elements of hierarchic structural module 4x4 are modeled and they may be combined thus producing larger roofed areas. Notwithstanding the number of smaller elements, the scheme with slanting pendants has obvious remarkable drawbacks: too long columns, which entail too large top corner vertical displacements of the bottom level standard elements, as well as too large horizontal forces in these due to the angle between the pedants and vertical axis, which in its turn requires strengthening of the standard element. Roofing scheme with vertical pedants is assumed as optimum one.

Not to exclude the possibility of expanding the structure in longitudinal direction of the module, transversal direction variant of the higher level cable structure is assumed, and to reduce the total height of structures, the upper part is lowered under the level of primary elements, at the same time lifting the corners of the primary elements by means of vertical latticed post fixed to these cables.

Stressing cables change to load – bearing ones and vice versa. Camber values for the numerical experiment are assumed as 1/20; 1/10; 1/5; 1/2.5 part of the span in compliance with the literature recommendations and the existing structures. Calculation scheme of the primary element is illustrated in Figure 2.

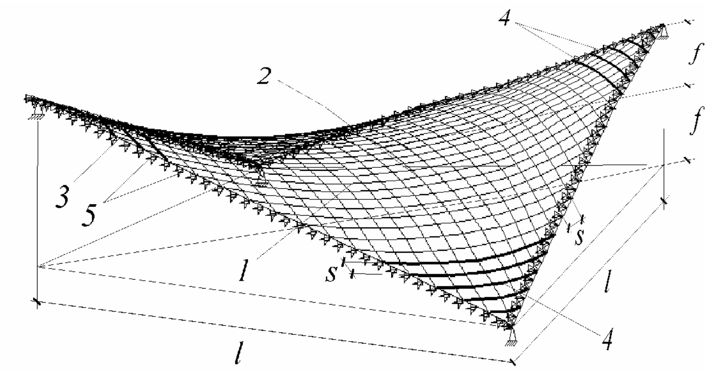

Figure 2. Calculation scheme of primary element.
1, 2 – load-bearing/stressing cables; 3 – edge cable; 4 – LCP Vectran® composite cable; 5 - symmetry boundary conditions; *l* – span; *f* – initial deflection of load-bearing/stressing cables; *S* - load-bearing/stressing cables spacing; fabric covering have not been conventionally displayed.

In the middle part of hierarchic cable structure the board element of the primary element is shaped as an upright cable, which is symmetrically loaded with horizontal forces developed from the load – bearing or stressing cables of adjoining primary elements. In case of uniformly distributed load these forces are intercompensated, which open the way to reduce the material consumption at the expense of the board element. The above – mentioned scheme enables to apply boundary conditions of symmetry during calculations of the primary element and brings closer its action to the element with rigid supporting contour, which considerably reduces the deformability of the primary element. The board element of outer primary elements is modeled as a stretched on cable loaded with the large uncompensated horizontal load coming from the cable net. We choose the camber of these elements equal to 1/15 part of span, basing on the assumption that the board element can be treated as freely suspended cable loaded with distributed load. In this case modeling of an upright rigid board element with length size 35 m is problematic due to large strains. To reduce the total deformability of the primary element and, basing on our previous investigations, the board element is modeled as a cable truss. The stressing value of the board elements is assumed as 50% from the design strength to avoid from slack of the edge element when only permanent load and stressing of the cable net act.

The step of the cable net is assumed 1.77 m depending on the constructive condition lest rain bags should form, because the fabric or some other kinds of roofing in cable net structures mainly provide the transfer of external loading to the cable net.

Prestressing is assumed for primary dimensioning of primary element of cable net on identical level for all cable nets in compliance with literature recommendations (Walton 1996) which makes 22.5 % from the tensile strength (50 % from the design strength at material safety factor equals to 2.0). It is required to uniformly resist the opposite direction loads, because each group of cables can work as stressing one depending on the direction of loads. To prevent the increase of the remarkable relaxation effect for steel wires, it is assumed that stresses of permanent load and stressing should not exceed 45% from the cable tensile strength at tension.

By combining the primary elements of hierarchic structure, the common board element of two adjoining primary elements remains upright in plan, but in side projection it assumes the form of S – shape curved line, which causes the emergence of cable pendants in corners of the primary element at minimum loadings. To compensate the loss of prestressing, corners of the primary element shall be made from composite cables with enlarged limiting deformation properties, which make up 7 % from the total length of cables. One of such materials is recently produced Liquid Crystal Polymer (LCP) Vectran® developed on the basis of polyester molecules with regularly oriented structure along the longitudinal direction of fibers. Fibers made of this material are characteristic by large strength and increased deformation with minimum creep probability.

Acting as a component of hierarchic structure, the primary element subjected to the influence of higher level structures loaded with reaction forces of the primary element, which appear in the displacement of the supporting nodes of these elements in the opposite direction.

In order to assess the maximum permissible displacements of the primary element supporting nodes without cable slack (which simultaneously shows how strong should be a higher level cable structure to provide the displacements of supporting nodes), a numerical experiment for three level loading of the above mentioned variants of the primary elements was accomplished. It should be observed that the real distribution of aerodynamic coefficients for the structure in question is not established, because model investigations in wind tunnel or computer aided simulation of the exposure of wind flow by using the mathematical appliance of fluid mechanics (Halfmann et al. 2002) are needed.

The computing is accomplished by means of finite element method software ANSYS 6.0 University version applying for the cable modeling the universal spatial bar finite element LINK10 with three degrees of freedom in each node with specific bilinear stiffness matrix, which defines that the element works at tension only. The experiment was made by the iterative approximation method assuming the point of reference as the state when deadweight and prestressing loads work on the structure. Since uniform values of intensity of the wind both with positive and negative values can affect the primary element, which is identical with positive values of the snow load of Latvian conditions, structural performance is symmetric relatively horizontal plane and calculations are made only for the displacement of the upper points downwards.

Experimental results enable to draw a conclusion that limiting values of the supporting nodes displacements are not dependent on the load value, but on geometric parameters of the primary element. Regularity between the limiting values of supports nodes displacements of the primary elements and the initial curvature of the primary element is shown in Figure 3.

In order to prevent the formation of cable slack, a prestressing reserve within the limits of 10 to 40 percent from the total initial stressing shall be provided (suggest the increase of cable cross-sections by 1.25 to 5 times relatively primary element condition without nodal displacements). Larger or smaller reserve is not rational because it causes a rapid increase of material consumption for the primary element in case of larger reserve and that for higher level cable structure in case of smaller reserve.

Analysis of interaction of the primary element higher level cable structure has proved that rational initial camber area of the primary element lies from 0.05 to 0.2 part of the span, which is beyond rational curvature value of 0.3 part of the span of the primary element as separate structure without nodal displacements.

To reduce the total amount and complexity of the calculation, and to avoid from the convergence problems, the so-called substructuring method is used, which divides the structure into levels. In the first stage the required cross-sections of the primary elements cables and reaction forces on the supporting nodes are calculated, thereby collecting information (boundary conditions) for the next stage calculations of higher level cable structure.

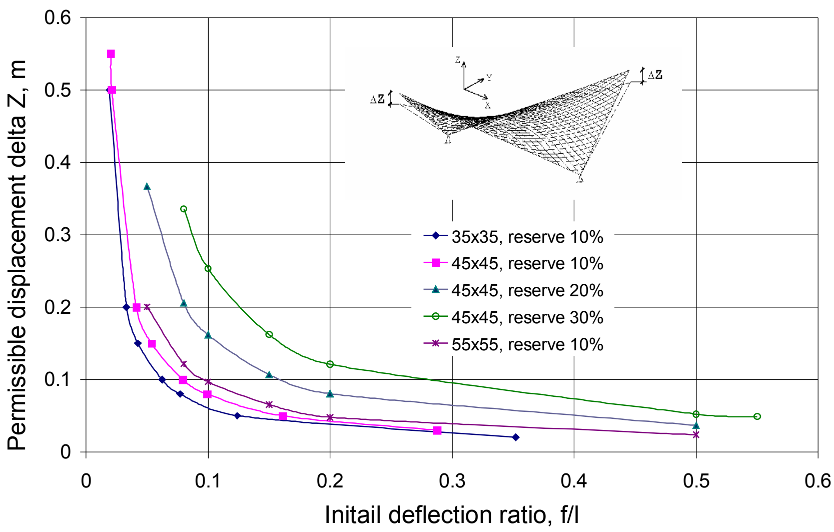

Figure 3. Permissible displacements of supports of primary element depending on initial deflection of primary element.

As a result of these calculations data for the next iteration are provided for calculation of the primary saddle-shaped element with new boundary conditions. Thus using of iterative approximation method the effect of interdependencies of separate structural elements and a higher level cable structure can be defined.

In the first stage of calculations for the initial dimensioning of the primary element cables, prestressing is assumed identical for all cables in the net, which makes up 40% to 10% from the design strength of material, taking into account the reserve accordingly up 10% to 40% to the displacement of the supporting nodes. At first the calculation were made to find cable cross-sections under the primary element under full load by method of stepwise approximation for each type of the primary element and for each load level providing that no stresses exceeding the design strength of the respective material should develop in any group of cables. In the next step calculation was made with the discovered cross-sections of cables and for the load of dead weight and prestressing. To develop the reference deformed state of structure closest to real structure after assembling and before exposure of live load, adjustment of nodal coordinates was made in compliance with this deformed condition.

In the next stage a full load is applied to the primary element with corrected geometry thus obtaining the reaction forces of the supporting nodes, i.e., boundary conditions for calculations of higher level cable structures. Using stepwise approximation are obtained the cable cross-sections of higher level structures on condition that at nodes of applying the load from primary elements shall not exceed the values of allowable displacements for the bearing nodes of the primary element.

3 CALCULATION RESULTS AND RECOMMENDATIONS

Since mat consumption of whole hierarchic structure is dependent on many parameters, to determine the rational form from the point of view of material consumption, the numerical experiment analysis of the behavior of hierarchic cable structures was accomplished using the values of variable parameters: dimensions of the primary elements: 35 x 35m, 45 x 45m, 55 x 55 m, initial deflection values for the primary element are assumed as follows: 1/20; 1/10; 1/5; 1/2.5 part of the span, initial deflection values for a higher level cable structure: 1/10; 1/5; 1/2.5 part of the span. For the 3 levels of loads: 1 – dead load and occasional snow load (0.48 kN/m^2), 2 - dead load and negative wind suction in accordance with recommendations (Davenport 1995) where the load values proportional to the distribution of actual aerodynamic coefficients, are replaced by uniformly distributed load (0.76 kN/m^2), 3 - dead load and negative wind suction with twice increased wind speed (1.6 kN/m^2) in compliance with literature recommendations which exceed by intensity snow load values of building codes valid in Latvia.

By using FEM software ANSYS University for each experimental point value determination, and software STATISTICA for the D-optimal experimental plan calculation and for analysis of the results, the correlation between the main geometrical characteristics of the roof and the material consumption required for the roofing of one unit of area is determined as second-order polynomial:

$$C = b_o + b_1 f + b_2 F + b_3 l + b_4 p + b_5 r + b_{12} f F + b_{13} f l + b_{14} f p + b_{15} f r + b_{23} F l + b_{24} F p + b_{25} F r + b_{34} l p + b_{35} l r + b_{11} f^2 + b_{22} F^2 + b_{33} l^2 + b_{44} p^2 + b_{55} r^2$$

where C - material consumption required for the roofing per unit of the area, kg/m^2; f - initial deflection ratio for the primary element; F - initial deflection ratio for a higher level cable structure; l - span of the primary element, m; p - calculation load, kN/m^2; r - reserve of prestressing, %.

The response function coefficients (Table 1) are determined on the results of numerical experiments by least squares method.

Table 1. Regression coefficients of response function

b_o	650.6	b_{13}	2.2	b_{35}	-393.3
b_1	-21.2	b_{14}	-22.7	b_{45}	1973.8
b_2	-80.9	b_{15}	-17.3	b_{11}	0.4
b_3	-130.9	b_{23}	151.0	b_{22}	48.8
b_4	-987.6	b_{24}	-257.7	b_{33}	741.9
b_5	-1445.0	b_{25}	-236.1	b_{44}	3095.1
b_{12}	4.8	b_{34}	-911.6	b_{55}	3377.8

Analysis shows that the important effect is achieved by two following parameters: the initial deflection ratio of higher level cable structure and the prestressing reserve of primary element, influence of which on mat consumption for one case is shown in Figure 4. The rational values for this particular span case 180m are: Initial deflection ratio 0.288 and Prestrressing reserve 0.302.

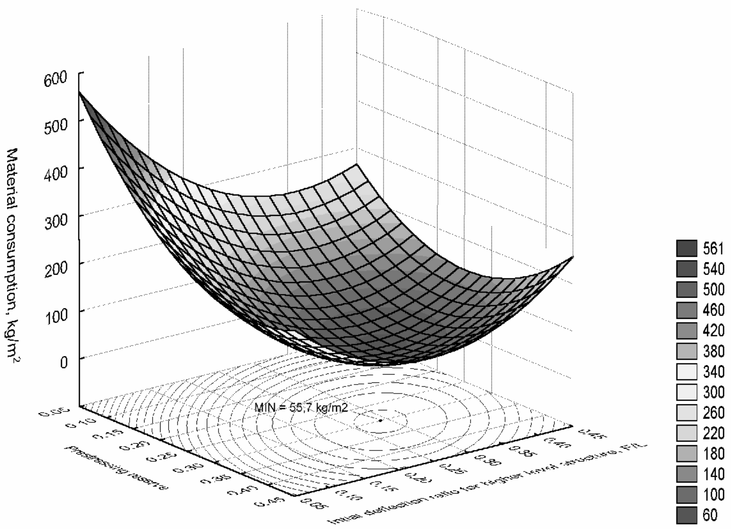

Figure 4. Material consumption of hierarchic cable structure depending on initial deflections ratio of higher level structure and the prestressing reserve of primary element (span L=180m (45x4), load 1.5 kN/m^2).

Comparing the results with other existing long span structures (Fig. 5) it can be concluded that material consumption of such type of structures lies in the rational material consumption area.

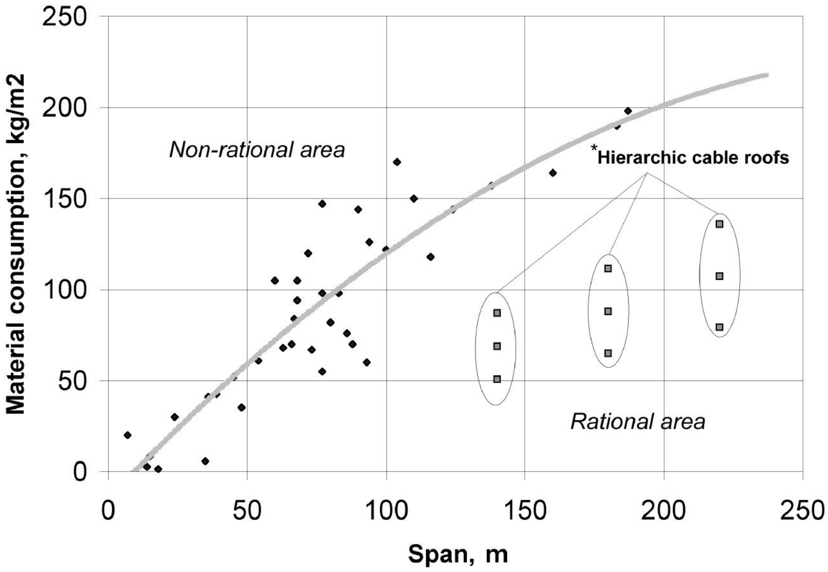

Figure 5. Material consumption depending on the span of structures
*hierarchic cable roof material consumption is shown for three level loading

Summing up the analysis of structural performance, the following recommendations for rational hierarchic structures have been worked out:

- from the constructive point of view most rational module of hierarchic cable structure should have dimensions 4x4 primary elements, the largest one is characterized by irrational material consumption for the higher level cable structure;
- to reduce the total height of structures, the upper higher level structures cables are lowered under the primary element surface by simultaneous lifting up of the primary elements corners by means of vertical latticed posts fixed to these cables;
- the bottom higher level structure shall be shaped with curvature upwards, because the plane bottom tie net is typical of irrational material consumption (Pakrastinsh & Rocens 2003);
- the roofing scheme with vertical pendants shall be designed since the application of inclined ones may require the strengthening of the primary elements;
- not to exclude the probability of extension of structural module in longitudinal direction, transversal direction variant of the higher level cable structure is recommended;
- to prevent the appearance of cable slack of the primary element, a reserve within the limits of 10 to 40 percent from the total initial stressing shall be provided in the prestressing of cables;
- for future development of structures, merging of the bottom and upper higher level cable structures into the cable trusses shall be provided , which can reduce the supporting nodes displacements and the proportion of material consumption of these structures;
- structural bracing between the corners of the primary elements shall be designed in order to distribute the effect caused by concentrated forces and horizontal wind loads.

4 CONCLUSIONS

By means of the accessible software internal forses, displacements and material consumption of complicated intersubordinated hierarchic cable roofings can be computed and the effect of displacements of separate structural elements on other structural elements can be determined. Hierarchic cable roofings can be usefully applied for large span structures (e.g. sports grounds, concert halls, parking – lot roofings) in areas with insignificant snow load. These structures can be also used as completely or partially dismountable translucent provisional coverings for covered agricultural areas.

A new and rational from the point of view of material consumption hierarchic cable roofing structure enabling to reduce the material consumption by 1.5 – 2 times comparing with other large – span structures has been worked out.

A saddle–shaped primary element structure has been developed and dimensioned, and selection of materials for components is accomplished. A determination method of permissible displacements of corners of the primary element structure acting as a component of hierarchic structure has been developed.

It has been proved that the primary elements of hierarchic cable structures cannot be modeled without the application of composite cables with increased limit of elongation.

It has been proved that due to the iteration of the primary element and higher level cable structure the rational initial curvature area of the primary element lies from 0.05 to 0.2 part of the span, which is beyond rational curvature value of 0.3 part of the span for a separate primary element.

Computing methods of the cable material consumption per covered unit of the area depending on geometric parameters and nodal displacements, which is based on the combination of submodels and iteration methods has been developed.

Second-order polynomial correlation between the main geometric parameters of hierarchic cable structures and the material consumption of cables per roofed unit of the area has been determined, and recommendations for modeling rational hierarchic structures are presented.

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