Volume estimation of 3D object modelled by parametrical iso-surfaces

A. Sisojevs*, A. Glazs
Images Processing and Computer Graphics Department,
Riga Technical University, Latvia
**E-mail: alexiv@inbox.lv*

Introduction. The problem of medical objects 3D modelling and their further analysis is an actual task of biomedical engineering and computer graphics. This work describes a new method of free-form 3D objects volume estimation, with a medical object used as an example. The approach of 3D medical object modelling described in [4]. In this case 3D model of medical object is set of Bezier surfaces. As is known the surface is calculated using the following formula [1]:

$$S_{Bezier} = \sum_{i=0}^{p}\sum_{j=0}^{q} P_{i,j} \cdot B_{i,p}(u) \cdot B_{j,q}(v), \tag{1}$$

where: S_{Bezier} – Bezier surface; p and q – surface degree in each parametrical direction; $P_{i,j}$ – array of control points; $B_{i,p}(u)$ and $B_{j,q}(v)$ – Bernstein polynomials in each parametrical direction.

The Bernstein polynomials are calculated as follows [1]:

$$B_{i,p}(u) = \binom{p}{i} \cdot u^i \cdot (1-u)^{p-i}, \tag{2}$$

The known methods of object volume estimation do not take into consideration the 3D structure of the object, meaning that the method of objects volume estimation using reconstructed 3D model is a more precise approach. The most common methods are the Trapezoidal estimation and Cavalieri's method, both described in [3]. These methods use the 2D data obtained from segmented medical images. Unfortunately, this could lead to miscalculations in volume estimation, because such methods have no information about the form of the pathological region. This means that there is no information about the pathology zone between the medical slices. According to this, in work [4] it was proposed to use the reconstructed 3D model of the medical object for volume estimation. The input data for the volume estimation algorithm is an array of Bezier patches. The patches are aligned so that the z axis is in the centre of the medical object. Based on this, the object's volume estimation formula is described as a sum of curvilinear prisms volumes:

$$V_{obj} = \sum V_{prism}, \tag{3}$$

Each prism is created based on Bezier patch by constructing a perpendicular from each point of the patch to the z axis. For the calculation the volume of prism a surface integral is used [4]:

$$V_{prism} = \frac{1}{2}\int_0^1\int_0^1\left(x\frac{\partial y}{\partial u}\frac{\partial z}{\partial v} - x\frac{\partial y}{\partial v}\frac{\partial z}{\partial u} + \frac{\partial x}{\partial v}y\frac{\partial z}{\partial u} - \frac{\partial x}{\partial u}y\frac{\partial z}{\partial v}\right)dudv\ , \tag{4}$$

where: x, y and z – coordinates of surface $S_{Bezier}(u,v)$.

Proposed method. In this work (in contradistinction to work [4] where the graphical integration was proposed) the new approach of volume integral (4) calculation is proposed. Using equations (1) and (4) after transformation the following result is obtained:

$$\begin{aligned} V_{prizm} &= \frac{1}{2}\sum_{i=0}^{p}\sum_{j=0}^{q}\sum_{k=0}^{p}\sum_{l=0}^{q}\sum_{m=0}^{p}\sum_{n=0}^{q}\Big[\left(X_{i,j}\cdot Y_{k,l}\cdot Z_{m,n}\right)\cdot \\ &\cdot\left(\left(\int_0^1 B_{i,p}(u)\cdot\frac{dB_{k,p}(u)}{du}\cdot B_{m,p}(u)du\right)\cdot\left(\int_0^1 B_{j,q}(v)\cdot B_{l,q}(v)\cdot\frac{dB_{n,q}(v)}{dv}dv\right)-\right. \\ &-\left(\int_0^1 B_{i,p}(u)\cdot B_{k,p}(u)\cdot\frac{dB_{m,p}(u)}{du}du\right)\cdot\left(\int_0^1 B_{j,q}(v)\cdot\frac{dB_{l,q}(v)}{dv}\cdot B_{n,q}(v)dv\right)+\ , \\ &+\left(\int_0^1 B_{i,p}(u)\cdot B_{k,p}(u)\cdot\frac{dB_{m,p}(u)}{du}du\right)\cdot\left(\int_0^1 \frac{dB_{j,q}(v)}{dv}\cdot B_{l,q}(v)\cdot B_{n,q}(v)dv\right)- \\ &\left.-\left(\int_0^1 \frac{dB_{i,p}(u)}{du}\cdot B_{k,p}(u)\cdot B_{m,p}(u)du\right)\cdot\left(\int_0^1 B_{j,q}(v)\cdot B_{l,q}(v)\cdot\frac{dB_{n,q}(v)}{dv}dv\right)\right)\Big] \end{aligned} \tag{5}$$

Using equations (2) and (5) after transformation the result takes form:

$$\begin{aligned} V_{prizm} &= \frac{1}{2}\sum_{i=0}^{p}\sum_{j=0}^{q}\sum_{k=0}^{p}\sum_{l=0}^{q}\sum_{m=0}^{p}\sum_{n=0}^{q}\Big[\left(X_{i,j}\cdot Y_{k,l}\cdot Z_{m,n}\right)\cdot \\ &\cdot\binom{p}{i}\cdot\binom{q}{j}\cdot\binom{p}{k}\cdot\binom{q}{l}\cdot\binom{p}{m}\cdot\binom{q}{n}\cdot \qquad , \\ &\cdot\left(T(a,p,k)\cdot T(b,q,n) - T(a,p,m)\cdot T(b,q,l) + \right. \\ &\left. + T(a,p,m)\cdot T(b,q,j) - T(a,p,i)\cdot T(b,q,n)\right)\Big] \end{aligned} \tag{6}$$

where: $a = i + k + m$ and $b = j + l + n$.

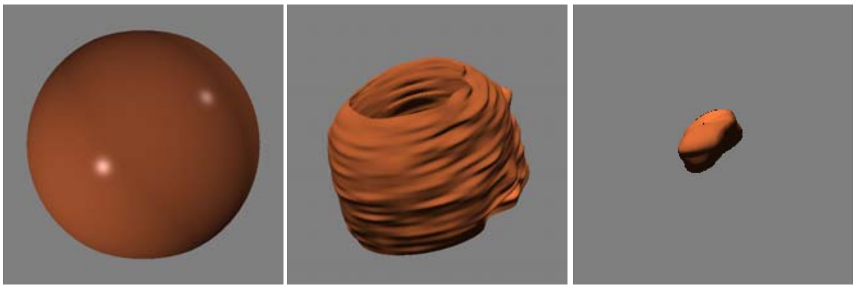
The function $T(a,p,k)$ can be described as follows:

$$T(a,p,k) = \begin{cases} -\dfrac{1}{3} & if \quad a = 0 \\ tmp & if \quad a \in [1;3\cdot p - 1] \\ \dfrac{1}{3} & if \quad a = 3\cdot p \end{cases}, \tag{7}$$

where:

$$tmp = \sum_{r=0}^{3\cdot p-a-1} (-1)^r \cdot \binom{3\cdot p-a-1}{r} \cdot \left(\frac{k}{r+a} - \frac{p}{r+a+1} \right), \quad (8)$$

Experimental results. In this work the proposed method was compared with the methods described in [3]. For comparison three objects were chosen: one modelled object (sphere) with volume 40478780,46 and two medical objects what described in works [2] and [4]. All objects are shown in Fig. 1.



Model object Head's model Pathology model

Fig. 1. Objects in experimental research

For objects modelling bi-cubic Bezier patches are used. In this case the value of polynomials degree is p=3 and q=3. Consistent function $T(a,p,k)$ values in Table 1 are given.

Table 1. Function $T(a,p,k)$ values for case p=3

a	0	1	2	3	4	5	6	7	8	9
***T*(*a*,3,*k*)**	$-\frac{1}{3}$	$\frac{1}{8}\cdot k-\frac{1}{24}$	$\frac{1}{56}\cdot k-\frac{1}{84}$	$\frac{1}{168}\cdot k-\frac{1}{168}$	$\frac{1}{280}\cdot k-\frac{1}{210}$	$\frac{1}{280}\cdot k-\frac{1}{168}$	$\frac{1}{168}\cdot k-\frac{1}{84}$	$\frac{1}{56}\cdot k-\frac{1}{24}$	$\frac{1}{8}\cdot k-\frac{1}{3}$	$\frac{1}{3}$

Table 2 shows results of model and medical objects volume estimation, obtained by the proposed method, Cavaleri & trapezoidal method and graphical integration method [4].

Table 2. Experimental results

Methods	Model object	Head's model	Pathology model
Proposed method	40478780,46	2802491,60	9157,55
Cavaleri & trapecoidal methods	40171279,48	2825806,26	9420,89
Graphical integration (10x10)	40475553,47	2802798,76	9161,02
Graphical integration (100x100)	40478746.86	2802494,68	9157,58
	Difference from proposed, %		
Cavaleri & trapecoidal methods	0,760	0,832	2,88
Graphical integration (10x10)	$7{,}97\times10^{-3}$	$1{,}10\times10^{-2}$	0,0379
Graphical integration (100x100)	$8{,}30\times10^{-5}$	$1{,}10\times10^{-4}$	$3{,}79\times10^{-4}$

In case of model object the proposed method result is equivalent to the object's previously known value. Experimental results show that the proposed method gives best result for value precision.

Conclusions. In this work the method of volume integral calculation and volume estimation is proposed, using the 3D object model as initial data. It is more correct for medical engineering tasks than the known methods that are based on 2D image data, because existing methods have no information about the object between slices.

Acknowledgement. This work has been supported by the European Social Fund within the project "Support for the implementation of doctoral studies at Riga Technical University".

References

1. Rogers D., Adams J Mathematical Elements for Computer Graphic, 2nd Ed., McGraw-Hill, Boston, MA, 1990.
2. Sisojevs A., Glazs A. Efficient approach of medical objects 3D visualization// Conference Biomedical Engineering Proceedings. Kaunas, Lithuania, 2008. – P. 222–224.
3. Smitha S., Revathy K., Kesavadas C. Segmentation and Volume Estimation of Brain tissues from MR Images//Proceedings of IMECS. Hong Kong, 2006. – P 543–547.
4. Sisojevs A., Krechetova K., Glazs A. 3D Modeling of Free-Form Object (Interpolation, Visualization and Volume Estimation)// The Conference WSCG Communication Papers proceedings. Plzen, Czech Republic, 2009. – P. 125 – 128.

Volume estimation of 3D object modelled by parametrical iso-surfaces

A. Sisojevs, A. Glazs
Images processing and computer graphics department, Riga Technical University, Latvia

Medical objects volume estimation is an actual task for biomedical engineering and medical diagnostic. At the same time, the input data for this task is often described in form of 2D layers. In this work a new approach for volume estimation of a 3D model is proposed. The proposed method is based on precise integral calculation of 3D models volume. The experiments show that this approach gives best result in comparison with other approaches.