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APPLICATION OF VARIATION METHODS FOR CALCULATION ELASTOMERIC ELEMENTS OF A DIFFICULT CONFIGURATION

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Abstract: *For an efficient utilization of elastomeric elements in modern mechanical engineering, including vibration insulation, it is necessary to be able to calculate the characteristics of compressive stiffness of elastomeric elements. Many elastomeric elements have the geometrical form of a complex configuration. In this work the design procedure the compressive stiffness characteristics of type "force – settlement", for elastomeric elements of any geometrical form is offered at static loading. . In the given work for reception of the approached decisions it is offered to use direct methods using functional considered by V.Prager [1] and converted to weakly compressible materials. Methodology use is shown on an example of calculation of the concrete rubber absorber.*

Key words: *compressive stiffness, rubber, shock-absorber, subareas.*

1. PROBLEM DESCRIPTION

For an efficient utilization of elastomeric elements in modern mechanical engineering, including vibration insulation, it is necessary to be able to calculate the characteristics of compressive stiffness of elastomeric elements. Many elastomeric elements have a complex geometric configuration, which does not permit the application of variational methods of calculation [2, 3], effectively used in the calculation of elastomeric elements simple geometry. But for the elastomeric elements of complex configuration or consisting of well contacting parts made of various materials, using functionals [2, 3] virtually impossible to choose a continuous function of displacement required to satisfy the geometric conditions and the conditions of continuity throughout the volume of complex elastomeric element. In this paper considered the method of calculating the rigidity characteristics under static load, type of force - settlement, for the elastomeric elements of any geometric shapes. Deformations are assumed to be small. It is proposed to use the variational principle proposed V.Prager [1] using discontinuous displacement function and modified for weakly compressible and incompressible materials.

2. DECOMPOSITION OF THE VOLUME OF THE PRODUCTS ON A SUBAREAS

Let's consider the elastomeric element with the complex geometrical form with volume V and the area of a surface F:

$$V = \sum_{n=1}^{N} V_n \quad F = \sum_{n=1}^{N} F_n \qquad (1)$$

where:
V_n - one-coherent regular subareas;
N - number of subareas, received as a result of crushing elastomeric element;
F - total surface area of all subdomains elastomeric element.

The surface limiting n-th subarea, looks like

$$F_n = F_\sigma^n + F_u^n + \Gamma_n \qquad (2)$$

where:
F_σ^n – loading surface;

F_u^n – surface of the fixing;
Γ_n – surface area of contact subarea partitioning elastomeric element.

On the crushing surface Γ_n must be satisfied the conditions of continuity:
- of displacements:

$$u_i^n = u_i^{n+1} \qquad (3)$$

- and of stresses:

$$\sigma_{ij}^n m_j^n = -\sigma_{ij}^{n+1} m_j^{n+1} \qquad (4)$$

Where the index „n" specifies a current issue of a subarea of crushing and m_j^n and m_j^{n+1} - directing cosines normals external, accordingly to V_n and V_{n+1} on Γ_n and Γ_{n+1}. If you use only the external normal to V_n, which at Γ_{n+1} for V_{n+1} is internal, then $m_j^{n+1} = - m_j^n$ and the condition (4) reads:

$$\sigma_{ij}^n m_j^n = \sigma_{ij}^{n+1} m_j^n \qquad (5)$$

Hereinafter, for brevity, over repeated lower indices are summed, and the comma denotes a partial derivative.
Contact surface Γ_n may be artificial in the geometric decomposition of the elastomeric element, or natural, if the physical and mechanical characteristics of the material (G, μ) at the contact surface change abruptly, that is, a volume V composed of different materials.
Partition of the elastomeric element in the subareas extends the permissible class of unknown functions in the piecewise smooth and piecewise continuous functions with piecewise smooth and piecewise continuous derivatives [1, 4]. Discontinuity of displacements and efforts on the surface Γn denote:

$$u_i^n - u_i^{n+1} = \{u_i^{\,n}\}, \qquad (6)$$
$$\sigma_{ij}^n m_j^n - \sigma_{ij}^{n+1} m_j^n = \{\sigma_{ij}^n m_j^n\}$$

If the surface areas Γ_n of subareas V_n displacement components and efforts (all or only some) does not satisfy the conditions of continuity (3) and (4), then, using the designation (6), we can write:

$$\{ \sigma_{ij}^n m_j^n u_i^n\} = \{ \sigma_{ij}^n m_j^n \}^{I} u_i^n + \sigma_{ij}^n m_j^n \{u_i^n\}^{II} \quad (7)$$

where the indices I and II, respectively, indicate that the summation extends only to the displacement components and efforts that do not satisfy the conditions on the surface Γ_n.
Suppose that in each subareas partitioning the have the required properties of continuity and differentiability.
Variational principle for discontinuous functions during the fragmentation of the field on a subareas is given in [1]. Using the method of Lagrange multipliers, can be generalized to the case of weakly compressible and incompressible materials, which include majority of elastomeric materials.

3. MATHEMATICAL MODEL

When solving boundary value problems of static elasticity theory for incompressible and weakly compressible materials easier as the unknown functions to choose displacement u_i and the function of hydrostatic pressure s, which, for small strains, leads to a mathematical model [2,3]:
Equation of equilibrium:

$$G\left[\nabla^2 u_i + \frac{3}{2(1+\mu)} s_i\right] + f_i = 0 \qquad (8)$$

Volumetric deformation:

$$u_{j,j} = \frac{3(1-2\mu)}{2(1+\mu)} s \qquad (9)$$

Deformations:

$$\varepsilon_{ij} = 0{,}5\,(u_{i,j} + u_{j,i}) \qquad (10)$$

Stress

$$\sigma_{ij} = G\,[u_{i,j} + u_{j,I} + 3\mu/(1+\mu)\, s\, \delta_{ij}] \qquad (11)$$

Forces boundary conditions:

$$G[u_{i,j} + u_{j,I} + 3\mu /(1+\mu)\, s\, \delta_{ij}]\, n_j = P_i \quad (12)$$

Displacements boundary conditions:

$$u_i = u_{0_i} \quad (13)$$

When partition the element into a subarea to a mathematical model (8) - (13) to add conditions for docking (3) and (4).

In determining the integral characteristics, of type “force – settlement”, of the elastomeric element boundary problem (8) - (13) (without crushing the elastomeric element in the subareas) easier to solve a variational method using the Ritz procedure for the functional [2, 3]:

$$J(u_i^n, s^n) = G\int_V [\frac{1}{2}\left(u_{i,j}^n u_{j,i}^n + u_{i,j}^n u_{j,i}^n\right) + \\ + \frac{3\mu}{1+\mu} s^n u_{i,i}^n - \frac{9\left(1-2\mu_n\right)}{4\left(1+\mu_n\right)^2} s^{n\,2}]dV \quad (14)$$

Using the variational principle of V.Prager [1] and applying the method of undetermined Lagrange multipliers with a functional (14) be the boundary value problem (8) - (13) with the conditions of the joining (3) and (4) replaced by the variational problem with discontinuous function of demand on the surfaces of the partition Γ_n for the functional:

$$J^*(u_i, s) = \sum_{n=1}^{N} J(u_i^n, s^n) - \sum_{n=1}^{N-1} G_n \int_{\Gamma_n} [(u_{i,j}^n + u_{j+i}^n) + \\ + \frac{3\mu_n}{1+\mu_n} s^n \delta_{i,j} m_j \left\{u_i^n\right\}'' d\Gamma_n \quad (15)$$

where in each subare:

u_i^n – displacement;

s^n - function of hydrostatic pressure;

G_n - modulus of elasticity in shear;

μ_n - Poisson's ratio.

Using the functional (15), the choice of displacement functions u_i^n only need to follow geometric boundary conditions (13), as a function of hydrostatic pressure s_n in each subarea can be selected independently, not caring about its continuity at the boundary of the partition Γ_n.

As an example, consider obtaining dependence “force – settlement” to the hollow cylindrical absorber with core under axial compression (Fig. 1).

Elastomeric layer is still sealed with absolutely rigid top and bottom plates and a non-deformable inner core. For this shock absorber is difficult to write expressions for the displacement common to all elastomeric layer.

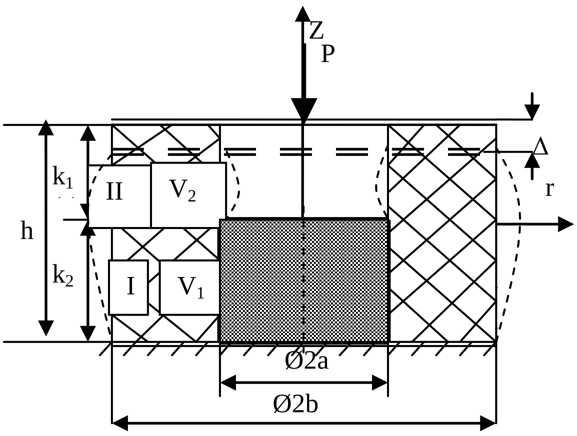


Fig. 1. The hollow cylindrical absorber with the arrester

The considered absorber is broken on two parts on border of a thrust block in parallel a shaft or. At use functional $J^*(u_i, s)$ (15), choosing u_i^n and s_n, it is enough to satisfy to geometrical boundary conditions on external surface F_u . Shock absorber is divided into two subareas (see Figure 1). All functions with an index „1” it is carried to a subarea I, and with an index „2” to a subarea II. We believe that the geometry of the elastomeric layer can not take into account the compressibility of the elastomer, that is, believe that the Poisson coefficient μ = 0,5.

The main boundary conditions will be:

$$\begin{gathered} u_1(r,-k_2) = u_2(r,k_1) = 0 \\ u_1(a,z)_{|0\geq z\geq -k_2} = w_1(a,z)_{|0\geq z\geq -k_2} = 0 \\ w_1(r,-k_2) = 0 \\ w_2(r,k_1) = -\Delta \end{gathered} \quad (16)$$

where: the functions u_i and w_i - displacement, respectively, on the axis of r and z.

Dependence "force – settlement" it is defined from the equation of balance of the top base of the absorber:

$$2\pi\int_a^b \sigma_{2zz|z=k_1} r dr = -P \qquad (17)$$

On a surface of splitting of a condition of ideal contact piece look like:

$$\begin{aligned} u_1(r,0) &= u_2(r,0) \\ w_1(r,0) &= w_2(r,0) \\ \sigma_{1rz}(r,0) &= \sigma_{2rz}(r,0) \\ \sigma_{1zz}(r,0) &= \sigma_{2zz}(r,0) \end{aligned} \qquad (18)$$

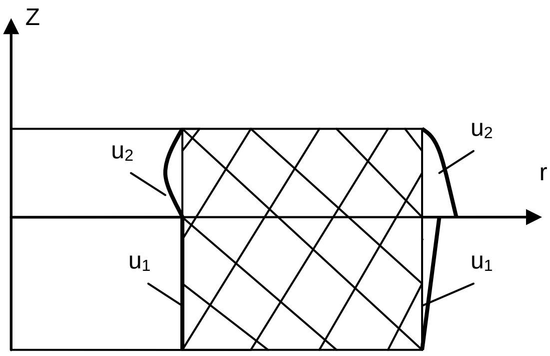
We choose conveyances u_n, w_n and function s_n whenever possible in the most simple kind with the account only conditions (11) and prospective character of deformation.

$$\begin{aligned} u_2 &= B_1 rz(z-k_1) + B_2(r-a)(z-k_1) \\ w_2 &= -\frac{\Delta z}{k_1} + A_1(r-a)(z-k_1) \\ s_2 &= C_2 \\ u_1 &= B_3(r-a)(z+k_2) \\ w_1 &= A_2(r-a)(z+k_2) \\ s_1 &= C_1 \end{aligned} \qquad (19)$$

where
$A_1, A_2, B_1, B_2, C_1, C_2, \Delta$ – unknown constants.

Functions (19) have on section height approximately following appearance for conveyances at r = a; b:

for u



for w

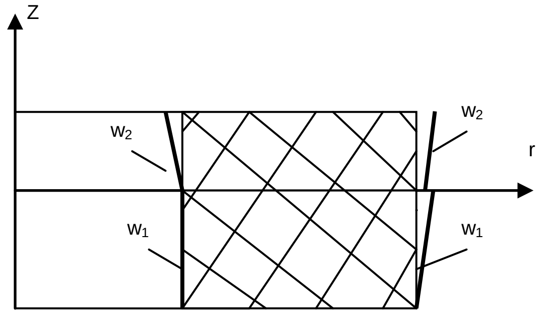


Fig. 2. Expected character of deformation

After integration it is received that functional J* depends only on unknown constants:

$$J^* = J\left(A_1, A_2, B_1, B_2, B_3, C_1, C_2, \Delta\right) \qquad (20)$$

From a condition stationarity

$$\frac{\partial J^*}{\partial\left(A_1, A_2, B_1, B_2, B_3, C_1, C_2, \Delta\right)} = 0 \qquad (21)$$

We receive system of the algebraic equations. For dependence "force – settlement":

$$\Delta = \frac{Pk_1}{2\pi G a^2(1-\alpha^2)}\frac{D_1}{D}, \quad \alpha = \frac{a}{b} \qquad (22)$$

where:
D, D_1 - determinants of algebraic equations (21), an expression which, due to the complexity of writing, are not given.

For absorber: h = 3 cm, a = 2 cm, b = 4,1 cm, k_2 = 1,4 cm, G = 7 kg/cm2, in Figure 3 shows the results: "1" line - the formula (22) (using MatCad); "2" line-experiment.

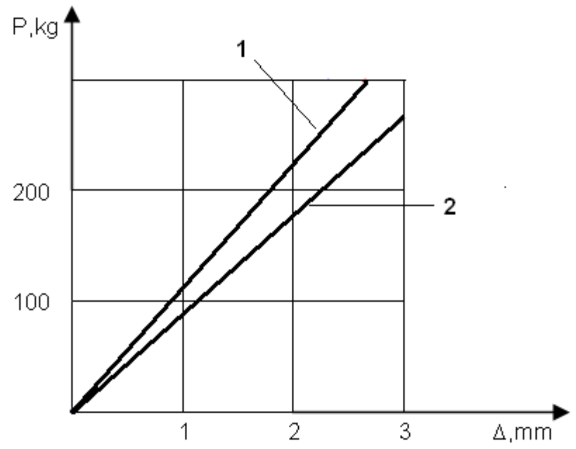


Fig. 3. Dependence force - settlement for the hollow cylindrical absorber with the arrester

If to accept that $k_2 = 0$ then from expressions (21) and (22) we will receive the absorber without an internal thrust block.

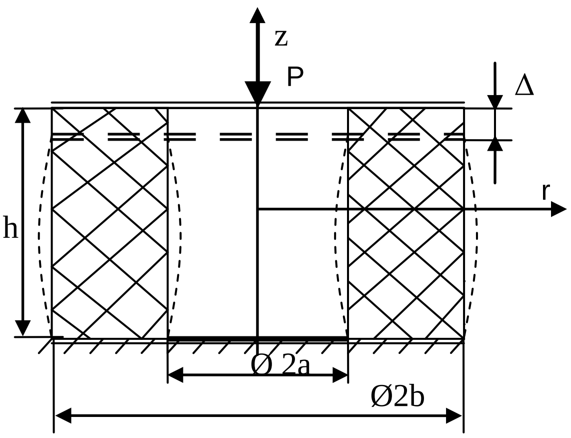


Fig. 4. The hollow cylindrical absorber without the arrester

In this case the dependence of the force - settlement will have the form:

$$\Delta = \frac{Ph}{2\pi G b^2 (1-\alpha^2)} D_3^{-1} \qquad (23)$$

where

$$D_3 = 1{,}5 + 0{,}62\rho^2(1+\alpha^2)\left[1 + \frac{\rho^2(1-\alpha^2)\alpha^2}{0{,}26(1-\alpha^2) - \rho^2\alpha^2 \ln\alpha}\right]$$

$$\alpha = \frac{a}{b}; \ \rho = \frac{b}{h}$$

Figure 5 The predicted results are: “2” line - according to the formula (23); “1” line - solution obtained in work [2].

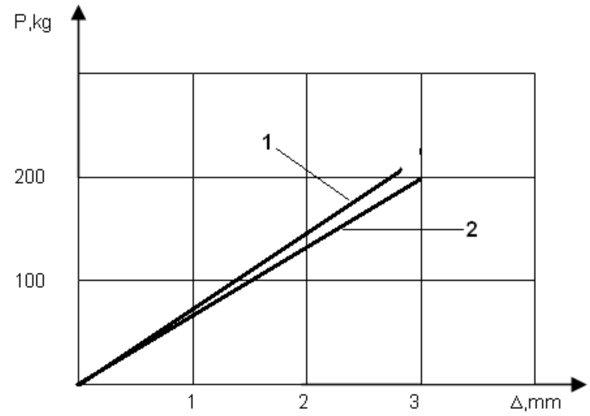


Fig. 5. Dependence force - a deposit for the hollow cylindrical absorber without the arrester

4. CONCLUSION

The proposed method allows in case of partition of the investigated area V on subarea V_n, using functional (15) and discontinuous functions sought to obtain integrated dependences of type “force – settlement”.

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