7th International DAAAM Baltic Conference
"INDUSTRIAL ENGINEERING"
22-24 April 2010, Tallinn, Estonia

NONLINEAR DYNAMICS AND RARE ATTRACTORS IN DRIVEN DAMPED PENDULUM SYSTEMS

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Abstract: *The pendulum systems are widely used in the engineering, but their qualitative behavior hasn't been investigated enough. Therefore the aim of this work is to study new nonlinear effects in three driven damped pendulum systems, which are sufficiently close to the real models used in dynamics of the machines and mechanisms. In this paper the existence of new bifurcation groups, rare attractors and chaotic regimes in the driven damped pendulum systems is shown.*
Key words: pendulum systems, complete bifurcation analysis, method of complete bifurcation groups, rare attractors, chaos, domains of attraction.

1. INTRODUCTION

Recent researches in nonlinear dynamics show, that so-called rare attractors (RA) exist in all typical and well studied models [1-3], but are unnoticed by the traditional methods of analysis. The systematic research of rare attractors is based on the method of complete bifurcation groups (MCBG) [2]. This method allows conducting more complete global analysis of the dynamical systems. The main features of this method are illustrated in this work by three driven damped pendulum systems.

Our aim is to build complete bifurcation diagrams and to find unknown rare regular and chaotic attractors using complete bifurcation analysis for some important parameters of the model: amplitudes and frequency of excitation. For complete bifurcation global analysis we have used the MCBG, Poincaré mappings, mappings from a line and from a contour, basins of attraction, etc. [1-3].

The main results of this work are presented by complete bifurcation diagrams for variable parameters of the driven damped pendulum systems. We consider three pendulum models: a) model with an additional linear restoring moment and with the periodically vibrating in both directions point of suspension, b) model with a linear restoring moment and with the external periodic moment of excitation, c) model with a sliding mass and with the external periodic moment of excitation. By building the complete bifurcation diagrams with stable and unstable periodic solutions, we have found different new bifurcation groups with their own rare regular and chaotic attractors. All results were obtained numerically, using original software.

2. MODELS AND RESULTS

The first pendulum model is shown in Fig.1a. The system has additional linear restoring moment with the harmonically vibrating in both directions point of suspension. The system has three equilibrium positions (Fig.1d). Backbone curves and restoring moment for the system are shown in Fig.1b,1c. Similar models have been examined in some works [4-10]. The aim of our bifurcation analysis is to find new rare attractors and new bifurcation groups. Some results of complete bifurcation analysis for this model were presented in one of the previous works [11].

The first mathematical pendulum model is described by following equation of motion:

$$mL^2\ddot{\varphi} + b\dot{\varphi} + c\varphi + mL[\mu + \ddot{y}(\omega_2 t)]\sin\varphi - mL\ddot{x}(\omega_1 t)\cos\varphi = 0 \quad (1)$$

where m – pendulum mass; L – pendulum length; μ – gravitational constant; φ – angle, read-out from a vertical; $\dot{\varphi}$ – angular velocity of the pendulum, $\dot{\varphi} = d\varphi / dt$; b – linear damping coefficient; c – linear stiffness coefficient; $\ddot{x}(\omega_1 t) = -A_2\omega^2 \cos \omega t$, $\ddot{y}(\omega_2 t) = -A_1\omega^2 \sin \omega t$ - suspension point acceleration in horizontal and vertical direction due to external excitation.

The results of bifurcation analysis of the model (1) are represented in Figs.2-5. Five different 1T bifurcation groups and one 2T bifurcation group have been found (Fig.2).

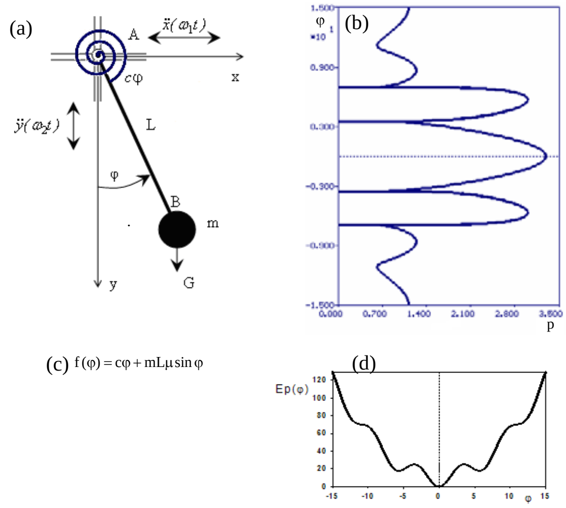

Fig. 1. The driven damped pendulum system with an additional linear restoring moment and with the periodically vibrating point of suspension in both directions. (a) physical model; (b) backbone curve; (c) restoring moment; (d) potential well.

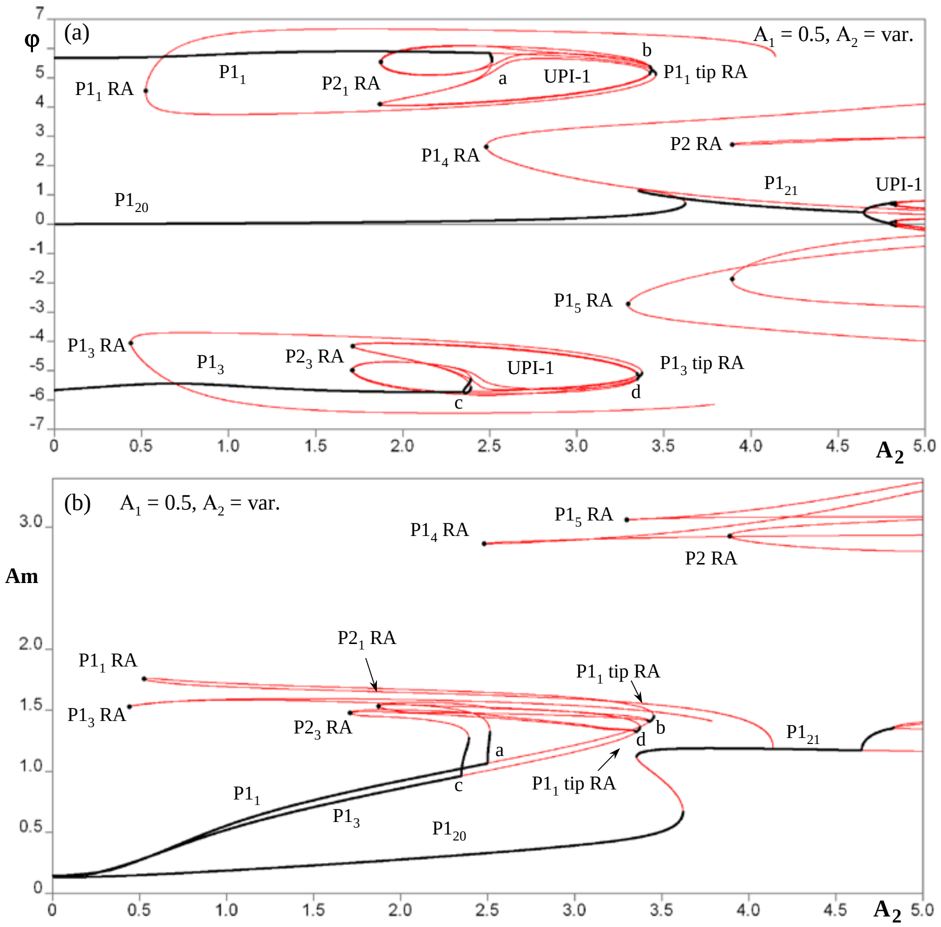

Fig. 2. The driven damped pendulum system (Eq. 1) with linear restoring moment and with the periodically vibrating point of suspension in both directions. (a), (b) Bifurcation diagrams: state (φ, Amplitude) of the fixed periodic points versus vertical external force amplitude A_2. There are five 1T and one 2T bifurcation groups. The system has many rare attractors of different kinds. Parameters: m = 1, L = 1, b = 0.2, c = 1, μ = 9.81, ω = 1.5, A_1 = 0.5, A_2 = var.

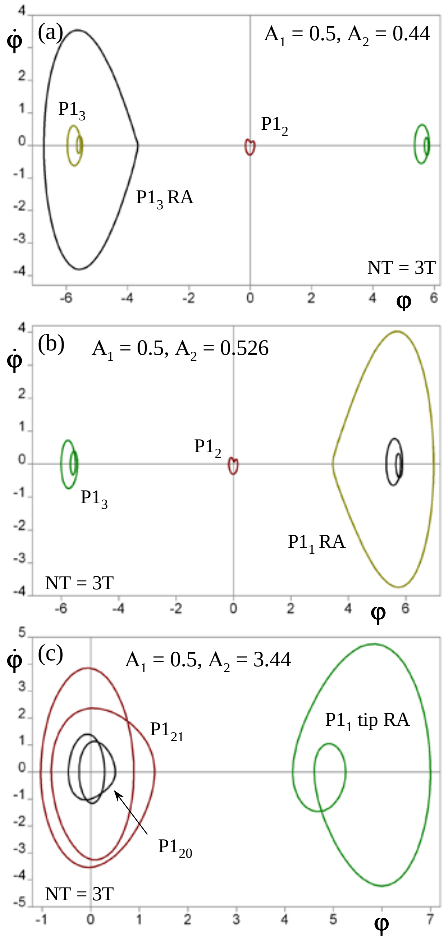

Fig. 3. Coexistence of P1 usual and rare attractors P1 RA for three cross-sections (see Fig.2). (a) Phase projections for A_2 = 0.44. The rare attractor $P1_3$ RA has the fixed point FP (-4.05606/1.17632). (b) The same for A_2 = 0.526. The rare attractor $P1_1$ RA has the FP (4.56968/-2.60245). (c) The same for A_2 = 3.44. The rare attractor $P1_1$ tip RA has the FP (5.23616/-0.315143). Parameters: m = 1, L = 1, b = 0.2, c = 1, μ = 9.81, ω = 1.5, A_1 = 0.5, A_2 = var.

Two of these groups are topologically similar and have rare attractors of a tip kind $P1_1$ RA and $P1_3$ RA. Two period one brunches near A_2 = 4 are not completed, because of problems of singularity. Other three 1T bifurcation groups have their own rare attractors $P1_4$ RA and $P1_5$ RA, which are stable in small parameter regions.

Form some cross-sections of bifurcation diagrams (see Fig.2) the dynamical characteristics are represented in Figs.3,4. All attractors are of tip kind so each of them has not only periodic attractors, but also chaotic attractors as well.

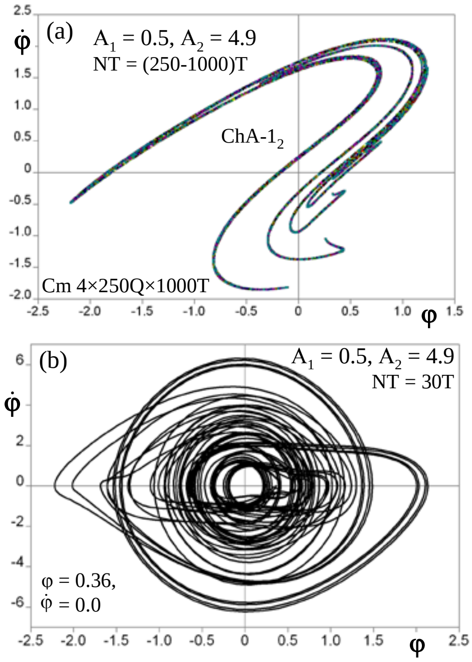

Fig. 4. Chaotic attractor in the driven damped pendulum system for cross-section A_2 = 4.9 of bifurcation diagram Fig.2: (a) Poincaré map - Cm 4×250Q×(250-1000)T; (b) phase projection with NT = 30T. Parameters: m = 1, L = 1, b = 0.2, c = 1, μ = 9.81, ω = 1.5, A_1 = 0.5, A_2 = 4.9

The second pendulum model is shown in Fig.5a. The equation for this model is:

$$\ddot{\varphi} + b\dot{\varphi} + (a\varphi + a_1 \sin 2\pi\varphi) = h_1 \cos \omega t \quad (2)$$

where φ – angle, read-out from a vertical; $\dot{\varphi}$ – angular velocity of the pendulum, b – linear damping coefficient; a – linear stiffness coefficient; a_1 – coefficient which include pendulum length and gravitational constant; $h_1 cos(wt)$ – harmonical moment enclosed in a point of support.

For the given system with one equilibrium position the complete bifurcation analysis was made. Results of the analysis are shown in Fig. 6. Feature of this system is the unexpected isolated P1 isle, amplitudes of vibrations of which are much greater than ones of the usual P1 regime. Also for these three attractors P1 the domains of attraction were constructed (see Fig. 7) for cross-section ω = 0.347 of bifurcation diagram in Fig.6.

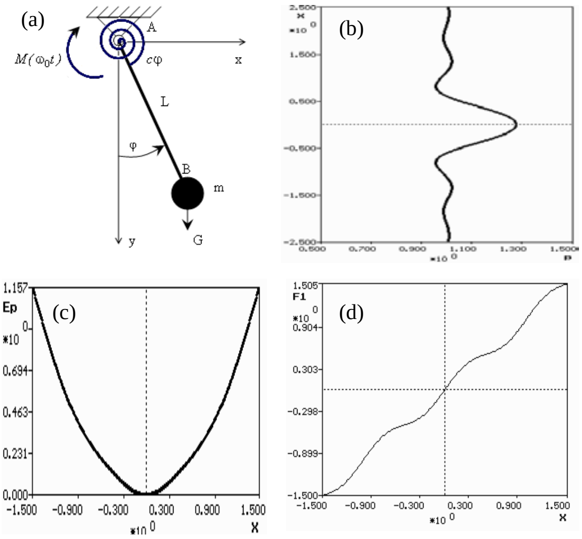

Fig. 5. The driven damped pendulum system with a linear restoring moment and with the external periodic excited moment. (a) physical model; (b) backbone curve; (c) potential well; (d) restoring moment.

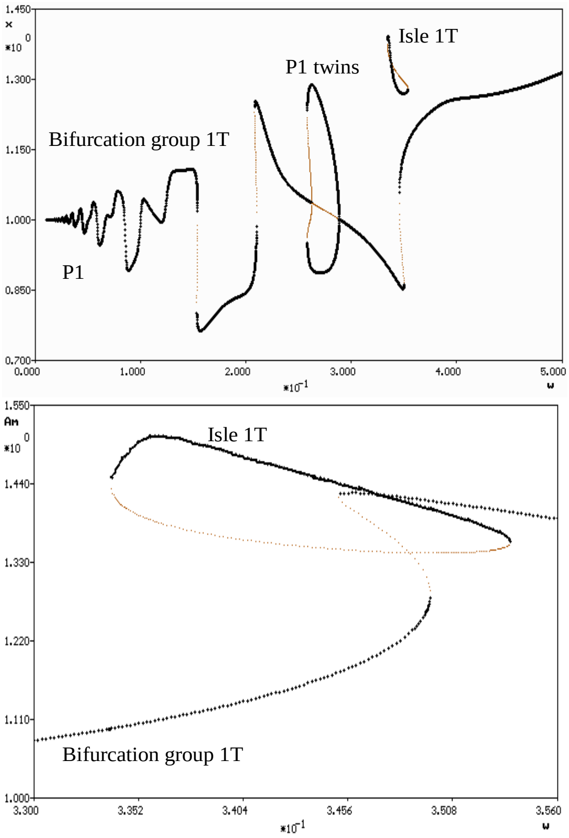

Fig. 6. The driven damped pendulum system (Eq. 2) with a linear restoring moment and with the external periodic excited moment. The system has two 1T bifurcation groups: usual P1 and isle P1. Parameters: b = 0.1, a = 1, a_1 = 0.1, h_1 = 1, ω = var., k = 7.

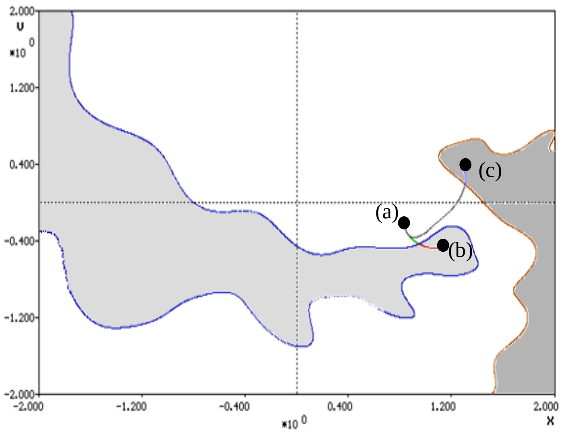

Fig. 7. Domains of attraction for three attractors P1. Parameters: b = 0.1, a = 1, a_1 = 0.1, h_1 = 1, ω = 0.347, k = 7.

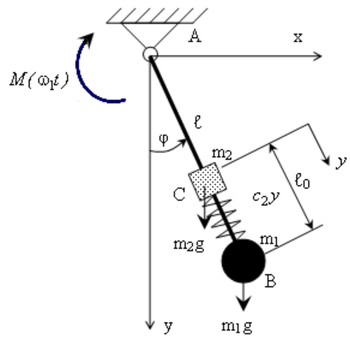

Fig. 8. The driven damped pendulum model with a sliding mass and with the external periodic excited moment.

Equations of motion for the third mathematical pendulum model with a sliding mass and with the external periodic excited moment (Fig. 8) are such:

$$\begin{cases} \left[m_1 l^2 + m_2 (l - l_0 + y)^2\right]\ddot{\varphi} + b_1 \dot{\varphi} + \\ + 2 m_2 (l - l_0 + y)\dot{\varphi}\dot{y} + m_1 \mu l \sin\varphi + \\ + m_2 \mu (l - l_0 + y)\sin\varphi = M(\omega_1 t) \\ m_2 \ddot{y} + b_2 \dot{y} + c_2 y - m_2 (l - l_0 + y)\dot{\varphi}^2 + \\ + m_2 \mu (1 - \cos\varphi) = 0 \end{cases} \quad (3)$$

where φ – angle of rotation, read-out from a vertical line; $\dot{\varphi}$ – angular velocity; y – displacement of the sliding mass, read-out from a quiescent state; $\dot{y}$ – velocity of the sliding mass; m_1 – pendulum mass, l – length of the pendulum; m_2 – second mass, l_0 – quiescent state of the sliding mass; μ – gravitational constant; b_1, b_2 – linear damping coefficients of the pendulum and the sliding mass; c_2 – linear stiffness coefficient of the sliding mass;

$M(\omega_1 t) = h_1 \cos \omega_1 t$ – external periodically excited moment; h_1 and ω_1 – amplitude and frequency of excitation.

Similar models have been studied in works [4,5,10]. Results for this system are represented in Figs.9,10. One 1T and one 5T bifurcation group with Andronov-Hopf bifurcations, several symmetry breakings, period doublings and rare attractor P5 RA (see Fig.10a) have been found in the third model. In the Fig.10b there are 1T and 2T bifurcation groups with several symmetry breakings, period doublings, folds and tip type rare attractors. Global chaos ChA-1 have been found in this pendulum system using Poincaré mapping Cm 4×50Q×250T from a contour. These Figs. show that method of complete bifurcation groups allows finding new unnoticed before regimes also in the system with two-degree-of-freedom. Thus, the application of this method for global analysis of forced oscillations is also possible for systems with several degree-of-freedoms.

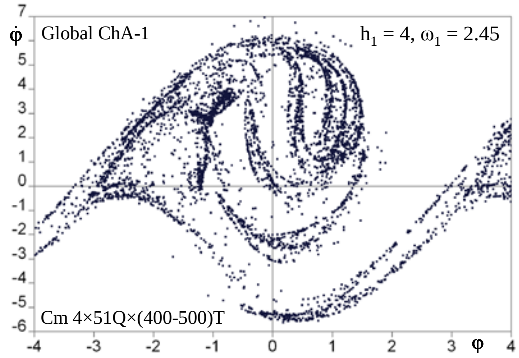

Fig. 9. Global chaos ChA-1 on Poincaré map Cm 4×50Q×250T in the pendulum system (Eq. 3) with a sliding mass for cross-section of bifurcation diagram (see Fig.10). Parameters: $m_1 = 1$, $m_2 = 0.1$, $l = 1$, $l_0 = 0.25$, $b_1 = 0.2$, $b_2 = 0.1$, $c_2 = 2$, $\mu = 10$, $h_1 = 4$, $\omega_1 = 2.45$.

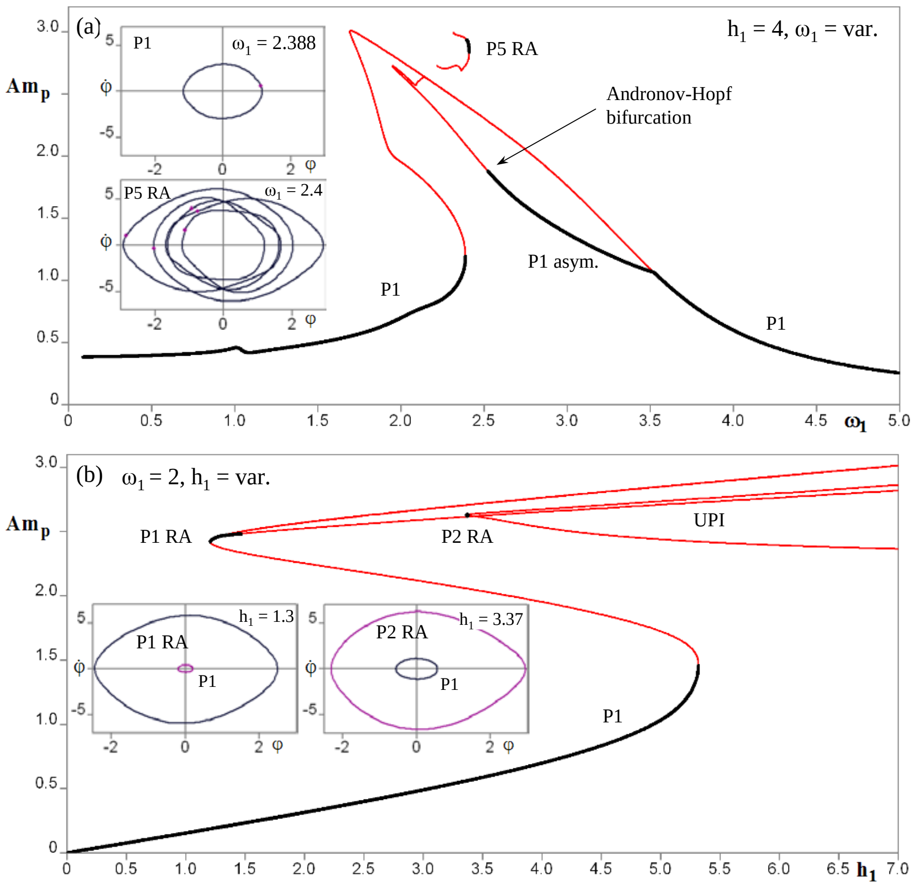

Fig. 10. The driven damped pendulum system (Fig.8 and Eq. 3) with a sliding mass and with the external periodic excited moment. Bifurcation diagrams – amplitude of the pendulum Am_p vs. frequency ω_1 and amplitude h_1 of excitation: (a) $h_1 = 4$, $\omega_1 =$ var.; (b) $\omega_1 = 2$, $h_1 =$ var. The pendulum system has several bifurcation groups with their own tip type rare attractors. Parameters: $m_1 = 1$, $m_2 = 0.1$, $l = 1$, $l_0 = 0.25$, $b_1 = 0.2$, $b_2 = 0.1$, $c_2 = 2$, $\mu = 10$.

3. CONCLUSION

The pendulum systems are widely used in the engineering, but their qualitative behavior hasn't been investigated enough. Therefore in this work the new nonlinear effects in three driven damped pendulum systems, which are sufficiently close to the real models used in dynamics of the machines and mechanisms, were shown. These results were obtained using the method of complete bifurcation groups. Only the method of complete bifurcation groups allows to find rare periodic and chaotic regimes systematically. These regimes can lead to small breakages of machine and mechanisms and to global technical catastrophes, because they are unexpected and usually have large amplitudes.

Acknowledgments
This work has been supported by the European Social Fund within the project „Support for the implementation of doctoral studies at Riga Technical University" 2009-2010 and by the Latvian Council of Science under the grants R7329, R7942, 06.1965, and 09.1073 (2005-2010).

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