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4th International Conference
APLIMAT 2005

REGRESSION MODEL FOR THE PENSION FUND'S ADDITIONAL PENSION CAPITAL

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Abstract. The aim of the article is to adapt algorithms for private pension fund additional capital valuation to real time model and to estimate acquired model. Earlier considered formula
kb = ks + c – e + (ks + c - e)/(K + A)x R
we have transferred to formula for real time condition

$$R_t = (K_{t-1} + c_t - e_t)\left(\frac{K_t}{\frac{n_t}{n_{t-1}} K_{t-1} + c_t - E_{adm}} - 1\right)$$

We are going to search and estimate regression equation of this model and then to use the autoregressive model for R for the forecasting of PPF resources. We are using time series from Latvia's market data (years 2001-2003)

1 Introduction

In last few years there is very quick development of Private Pension Fund (PPF) business in Latvia. Only in year 2001 PPF contributions have rose 2.9 times and received 4,807 millions Lats (about USD 8 millions). In PPF growing also heighten necessity of qualitative computer programs for accounting purposes. More ever computer programs must include not only these technologies that are used in Latvia by today. Programs must include technologies that are unfamiliar for Latvia now but will be meaning at next future. Thereby this approach can stimulate rapid growth of pension business as it was in Great Britain in 1950ties. In those days parallel with developing of computer sciences Pension funds offered to theirs clients new derivative pension fund products, which stimulate quick growing of PPF assets.

To establish software according these requirements is essential to look through algorithms applied in other countries and especially in UK. Then these algorithms can be adapted to Latvia market according Latvia market requirements and local legislation. Valuation of PPF additional capital is one of essential part of PPF valuation and we will concern to it. The aim of this proceeding is to adapt these algorithms to real time model and to estimate acquired model.

2 The statement of the problem

To calculate the saved additional pension capital in Pension Fund, payments of the participants pension plan recalculate as determined numbers of a share (unit), which is equated to the relation between size of payments and meaning of one share appropriate pension plan by the beginning of a quarter. Hence, it is possible easily to calculate value of a money sum, which collects on individual account of each of the participant for certain time.

The additional pension capital accumulated in the individual account of a pension plan participant shall be calculated at the end of each quarter, based on the profit-and-loss account of the pension plan during the particular quarter:

$$k_b = k_s + c - e + \frac{k_s + c - e}{K + A} \times R, \quad (1)$$

where

k_b - the additional pension capital accumulated in the individual account of the pension plan participant at the end of the current quarter;

k_s - the additional pension capital accumulated in the individual account of the pension plan participant at the end of the preceding quarter;

c - instalments made by the pension plan participant and instalments made for his/her benefit in accordance with the pension plan, as well as wire transfers from another pension fund or pension plan in the respective quarter;

e - administrative costs applicable to the pension plan participant in the respective quarter;

K - item "Pension plan capital in the accounting period" of the profit-and-loss account of the pension plan in the respective quarter;

A - item "Instalments and disbursements of the pension plan" of the profit-and-loss account of the pension plan in the respective quarter;

R - item "Pension plan operation result" of the profit and loss account of the pension plan in the respective quarter.

In focus of technology is dividing of investment to equal parts, which we are called for units. After this the goal is to seek count of units corresponding to each unit holder. As new member enters to PPF to him was sold part of units by bid price and after this his unit portfolio will rise respectively by rise of all PPF investment portfolio. When this member dissolve from plan, then to him will be sold units by current offer price. After this we are going to search and estimate regression equation of this model.

3 The definition of the individual price

The main approach in consideration of this problem is the distribution of the investments in equal shares (or uniform price) on all participants of pension fund. The purpose of such approach - to define that part of means of fund, which belongs to each participant, and also to supervise process of change of this share at reception of the new participants or leaving of the participants.

A major task of definition of the additional pension capital is of observance of interests of the participants (fair distribution of results of the investment income). Inadmissible the situation is, when the participants who have entered fund in the beginning of its activity, are compelled to pay the initial charges of fund. The ideal decision of this problem would be distribution of these charges of fund to all participants fifty-fifty.

4 The mathematical model

According with initial task we should applied the formula (1) for recalculation "on-line" mode, but not only in the respective quarter.

$k_b \sim k_t$; $k_s \sim k_{t-1}$,

where k_{t-1} means the pension capital calculated for the prior time and $k_0=0$.

Due to $K = \sum_i k_s$, where i means PF participant with number i, then $K = \sum_i k_{t\,i}$.

We know, what administrative payments are connected to each participant of the pension plan during one period (for example, within one year). It is so-called periodic payments. Further, from the formula (1) we know also what with each participant the so-called expendable payments (for example, payments connected to expendable transfer of money to the account of pension fund) are connected. Therefore at any moment time t the current administrative and other payments of one client can be define, carrying out linear interpolation to periodic payments, and also summarizing expendable payments, therefore

$e \sim e_t$.

Is similar a total sum of payments in fund and the general payments from fund can be defined if to summarize payments and payments for each participant of pension fund separately, i.e. $A \sim A_t$.

Thus, we still had the unique factor R, which is necessary for defining in a mode on-line. It is difficult enough to define this variable precisely at any moment of time; therefore here it is necessary to apply the approached methods. In effect of calculation R_t is connected to other very important question of work of pension fund. Namely, a bigger problem relates to the fact that the calculations consider only revenue flows in the funds (contributions, investment income, expenses). There is no reflection of capital gains (realised or unrealised).

Such investments of the capital are complex for estimating in a mode on-line.

In a basis of calculation of additional pension capital accumulated in the private pension fund on the individual accounts a principle of recalculation of money sums through units is incorporated. Before now this process is applied to account of the rests on the end of a quarter and on the end of a year. The main idea of a procedure of recalculation consists in following:

1. By the beginning of the first time period cost one unit is equal 1 Lats.
2. Unit prices are calculated quarterly.
3. Units are allocated at the next price calculated after receipt of contribution or due date of payment of claim.
4. Initial unit price is 1 Lats, which is used to allocate units for the first three months (of a year) contributions paid. Subsequent contributions are allocated at the unit price next following receipt of premiums.

Now the aim is to adapt these algorithms to real time model and to estimate acquired model. So, having entered the earlier stipulated designations for the formula for calculation of the additional pension capital accepts the following kind.

$$K_t = K_{t-1} + c_t - e_t + \left(\frac{K_{t-1} + c_t + e_t}{\sum K_{t-1} + \sum (c_t - e_t)} * R_t \right) \qquad (2)$$

For the profit and loss factor R_t it can be rewritten as:

$$R_t = (K_{t-1} + c_t - e_t)\left(\frac{K_t}{\frac{n_t}{n_{t-1}} K_{t-1} + c_t - E_{adm}} - 1\right) \qquad (3)$$

Here C_t - the contributions of the participant;
E_t - the payments by the participant;
E_{adm_t} - the administrative charges.

To our attention the following data on the participants of pension fund in the period with 01.01.2001 up to 31.03.2004. (see table 1) are offered.

Data for the period from 31.12.2000 to 31.03.2004.

t	n-t	K_t	c_t	e_t	E_adm_t	
Date	Count of individuals	Pension plan kapital	Contribution of the partic.	Payments by the particip.	Administr. charges	
31.12.2000	6992	5874134	1667553	39980	45013	R
31.01.2001	7040	5978075	103941	720	346	-39709.48
28.02	7238	6175568	197493	1416	1205	-162497.7
31.03	7247	6409806	212393	27736	13016	27086.676
30.04	7287	6534326	124520	12546	849	-34282.62
31.05	7337	6671253	136927	20348	1541	-42884.14
30.06	7430	6877912	137435	25730	12825	-2476.748
31.07	7625	7025788	147876	9345	4812	-171183.9
31.08	8861	7275281	249493	11724	2198	-981493.1
30.09	9121	7423404	156746	6819	11336	-204990.1
31.10	9204	9009958	1586554	19348	5376	-61617.35
31.11	9458	9248863	238905	33982	7895	-233780.2
31.12	17359	9496371	1483326	6282459	11444	-2159048
31.01.2002	17409	9790617	294246	22286	11349	-15941.39
28.02	17683	10125980	335363	2125	4360	-147521.5
31.03	17768	10666049	232958	8597	6466	263608.72
30.04	17844	10783216	305105	17244	8224	-224218
31.05	18105	10993145	318078	19085	7806	-254190.2
30.06	18595	11723084	307285	14726	8427	130057.45
31.07	18880	11961856	290334	20228	9305	-218461.3
31.08	18940	12045323	310105	31044	8796	-254603
30.09	19233	12736861	463932	13507	9482	49986.485
31.10	20041	12916543	335210	18003	9101	-654252.7
31.11	20050	13246033	370040	9361	8836	-37496.69
31.12	20064	13972331	303502	3792	9890	423338.42
31.01.2003	21101	14126135	356029	20018	9556	-870264.2
28.02	21836	15001963	302885	15341	9004	86894.23
31.03	22198	15163201	409978	14654	9575	-479958.6
30.04	23011	15948316	398875	25388	10035	-153439.8
31.05	23880	16244450	401022	36844	9963	-671313.8
30.06	24201	16639282	405117	42757	11227	-214196.2

31.07	24800	17162433	420010	43200	11244	-289893.9
31.08	25014	17544201	445366	40115	10618	-199062.9
30.09	25244	18012652	394680	38485	11837	-74921.59
31.10	25936	18399062	400350	31878	11887	-482342.3
31.11	26218	18876502	397240	36224	12122	-106457.5
31.12	26610	19315385	406473	46624	11935	-234032.4
31.01.2004	27300	20067037	445560	25118	13244	-176917.7
28.02	28775	20968846	480240	31876	13118	-616382.6
31.03	30084	21419863	389589	49028	12422	-840952.3

Table 1.

And the respective graph of profit/loss statement is shown in Chart1.

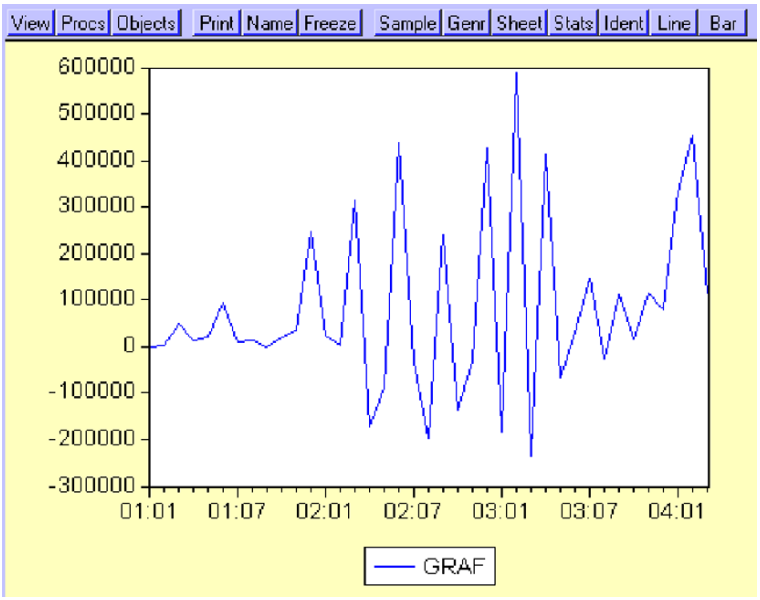

Fig. 1. The graph of the pension fund's operation result.

For further analysis we have used E-VIEWS software.

To find out the best ARMA model we have established corellogram to our time series. From Chart2 we can easily se that the best models will be AR(1), MA(1) or ARMA (1) models.

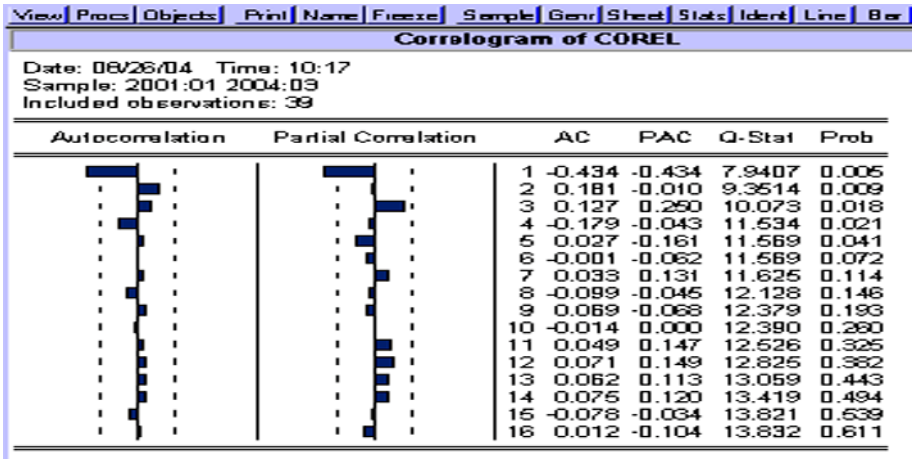

Fig. 2. The corellogram of the time series R

After comparing these models, we found that the best model is AR(1) due to less probability off negation of hypothesis "AR(1) is insignificant". The probability was only 0.6%.
After that we have analysed season effects on our model. We have find out that the model has seasonally changes in winter times. This finding led us to introduce new formula (4):

$$R_t = C + \alpha_1 r_{t-1} + \alpha_2 S_t + \varepsilon_t$$
$$S_t = \begin{cases} 1, & winter\ months \\ 0, & non\,winter\ months \end{cases} \quad (4)$$

We have estimated model's stationary errors with Unit root test and found that the time series for R did not have homogeneous outcome.
Also we have tested model's stability with Chow test – Chart3.

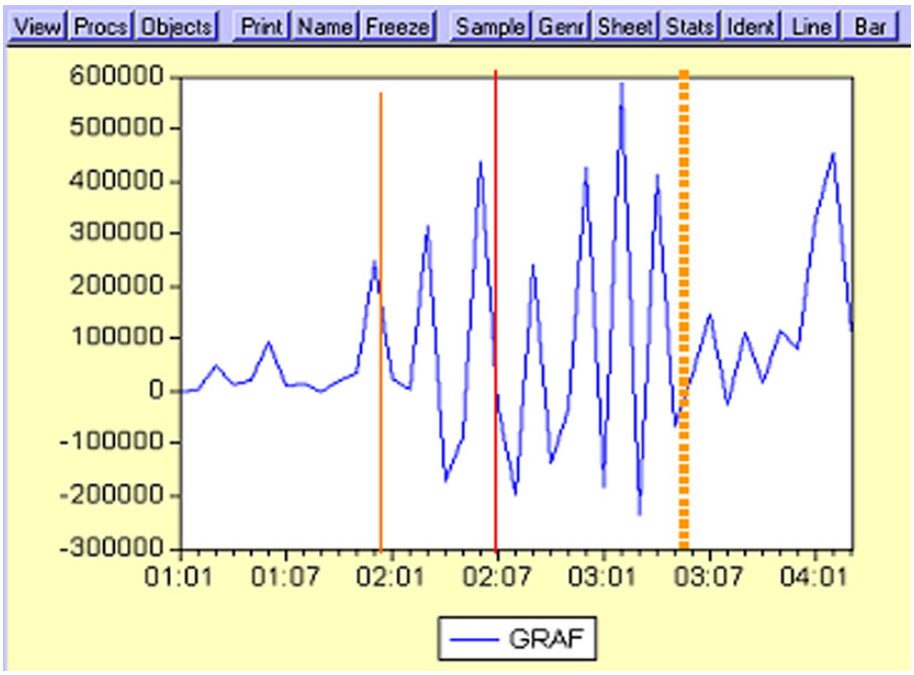

Fig. 3. The graph of the Chow test's procedure.

Firstly we have divided model in two equal parts and did not find any changes. After that we have analysed changes in two most critical moments January 2001 and July 2003. We have found that model have changes in July 2003 (dashed line in Chart 3) that it is due beginning of trend at that moment.

Financially we can explain these trends with funds developments. In first time (year 2001) in market there were only few participants, which led to high fluctuations in funds profit. Also there are large influences in middle period (year 2002-2003) because new funds come into market and always in beginning of operations fund results were unstable. But in the last period from July 2003 we can find stabilisation of market and also profit trend with tendency to grow up.

In the chart 4 we can see graph for functions with trend and without trend. We easily can see, that the main difference between functions is in last period.

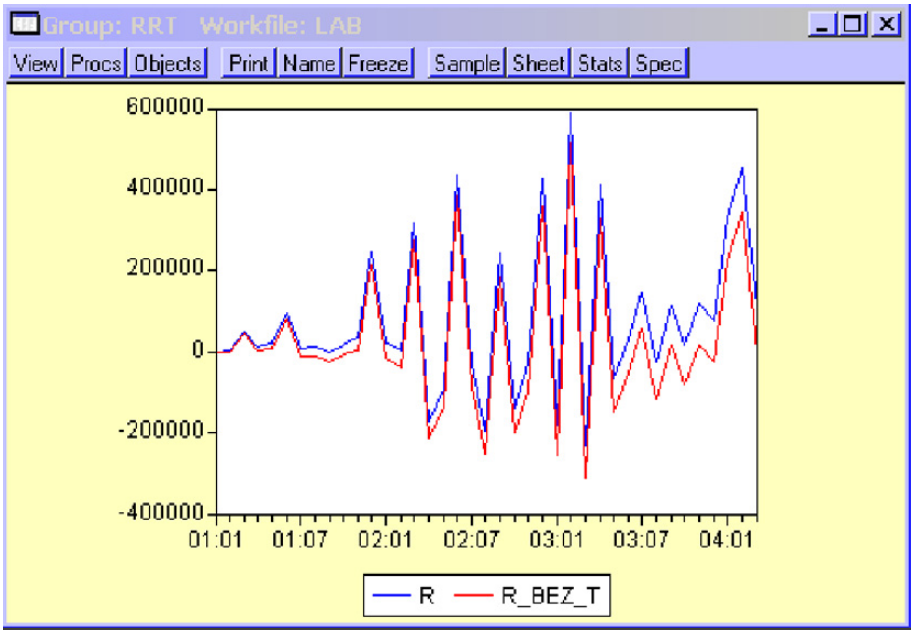

Fig. 4. The graph of the pension fund's operation result with and without trend.

We have found trend in last period also with method of local regression – Chart5

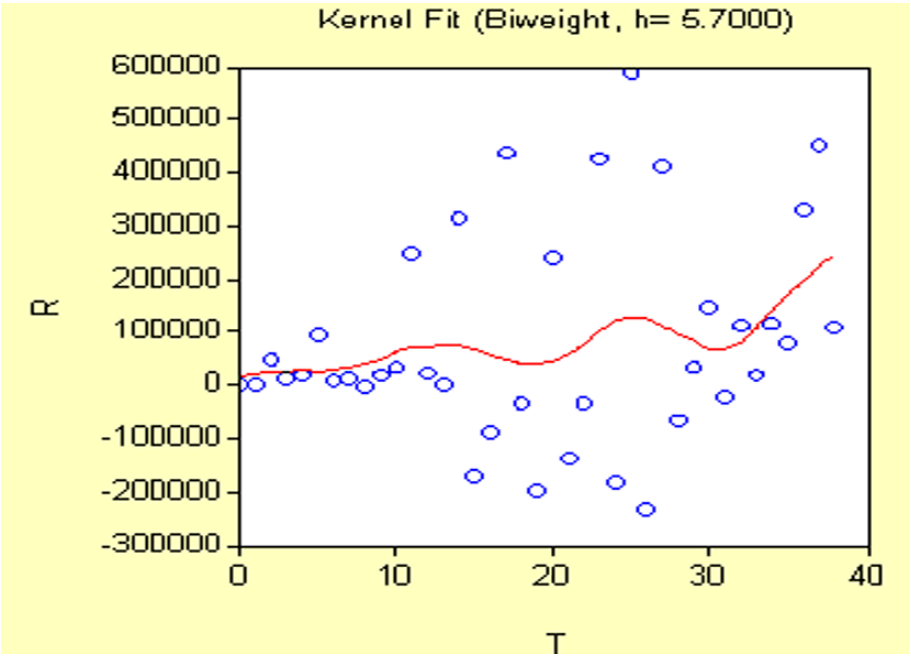

Fig. 4. The graph of the local regression analysis for the R.

After that we have forecasted future R values in Chart6. We can see that forecast in slow but profit growing.

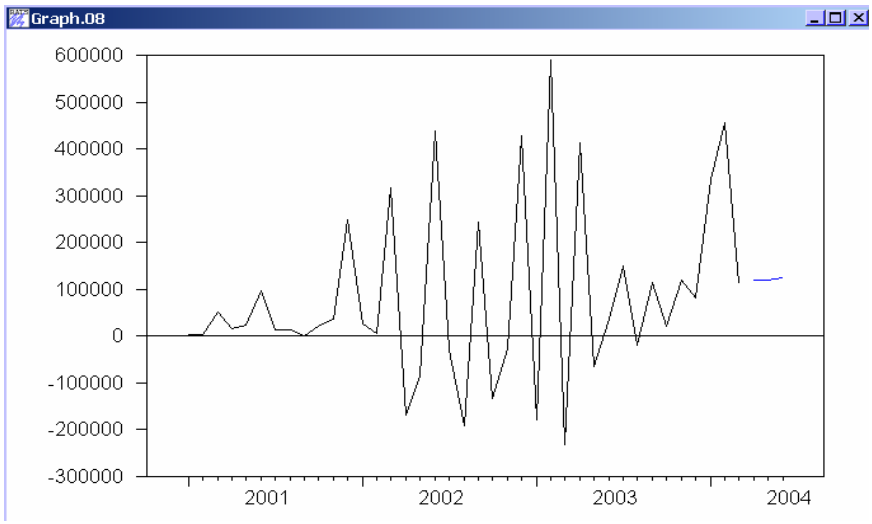

Fig. 5. The graph with the time series R forecasting

Conclusion

We have found and estimated autoregressive model for pension fund additional capital in real time condition which have following characteristics:

- time series in last periods has trend
- time series has seasonality
- time series is stationery
- time series did not has heteroskedastic
- Model is adaptive

The algorithm that we have used is good tool for pension fund stability analysis. Supervisory authorities can use it as procedure for particular pension fund stability analysis or overall pension fund market tendencies analysis.

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